Records Arithmetick:

OR,

THE GROUND OF ARTS;

TEACHING

The perfect work and practice of Arithmetick, both in whole Numbers and Fractions, after a more easie and exact form then in former time hath been set forth: Made by M. *Robert Record*, D. in Physick.

Afterward, augmented by M. John Dee.

And since enlarged with a third part of Rules of Practise, abridged into a briefer method then hitherto hath been published, with divers necessary Rules incident to the Trade of Merchandise: with Tables of the valuation of all Coyns, as they are currant at this present time,

By JOHN MELLIS.

And now diligently perused, corrected, illustrated and enlarged; with an *Appendix* of figurative Numbers, and the extraction of their Roots, according to the method of *Christian Urstitius*: with Tables of Board and Timber measure; and new tables of Interest, after 10. 8. and 6. per 100; with the true value of Annuities to be bought or sold present, Respited or in Reversion: the first calculated by *R. C.* but corrected, and the latter diligently calculated by *Ro. Hartwell Philomathemat.*

Scientia non habet inimicum nisi ignorantem.

Fide —————— sed —————— Vide.

Sum ex libris Na: Clutterbuck

LONDON:

Printed by *James Flesher*, and are to be sold by *Joseph Cranford*, at the signe of the Kings head in S^t. *Pauls* Church-yard. 1658.

THat which my friend hath well begun
For very love to Common-weal,
Need not all whole to be new done,
But now encreaſe I doe reveal.

Something herein I once redreſt,
And now again for thy behoof,
Of zeal I doe, and at requeſt,
Both mend and adde, fit for all proof.

Of numbers uſe, the endleſſe might,
No wit nor language can expreſſe:
Apply and try both day and night,
And then this truth thou wilt confeſſe.

J. Dee.

The Books Verdict.

TO pleaſe or diſpleaſe ſure I am,
But not of one ſort to every man;
To pleaſe the beſt ſort would I fain,
The froward diſpleaſe ſhall I certain;
Yet wiſh I will though not with hope,
All eares or mouths to pleaſe or ſtop.

To the most mightie Prince, *Edward* the 6th by the grace of God, King of *England*, *France*, and *Ireland*, &c.

THe Excellency of mans nature being such, as it is by Gods divine favour (most mighty Prince) not only created in highnesse of degree far above all other corporall things, but by perfection, reason, and search of wit, much approaching toward the image of God, as not onely the holy Scriptures doe testifie, but also those naturall Philosophers, which exactly did consider the nature of man, and namely the far reach and infinite compasse of the words of the mind, were inforced to confesse, that man scarcely was able to know himself. And if he would duely ponder the nature of himself, he would finde it so strange, that it might seem to him a very miracle: And thereof sprang that saying; *Magnum miraculum est homo, maximum miraculum sapiens homo.* For undoubtedly, as man is one of the greatest miracles that ever God wrought, so a wise man is plainly the greatest.

And therefore was it that some did account the head of a man the greatest miracle in the world, because not onely of the strange workmanship that is in it, but much more of the efficacy of reason, wit, memory, imagination, & such other powers, and works of the minde, which can more easily conceive any thing in a manner then understand it self. Amongst all the creatures of God, it findeth none more difficult to be perceived then the same powers of it self; whereby it doth conceive and judge: as it may be well conjectured by the diversity of opinions, that the wisest Philosophers did utter touching the spirit of man, and the substance of it: whereof I now intend to make no rehersal; but who so listeth to read

thereof, may finde it largly set forth, not onely in *Aristotle* his bookes *de Anima*, but also in *Galen* his booke called *Historia Philosophica*: and again in *Plutarch's* work, *De Philosophorum placitis*, whose words are also repeated of *Eusebius* in the xv. book, Τῆς εὐαγγελικῆς προπαρασκευῆς, unto whom I remit them that have desired to understand intricate difficulty of knowing our own selves, as touching our best part, and that part whereby we deserve to bear the name of men.

This matter seemed so obscure and difficult in knowledge; that *Galen* who for his excellent wisdome and judgement in naturall works, is called of many men, a Miracle in Nature, yet in searching the nature and substance of the spirit of man, he not only confesseth himself ignorant, but counteth it plain temerity to attempt to finde it. So farre above the hope of mans knowledge is that part whereby man doth know and judge of things. And although the ignorant sort (which hate all things that they know not) doe litle esteem the profoundnesse of a mans spirit and reason, the chief power and faculty of it: yet as there is a kind of fear and obedience of unreasonable beasts unto man, by the working power of God, so is there in those small reasoned persons a certain kind of reverence toward wisdome and reason, which they do shew oftentimes, and by power of perswasion, are inforced to obey reason, will they nill they. And hereby it came to passe, that the rudenesse of the first age of man was brought unto some more civill trade, as it is well declared by *Cicero*, in the beginning of his first book, *De Inventione Rhetorica*, where he saith thus, *Nam fuit quoddam tempus quum in agris homines passim bestiarum more vagabantur, & sibi victu ferino vitam propagabant, nec ratione animi quicquam, sed pleraq; viribus corporis administrabant. Nondum divinæ religionis, non humani ratio colebatur. Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos; non jus æquabile quid utilitatis haberet, acceperat: ita propter errorem atq; inscitiam cæca ac temeraria dominatrix animi cupiditas, ad se explendum viribus corporis abutebatur perniciosissimis satellitibus. Quo tēpore quidam, magnus videlicet vir & sapiens, cognovit quæ materia esset, & quanta ad maximas res opportunitas in animis inesset hominum, si quis eam possit elicere, & præcipiendo meliorem reddere. Qui dispersos homines in agris & in tectis sylvestribus abditos, ratione quadam compulit in unum locum, & congregavit: & eos in unamquamque rem inducens utilem atque honestam, primo propter insolentiam reclamantes, deinde propter rationem atque orationem*

studiosissime audientes, ex feris & immanibus, mites reddidit & mansuetos.

This long repetition of *Tullyes* words will seem tedious to them that love but little, and care much lesse for the knowledge of reason, but unto your Majesty (I dare say) it is a dilectable rememberance, and unto me it seemed so pleasant, that I could scarce stay my pen from writing all that mine eyes did so greedily read.

This sentence of *Cicero* am I loath to translate into English, partly for that unto your Majesty it needeth no translation, but especially knowing how far the grace of *Tullyes* eloquence doth excell any English mans tongue, and much more exceedeth the basenesse of my barbarous style: yet for the fruit of my sentence, I had rather unto my meer English Country men utter the rudenesse of my translation, then to defraud them the benefit of so good a lesson, trusting they will so learne to love reason, that they will also gladly and greedily embrace all good Sciences, that may help to the just furniture of the same, when they consider that informed reason was the onely instrument, or at least the chiefest means to bring men into civil regiment, from barbarous manners, and beastly conditions. "For the time was (saith *Tully*) that men "wandered abroad in the fields up and down like beasts, and used "no better order in feeding then they: so that by reasons rule "they wrought nothing, but most of their doings did they atchive "by fore of strength. At this time there was no just regard of "religion towards God, nor of duty towards man. No man had "seen right use of marriage, neither did any man know their own "children from other; nor no man had felt the commodity of "just Laws: so that through error and ignorance, willfull lust, "like a blind and heady ruler, abused bodily strength as a most "mortall minister for the satisfying of his desire. At that time "was there one which not onely in power, but also in wisdome was great, and he considered how that in the mind of "men was both apt instruments, and great occasion to the due "acomplishment of most weighty affaires, if a man could apply "them to use, and by teaching of rules frame them to better trade. "This man with perswasion of reason gathered into one place "the people that were wandring about the fields, and lay lurking in wild cottages, and woods, and bringing them in one common society, did trade them to all such things, as either were "profitable or honest, although not without repining at the first,

 by

" by reaſon then they had not been ſo accuſtomed before: Yet at
" length through reaſon and perſwaſion of words they obeyed
" him more diligently, and ſo of a wild and cruel people he made
" them curteous and gentile.

Thus hath *Tully* ſet forth the efficacy of reaſon and perſwaſion, how it was able to convert wild people to a mildneſſe, and to change their furious cruelneſſe into gentle courteſie: were it not now a great reproach in this our time (when knowledge reigneth ſo large) that men ſhould ſhew themſelves leſſe obſequious to reaſon? Unleſſe it may be thought that now every man having ſufficient knowledge of himſelfe, needeth not to harken to the perſwaſion of others.

Indeed he that thinketh himſelfe wiſe, will not eſteem the reaſon of any other, be he never ſo wiſe; ſo that of ſuch a one it may well be ſaid: He that thinketh himſelfe wiſer then he is, may juſtly be counted a double fool. Wherefore ſuch men are not to be permitted in open audience to talk, but muſt be put to ſilence, and be made to give ear to reaſon; which reaſon conſiſteth not in the multitude of words heaped raſhly together, and applied for one purpoſe, but reaſon is the expreſſing of a juſt matter with witty perſwaſions, furniſhed with learned knowledge: ſuch knowledge had *Moſes*, being expert in all learning of the *Ægyptians*, as the Scriptures declare, and therefore was able to perſwade the ſtubborn people of the Jews, although not without pain. Such knowledge, and ſuch reaſons did *Druys* ſhew which was the firſt Lawmaker of all the Weſt part of Europe. Like reaſon and wiſdome did *Xamolxis* amongſt the Goths. *Lycurgus* unto the Lacedemonians. *Zeleucus* to the Locrians. *Solon* to the Athenienſes, and *Dunwallo Molmutius* two thouſand years paſt amongſt the old Brittains of this Realm. And hereby came it to paſſe, that their Laws continued long till more perfect reaſon altered many of them, and wilfull power oppreſſed moſt of them.

Druys was ſon to King *Sarron*, and ſucceeded him in his kingdome.

At the beginning when theſe wiſe men perceived how hard it was to bring the rude people to underſtand reaſon, they judged the beſt means to attain this honeſt purpoſe, to depend of learning in every kinde: for by learning (as *Ovid* ſaith) *Pectora miteſcunt, aſperitáſque fugit*; Stout ſtomachs doe wax mild, and ſharp fierceneſſe is exiled. Therefore as *Beroſus* doth teſtifie, *Sarron* that was the third King over all this Weſt part of Europe, for to bring the people from beaſtly rage to manly reaſon: did erect ſchools of liberall Arts, which tooke ſo good ſucceſſe, that his name

continued

continued in that sort famous above two thousand years after: for *Diodorus Siculus* which was in the time of *Julius Cæsar*, maketh mention of the learned men of Goths or Celtes, and nameth them *Sarronides*, that is to say, *Sarron his Schollers and followers.*

Amongst these Arts that then were taught some did inform the tongue, and make them able both to utter aptly their minde, and also to perswade; as Grammar, Logick, and Rhetorick, although not so curiously as in this time; some other did appertain to the just order of partition of lands, the true using of Weights, Measures and reckonings of all sorts of bargains, and for order of building and sundry other uses; those were Arithmetick and Geometry. Again, to incourage men to the honour of God, they taught Astronomy, whereby the wonderfull works of God were so manifestly set forth, that no mans tongue, nor pen can in like sort expresse his infinite Power, his unspeakable wisdome, and his exceeding goodnesse toward man, whereby he doth bountifully provide for man all necessaries, not onely to live, but also to live pleasantly. And so was their confidence in Gods providence strongly stayed, knowing his goodnesse to be such, that he would help man as he could, & his power to be so great, that he would do nothing but that that was best. Besides these Sciences they taught also Musick, which most commonly they did apply partly to religious services, to draw men to delight therein, and partly to songs made of the manners of men, in praise of vertue and discommendation of vice, whereby it came to passe, that no man would displease them, nor doe any thing evill that may come to their hearing: for their songs made evill men more abhorred in that time then any excomunication doth in this time. The posterity of these Musitians continue yet both in Wales and Ireland, called *Bardes* unto this day, by the ancient name of *Bardus* the first founder.

This *Bardus Druydius* the 5. King of the Celtes, reigned 60. years, and died 1832 years before Christ.

And as these Sciences did encrease, so did vertue encrease thereby. Again as those Sciences did decay so vertue lost her estimation, and consequently was little in use: whereof to make a full declaration were a thing meet for a Prince for to hear, but it would require a peculiar Treatise. Wherefore at this present I count it sufficient, lightly to have touched this matter in generall words, and to say no more of the particularity thereof, but onely touching one of those Sciences, that is Arithmetick, by which not onely just partition of Lands was made, but also touching buying and selling, all Assises, Weights, and Measures were devis-

ed,and all reckonings and accounts driven; yea by proportion of it were the true orders of justice limited,as *Aristotle* in his Ethicks doth declare, and the degrees of estates in the common-wealth established; although that proportion be called Geometricall and not Arithmeticall, yet doth that proportion appertain to the art of Arithmetick, and in Arithmetick is taught the progression of such proportions, and all things thereto belonging. Wherefore I may well say, that seeing Arithmetick is so many wayes needfull unto the first planting of a Common-wealth, it must needs be as much required to the preservation of it also: for by the same means is any Common-wealth continued, by which it was erected and established. And if I shall in small matters in appearance,but indeed very weighty,put one example or two. What shall we say for the Statutes of this Realm, which be the onely stay of good order, in manner, now? As touching the measuring of ground by length and breadth, there is a good and an ancient Statute made by art of Arithmetick; and now it shall be to little use, if by the same art it be not practised and tried. For the assise of bread and drink,the two most common and most necessary things for sustentation of man, there was a goodly ordinance in the Law made, which by ignorance hath so grown out of knowledge, and use,that few men do understand it,and therefore the Statute books wonderfully corrupted,and the Commons cruelly oppressed: notwithstanding some men have written that it is too doubtfull a matter to execute those assises by those Statutes,by reason they depend of the standard of the coyne, which is much chang'd from the state of that time, when those Statutes were made. Thus shall every man read (that listeth) in the abridgement of the Statutes, in the title of Weights and Measures, in the seventh number of the English Book, where he should have translated a good ordinance which is set forth in the French Book: but no marvell if the Abridgment doth omit it seeing the great Book of Statutes doth omit the same Statute, as it hath done many other very good laws. And this is the fruit of ignorance to reject and condemn all that it understandeth not, although they use some cloaks for it, but such cloaks as being allowed, migh serve to repell all good laws; which God forbid.

Again, there is an ancient order for assise of fire, wood and coals, which was renued not many years past; and now how avarice and ignorance doth canvase that Statute, it is too pitifull to talk of, and more miserable to feel.

Further-

Furthermore, for the Statute of Coynage, and the standard thereof, if the people understood rightly the Statute, they should not nor would (as they often do) gather an excuse for their folly thereby: but as I said, these Statutes by wisdome and good knowledg of Arithmetick were made, & by the same must they be continued. And let ignorance no more meddle with the use of them, then it did with the making of them. Oh in how miserable case is that Realm, where the ministers and interpreters of the law are destitute of all good Sciences, which be the keyes of the lawes! How can they either make good lawes, or maintain them that lack the true knowledge whereby to judge them? And happy may that Realm be accounted, where the Prince himself is studious of learning, and desireth to understand equity in all laws. Therefore most happy are we the loving subjects of your Majesty, which may see in your Highnesse not onely such towardnesse, but also such knowledge of diverse arts, as seldom hath been seen in any Prince of such years, whereby we are enforced to conceive this hope certainly, that he which in those years seeketh knowledg when knowledge is least esteemed, and of such an age can discern them to be enemies both to his royall Person and to his Realms, which labour to withdraw him from knowledge to excessive pastime, and from reasonable study to idle or noysome pleasures, he must needs when he cometh to more mature years, be a most prudent Prince, a most just Governour, and a right Judge, not only of his subjects commonly, but also of the ministers of his Laws, yea, and of the Laws themselves: and to be able to conceive the true equity and exact understanding of all his Laws and Statutes, to the comfort of his good subjects, and the confusion and reproach of them which labour to obscure or pervert the equity of the same Laws and Statutes. How some of these statutes may be applyed to use, as well in this our time, as in any other time, I have peculiarly declared in this Book, and some other I have omitted for just considerations, till I may offer them first unto your Majesty to weigh them as to your Highnesse shall seem good: for many things in them are not to be published without your Hignesse knowledge and approbation; namely because in them is declared all the rates of alloyes for all standards from one ounce upward, with other mysteries of Mint matters, & also most part of the varieties of coyns that have been currant in this your Majesties Realm by the space almost of six hundered years last past, and many of them that were currant in the time that the Romans ruled here.

All which with the ancient deſcription of England and Ireland, and my ſimple cenſure of the ſame, I have almoſt compleated to be exhibited to your Highneſſe: In the mean ſeaſon moſt humbly beſeeching your Majeſty to accept this ſimple Treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your Majeſty is, that this amongſt other of my Bookes may paſſe under the protection of your Highneſſe, whom I beſeech God moſt earneſtly and daily, according to my duty, to advance in all honor, and Princely Regality, and to increaſe in all knowledge, juſtice and godly policy, Amen.

Your Majeſties moſt obedient
ſubject and ſervant,

Robert Record.

To

To the loving READERS

The Preface of Mr Robert Record.

SOre oft times have I lamented with my self the unfortunate condition of England, seeing so many great Clerks to arise in sundry other parts of the world, and so few to appear in this our Nation: whereas for pregnancy of naturall wit (I thinke) few Nations doe excelle Englishmen: But I cannot impute the cause to any other thing, then to the contempt, or misreguard of learning. For as Englishmen are inferior to no men in mother wit, so they passe all men in vain pleasures, to which they may attain with great pain and labour: and are slack to any never so great commodity, if there hang of it any painfull study, or travelsome labour.

Howbeit, yet all men are not of that sort, though the most part be, the more pity it is, but of them that are so glad, not onely with painfull study, and studious pain to attain learning, but also with as great study and pain to communicate their learning to others, & make all England (if it might be) partakers of the same; the most part are such that unneath they can support their own necessary charges, so that they are not able to beare any charges, in doing of that good that else they desire to do.

But a greater cause of lamentation is this that when learned men have taken pains to do things for the aid

of

of the unlearned, ſcarce they ſhall be allowed for their well doing, but derided and ſcorned, and ſo utterly diſcouraged to take in hand any like enterpriſe again. So that if any be found (as here are ſome) that doe favour learning and learned wits, and can be contented to further knowledge, yea onely with their word; ſuch perſons though they be rare, yet ſhall they encourage learned men to enterpriſe ſomething at the leaſt that England may rejoyce of. And I have good hope that England will (after ſhe hath taken ſome ſure taſte of learning) not onely bring forth more favorers of it, but alſo ſuch learned men, that ſhe ſhall be able to compare with any Realm in the world. But in the mean ſeaſon where ſo few regarders of learning are, how greatly they are to be eſteemed that doe favour and further it, my pen will not ſuffice at full to declare.

Therefore, gentile Reader, whereas I doe upon moſt juſt occaſion judge, yea and know aſſuredly, that there be ſome men in this Realm, which both love and alſo much deſire to further good learning, and am not well able to write their condign praiſe for the ſame, I think it better with ſilence to overpaſſe it, then either ſay too litile of it, or provoke againſt them the malice of ſuch other, which doe nothing themſelves that is praiſ-worthy, and therefore cannot abide to heare the praiſe of any other mans good indeed.

And conſidering their great favour unto learning, though I my ſelfe be not worthy to be reckoned in the number of great learned men, yet am I bold to put my ſelfe in Preſſe, with ſuch ability as God hath lent me, though not with ſo great cunning as many men, yet with as great affection as any man to helpe my country men, and will not ceaſe daily, (as much as my ſmall ability

will suffer me) to endite some such thing, that shall be to the instruction, though not of learned men, yet at the least of the vulgar sort, whose argument alwaies shall be such as it shall delight all learned wits though they doe not learne any great thing out of it.

But to speak of this present booke of Arithmetick, I dare not, nor will not set it forth with any words, but remit it to the judgement of all gentile Readers, and namely such as love good learning, beseeching them so to esteem it, as it doth seem worthy. And so either to accept the thing for it selfe, either at the least to allow my good endeavour. But I perceive I need not use any perswasions unto them, whose gentile nature and favourable mind is ready to receive thankfully and interpret to the best of all such enterprises attempted for so good an end, though the thing do not alwayes satisfie mens expectation. This considered, did bolden me to publish abroad this little Booke of the Art of numbering, which if you shall receive favourably, you shall encourage me to gratifie you hereafter with some greater thing.

And as I judge some men of so loving a mind to their native Countrey, that they would much rejoyce to see it prosper in good learning, and witty Arts: so I hope well of all the rest of Englishmen, that they will not be unmindfull of his due praise, by whose means they are helped and further'd in any thing. Neither ought they to esteem this thing of so little value, as many men of little discretion oftentimes doe. For who so setteth small price by the witty device and knowledge of numbering, he little considereth it to be the chief point, (in manner) whereby men differ from all brute beasts: for as in all other things (almost) beasts are partakers with us,

so in numbering we differ cleane from them, and in manner peculiarly, sith that in many things they excell us again.

The Fox in crafty wit exceedeth most men,
A Dog in smelling hath no man his peer.
To foresight of weather if you look then,
Many beasts excell men ; this is cleer.
The wittinesse of Elephants doth letters attain,
But what cunning doth there in the Bee remain?
The Emmet foreseeing the hardnesse of winter,
Provideth victuals in the time of summer.
The Nightingale, the Linet, the Thrush, the Lark,
In Musicall harmony passe many a Clark.
The Hedghog of Astronomy seemeth to know,
And stoppeth his cave where the wind will blow.
The Spider in weaving such art doth show,
No man can him mend, nor follow I trow.
When a house will fall, the Mice right quick
Flee thence before; can man doe the like?

Many things else of the wittinesse of Beasts and Birds might I here say, save that another time of them I intend to write, wherein they excell in manner all men as it is daily seen: but in number was there never beast found so cunning, that could know or discerne one thing from many, by daily experience you may well consider, when a Bitch hath many whelps, or a Hen many chickens: and likewise of other whatsoever they be, take from them all their young saving only one, and you shall perceive plainly that they miss none, though they will resist you in taking them away, and will seek them again if they may know where they be, but else they will

never

never misse them truly; but take away that one that is left, and then will they cry and complain; and restore to them that one, then are they pleased again. So that of number, this may I justly say, it is the onely thing almost that separateth man from beasts. He therefore that shall contemn number, declareth himselfe as bruitish as a beast and unworthy to be countin the Fellowship of men. But I trust there is no man so foul overseen, though many right smally doe it regard.

Therefore will I now stay to write against such, and return again to this my Book, which I have written in the form of a Dialogue, because I judge that to be the easiest way of instruction, when the Scholer may aske every doubt orderly, and the Master may answer to his question plainly. Why the Author wrote in Dialogue wise.

Howbeit I think not the contrary, but as it is easier to make another mans worke then to make the like; so there will be some that will finde fault because I writ in a Dialogue: but as I conjecture those shall be such as do not, cannot, or will not perceive the reason of right teaching, and therefore are unmeet to be answered unto, for such men with no reason will be satisfied.

And if any man object, that other books have been written of Arithmetick *already so sufficiently that I needed not now to put pen to the book, except I will condemn other mens writings: To them I answer, That as I condemn no mans diligence, so I know that no one man can satisfie every man: and therefore like as many doe esteeme greatly other bookes, so I doubt not but some will like this my booke above any other* English Arithmetick *hitherto written; and namely such as shall lack instructors, for whose sake I have so plainly set forth the Examples, as no booke that I have seen hath done hitherto*

therto: which thing shall be great ease to the rude Readers.

Therefore (gentile Reader) though this Booke can be but small aid to the learned sort, yet unto the simple ignorant (which needeth most helpe) it may be a good furtherance and mean unto knowledge.

And though unto the King his Majesty privately I doe it dedicate, yet I doubt not (such is his clemency) but that he can be content, yea, and much desirous, that all his loving Subjects shall take the use of it, and imploy the same to their most profit. Which thing if I perceive that they thankfully doe, and receive with as good will as it was written, then will I shortly with no lesse kindnesse set forth such introductions into Geometry *and* Cosmography, *as I have at times promised, and as hitherto in* English *hath not been enterprised, wherewith I dare say all honest hearts will be pleased, and all studious wits greatly delighted.*

I will say no more, but let every man judge as he shall see cause. And thus for this time I will stay my Pen, committing you all to that true fountain of perfect number, which wrought the whole world by number and measure: he is Trinity in Unity, and Glory, Amen.

Here

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The

The Contents.

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The Contents.

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x	10	*ten*	DC	600	*6 hundred*
xj	11	*eleven*	M	1000	*a thousand*
vij	12	*twelve*	MD	1500	*a thou. 5 hund.*
&c.	&c.	*&c.*	&c.	&c.	*&c.*

A Dialogue between the Master and the Scholer: teaching the *Art* and use of *Arithmetick* with *Pen.*

The Scholar speaketh.

SIR, such is your authority in mine estimation, that I am content to consent to your saying, and to receive it as truth, though I see none other reason that doth lead me there unto: whereas else in mine owne conceit it appeareth but vaine, to bestow any time privately in learning of that thing that every Childe may and doth learne at all times and houres, when he doth any thing himselfe alone, and much more when he talketh or reasoneth with others.

M. Lo, this is the fashion and chance of all them that seek to defend their blinde ignorance, that when they think they have made strong reason for themselvs, then have they proved quite contrary. For if numbring be so common (as you grant it to be) that no man can doe any thing alone, and much lesse talk or bargain with other, but he shall still have to doe with number: this proveth not number to be contemtible and vile, but rather right excel-

lent and of high reputation, sith it is the ground of all mens affairs, in that without it no tale can be told, no communication without it can be continued, no bargaining without it can duly be ended, or no businesse that man hath justly compleated. These commodities if there were none other, are sufficient to approve the worthinesse of number. But there are other innumerable, far passing all these, which declare number to exceed all praise. Wherefore in all great works are Clerks so much desired? Wherefore are Auditors so richly fed? What causeth Geometricians so highly to be inhaunced? Why are Astronomers so greatly advanced? Because that by number such things they finde, which else would far excell mans minde.

Scholar. Verily, Sir, if it be so, that these men by numbering, their cunning do attain, at whose great works most men doe wonder, then I see well I was much deceived, and numbring is a more cunning thing then I tooke it to be.

Master. If number were so vile a thing as you did esteem it, then need it not to be used so much in mens communication. Exclude number, and answer to this question: How many years old are you?

Scholar. Mum.

Master. How many dayes in a weeke? How many weekes in a yeare? What lands hath your Father? How many men doth hee keep? How long is it since you came from him to me?

Scholar. Mum.

Master. So that if number want, you answer all by mummes: How many miles to London?

Scholar.

Scholar. A peak full of plums.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking it maketh men dumb, so that to most questions they must answer mum.

Scholar. This is the cause sir, that I judged it so vile, because it is so common in talking every while: For plenty is not dainty, as the common saying is.

Master. No, nor store is no sore perceive you this? The more common that the thing is being needfully required, the better is the thing, and the more to be desired. But in numbering as some of it is light and plain, so the most part is difficult and not easie to attain. The easier part serveth all men in common, and the other requireth some learning. Wherefore as without numbring a man can doe almost nothing, so with the help of it you may attaine to all things.

Scholar. Yea sir, why then it were best to learn the Art of numbring, first of all other learning, & then a man need learn no more if all other come with it.

M. Nay, not so: but if it be first learned, then shall a man be able (I mean) to learn, perceive & attain to other Sciences; which without it he could never get.

Scholar. I perceive by your former words, that Astronomy and Geometry depend much on the help of numbring: but that other Sciences, as Musick, Physick, Law, Grammer, and such like, have any help of Arithmetick, I perceive not.

Master. I may perceive your great Clerklinesse by the ordering of your Sciences: but I will let that passe now, because it toucheth not the matter that

I

I intend, and I will shew you how Arithmetick doth profit in all these, somewhat grosly, according to your small understanding; omitting other reasons more substantiall.

Musick. First (as you reckon them) musicke hath not onely great help of Arithmeticke, but is made, and hath its perfectnesse of it: for all musicke standeth by number & proportion: And in Physicke, beside the

Physick. calculation of criticall dayes, with other things which I omit, how can any man judge the pulse rightly, that is ignorant of the proportion of numbers.

Law. And as for the Law, it is plain, that the man that is ignorant of Arithmeticke, is neither meet to be a Judge, neither an Advocate, nor yet a Proctor. For how can hee well understand another mans cause, appertaining to distribution of goods, or other debts or of summes of money, if he be ignorant of Arithmetick? This oftentimes causeth right to bee hindred, when the Judge either delighteth not to heare of a matter that hee perceiveth not, or cannot judge for lacke of understanding: this commeth by ignorance of Arithmeticke.

Grammer. Now, as for Grammer, mee thinketh you should not doubt in what it needeth number, sith you have learned that Nouns of all sorts, pronouns, Verbs and Participles are distinct diversly by numbers: besides the variety of Nouns of Number, and Adverbs. And if you take away number from Grammer, then is all the quantitie of Syllables lost. And many other wayes doth number help Grammer. Whereby were all kindes of Meeters found and made? was it not by number?

But

But how needfull Arithmeticke is to all parts of Philosophy, they may soon see, that doe reade either Aristotle, Plato, or any other Philosophers writings. For all their examples almost, and their probations depend of Arithmetick. It is the saying of Aristotle, that hee that is ignorant of Arithmeticke, is meet for no Science. And Plato his Master wrote a little sentence over his School-house door, Let none enter in hither (quoth he) that is ignorant of Geometry. Seeing hee would have all his Scholars expert in Geometry, much rather hee would have the same in Arithmeticke, without which Geometry cannot stand. Philosophy.

And how needfull Arithmeticke is to Divinity, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much do write of it. And if I should go about to write all the commodities of Arithmeticke in civill acts, as in governance of Common-weales in time of peace, and in due provision and order of Armies in time of war, for numbering of the Host, summing of their wages, provision of Victuals, viewing of Artillerie, with other Armour; beside the cunningest point of all, for casting of ground, for encamping of men, with such other like: And how many waies also Arithmeticke is conducible for all private Weales, of Lords and all Possessioners, of Merchants, and all other occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receivers, Stewards, Bailiffes, and such like, whose Offices without Arithmeticke are nothing: If I should (I say) particularly repeat all such commodities of the noble Science of Arithmeticke, it were Divinity. Armie.

enough

enough to make a verie great booke.

Scholar. No, no, sir, you shall not need: For I doubt not but this, that you have said, were enough to perswade any man to think this Art to be right excellent and good: and so necessary for man, that (as I thinke now) so much as a man lacketh of it, so much he lacketh of his sense and wit.

Master. What, are you so farre changed since, by hearing these few commodities in generall? by likelihood you would be farre changed if you knew all the particular Commodities.

Scholar. I beseech you Sir, reserve those Commodities that rest yet behinde unto their place more convenient: and if ye will be so good as to utter at this time this excellent treasure, so that I may be somewhat inriched thereby, if ever I shall be able, I will requite your pain.

Master. I am very glad of your request, and will doe it speedily, sith that to learn it you be so ready.

The duty of a Scholar.

Scholar. And I to your authority my wit doe subdue, whatsoever you say, I take it for true.

Master. That is too much, and meet for no man to be believed in all things, without shewing of reason. Though I might of my Scholar some credence require, yet except I shew reason, I doe it not desire. But now sith you are so earnestly set this Art to attaine, best it is to omit no time, lest some other passion coole this great heat, and then you leave off before you see the end.

Perseverance in study.

Scholar. Though many there be so unconstant of mind, that flitter and turn with every winde, which often begin, and never come to the end, I am

nons

none of this sort as I trust you partly know. For by my good will what I once begin, till I have it fully ended, I would never blin.

Master. So have I found you hitherto indeed, and I trust you will increase rather, then goe backe. For better it were never to assay, then to shrinke and fly in the mid way: But I trust you will not doe so; therefore tell me briefly: What call you the Science that you desire so greatly.

Scholar. Why sir, you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to have studied a Science, and yet cannot tell how it is named.

Scholar, Some call it Arsemetrick, and some Augrime.

Master. And what doe these names betoken?

S. That if it please you, of you would I learn.

Master. Both names are corruptly written: Arsemetrick for Arithmeticke, as the Greeks call it, and Augrime for Algorisme, as the Arabians found it: which doth betoken the Science of Numbring: for Arithmos in Greeke is called Number: and of it commeth Arihmeticke, the Art of Numbering: So that Arithmeticke is a Science or Art teaching the manner and use of Numbering: This Art may bee wrought diversly, with Pen, or with Counters. But I will first shew you the working with the pen, and then the other in order. Ἀριθμη-τική.

Scholar. This will I remember. But how many things are to bee learned to attaine this Art fully?

Master. There are reckoned commonly seven parts or works of it.

[Numeration,

Numeration, Addition, Subtraction, Multiplication, Diviſion, Progreſſion, and Extraction, of roots: to theſe ſome men adde Duplation, Triplation, and Mediation. But as for theſe thrée laſt they are contained under the other ſeven. For Duplation, and Triplation are contained under Multiplication, as it ſhall appeare in their place: And Mediation is contained under Diviſion, as I will declare in his place alſo.

Scholar. Yet then there remain the firſt ſeven kinds of Numbering.

Maſter. So there doth: howbeit if I ſhall ſpeake exactly of the parts of Numbering, I muſt make but five of them: for Progreſſion is a compound Operation of Addition, Multiplication and Diviſion. And ſo is the Extractions of roots. But it is no harme to name them as kinds ſeverall, ſéeing they appeare to have ſome ſeverall working. For it forceth not ſo much to contend for the number of them, as for the due knowledge and practiſing of them.

5 parts

Scholar. When you will that I ſhall name them, as ſeven kindes diſtinct: But now I deſire you to inſtruct me in the uſe of each of them.

Maſter. So I will, but it muſt be done in order: for you may not learn the laſt ſo ſoon as the firſt, but you muſt learne them in that order, as I did rehearſe them, if you will learne them ſpéedily and well.

Scholar. Even as you pleaſe. Then to begin: Numeration is the firſt in order, what ſhall I doe with it?

Maſter.

Maſter. Firſt, you muſt know what the thing is, and then after learn the uſe of the ſame.

Numeration.

NUmeration *is that* Arithmeticall *skill, whereby we may duely value, expreſſe, and read any* Number or Summe *propounded: or elſe in apt* Figures *and* Places *ſet downe any* Number *known or named.*

Scholar. Why? then me thinketh you put a difference betwéen the Value and the Figures.

Maſter. Yea ſo doe I. For the Value is one thing, and the Figures are another thing, and that cometh partly by the diverſity of Figures, but chiefly in the places wherein they be ſet.

Scholar. Then muſt I know here 3 things, the Value, the Figure, and the Place.

Maſter. Even ſo. But yet adde Order to them as the fourth. And firſt mark, that there are but tenne Figures that are uſed in Arithmetick; and of thoſe tenne, one doth ſignifie nothing, which is made like an o, and is privately called a Cypher, though all the other ſometime be likewiſe named. The other nine are called ſignifying Figures, and be thus figured.

A Cypher.

Figures.

1. 2. 3. 4. 5. 6. 7. 8. 9.

And this is their value.

i. ii. iii. iiii. v. vi. vii. viii. ix.

But here you must mark, that every Figure hath two values: One alwayes certain, that it signifieth properly, which it hath of his form; and the other uncertain, which he taketh of his place.

A place. A place is called the seate or roome that a Figure standeth in. And looke how many Figures are written in one summe, so many places hath that whole number. And that must be called the first place, that is next to the right hand, and so reckoning by order towards the left hand, so that that place is last that is next to the left hand. As for example. If there stood before you six men in a row, side by side, and you should tell them as they stand in order, beginning with the man that were next to your right hand; then he that were next him should be called your second, and so forth to the furthest from your right hand, which is the sixth and the last.

Scholar. I perceive you well: so might I reckon Letters or any other thing. As if I should write 8 Letters after this order, a, b, c, d, e, f, g, h. then must I say, h, is the first, g, the second, f, the third, e, the fourth, d, the fifth, c, the sixth, b, the seventh, a, the eight.

Master

Master. That is well done. And after the same sort use hereafter, that what I declare by one example, do you expresse by another : and so shall I perceive whether you understand it or no. And so passe over nothing, till you perceive it well, and be expert therein.

Schol. I pray you how many of these places be there in all?

Master. There is no certain number of them, but they are sometimes more, and sometimes fewer, according to the sum that is expressed. For so many as the figures are, so many are the places : and the last place is so called, not because it is the last of all other, but it is the last of that present summe, and it may bée the middle place in another summe.

Scholar. Me séemeth I perceive this very well, as touching the order of reckoning of the places : but as for the number of them, you say there is no certainty. Now there resteth to declare the value of the figures by the diversity of places, which you called the value uncertain. Value uncertain.

Master. But first let me hear whether you know perfectly the certain value. Value certain.

Scholar. Yes sir, as you wrote them, so I marked them.

Master. How write you then five?

Scholar. By this figure 5.

Master. And how six?

Scholar. Thus 6.

Master. Write these thrée numbers, each by it self, as I speake them, vii, iiii, iii.

Scholar. 7. 4. 3.

Master. How write you these foure other, ii.i.ix.viii.

Scholar. Thus (I trow) 2. 1. 6. 8.

Master. Nay there you misse: looke on mine example again.

Scholar. Sir true it is, I was to blame, I take 6 for 9, but I will beware hereafter.

Master. Now then take heed those certain values every figure representeth when it is alone written without other figures joyned to him. And also when it is in the first place, though many other doe follow: as for example, this figure 9 is ix, standing now alone.

Scholar. How is he alone, and standeth in the middle of so many letters?

Master. The letters are none of his fellows. For if you were in France in the middle of a thousand Frenchmen, if there were no English man with you, you would reckon your self to be alone.

Scholar. So it is. Then 9 without more figures of Arithmetick betokeneth ix. whatsoever other letters be about it.

Master. Even so, and so doth it, if it be in the first place joyned with other, how many soever do follow, as in this example. 3679. You see 9 in the first place, and doth betoken nine as it were alone.

Scholar. I perceive that, and doth not 7 that standeth in the second place (between 9 and 6 in the third place) betoken vii. and so 3 in the fourth place betoken three?

Master. These figures be as you have said, but their values are not so. For as in the first place every figure betokeneth his own value certain onely, so in

in the second place every figure betokeneth his own value certain tenne times: as in the example, 7 in the second place is seven times ten, and is lxx. And in the third place, every Figure betokeneth his own value an hundred times, so the 6 in that place betokeneth vi C. and in the fourth place every figure betokeneth his owne value a M. times, as in the aforesaid number 3 in the fourth place standeth for 3 M. and in the fifth place every figure standeth for his own value x M times, and in the sixth place a C M. times, and in the seventh place a M M. times, and in the eighth place x M M, so that every place exceedeth the former ten times.

Scholar. As thus: if I make this number at all adventures, 91359684. here are eight places. In the first place is 4 and betokeneth but foure: in the second place is 8 and betokeneth ten times 8 that is 80, in the third place is 6 and betokeneth six hundred: in the fourth place 9 is nine thousand, and 5 in the fifth place is x M times 5, that is fifty M. So 3 in the sixth place is a C M times 3 that is CCC M. Then 1 in the seventh place, one M M, and 9 in the eighth place, ten thousand thousand times 9, that is xc M M. But now I cannot easily nor quickly read it in order. A generall Rule.

Master. That shall you practise by this meanes. First, put a pricke over the fourth figure, and so over the seventh. And (if you have so many) over the tenth, thirteenth, sixteenth, and so forth, still leaving two figures between each two prickes. And those two roomes between the pricks are called Ternaries. Ternaries

Then begin at the last pricke, and see how many

figures are betwéen him and the end, which cannot passe thrée, reckoning himselfe for one: then pronounce them as if they were written alone from the rest, and at the end of their value, so many times thousands as your numbers have pricks.

After that come to the next thrée figures, and sound them as if they were apart from the rest, and adde to their value so many times thousands, as there are pricks betwéen them, and the first place of your whole number. And so do by every other thrée figures following, if you have more. As in example, 91359684 this was your number.

Put a prick over 9 in the fourth place, and over 1 in the seventh place, and then no more (for your places come not to ten) as thus: 91359684.

Now go to the last prick over 1, and take it and the figure 9 that followeth it, & value them alone.

Scholar. 91, that is xci.

Master. So it is, then adde for the number of your pricks twice M.

Scholar. that is, xci. thousand thousand.

Master. So it is. Then take the thrée other figures from one to the next Prick, and value them.

Scholar. 359, that is, CCC. lix.

Master. Now adde for the one prick, that is betwéen them and the first place, M.

Scholar. CCC. lix. thousand.

Master. Then come to the other 3 figures that remain.

Schol. 684, that is, vi C. lxxxiiii.

Master. Now have you valued all. And at the end of the last number you shall adde nothing, because there remaineth no pricke nor number after

it,

it, yet prove it in another number, as thus, 23086408910 5340.

Schol: 2308640891 05340. I have pricked them as you taught me, but I am in doubt whether I have done well or no, because of the Cyphers, for I remember you told me that they do signifie nothing, and therefore I doubt whether I should reckon them for a Figure in setting of the pricks; and againe, I know not wherefore they serve.

Master. That will I tell you now. Indeed they are of no value themselves, but they serve to make up the number of places, and so make the Figure following them to bee in a farther place, and therefore to signifie the more value: as in this example, 90 the Cypher is of no value, but yet he occupieth the first place, and causeth 9 to be in the second place, and so to signifie ten times 9, that is xc. so that two Cyphers thrust the Figure following them into the third place, and so forth. The use of Cyphers.

Scholar. Then I perceive in the example above, I have pricked well enough, for though that Cypher that is pricked signifies nothing, yet must he have the prick, because he came in the thirteenth place. Then will I prove to number that summe. First, there is 230, M, M, M, M. and then followeth 864, M, M, M. And what shall I now do? There is a Cypher in the third place, and no Figure after him, but they that I have reckoned.

Master. He did serve for them that you have already reckoned, to make them in a place further then they should bée, if he were away, and therefore now ye shall let him goe. And so doe alwayes when he occupieth that place next before any

pricke, which is the last of that Ternary, and a Cypher in the last place doth nothing.

Scholar. Then shall I say but 89, M, M.

Master. So, but go forth.

Scholar. 105 thousand. Now are all my pricks spent, and yet remain 340, so that I must value them, CCC. xl onely.

Master. Now can you reckon after this sort: and remember that every such room, so parted, is called a Ternary, or Trinity, for you have numbred or valued the summe most truly and by the aid of the pricks each denomination is distinct most plainly.

Trinity.

Scholar. What call you Denomination?

Denomination.

Master. It is the last value or name added to any summe. As when I say, an hundred two and twenty pounds: Pounds is the Denomination. And likewise in saying, 25 men: Men is the Denomination, and so of other: But in this place (that I spake of before) the last number of every Ternary, is the Denomination of it. As for the first Ternary, the Denomination is Vnites, & of the second Ternary, the Denomination is thousands, & of the third Ternary, thousand thousands, or Millions; of the fourth, thousand thousand thousands or thousand millions: & so forth.

Scholar. And what shall I call the value of the three figures that may bee pronounced before the Denomination, as in saying, 203000000. that is, two hundred three millions: I perceive by your words, that millions is the Denomination: but what shall I call CCiii. joyned before the millions?

Numerator.

Master. That is called the Numerator, or Valuer, and the whole summe that resulteth of them both, is called the Summe, Value or Number.

Summe or value.

Scholar.

Scholar. Now is there any thing else to be learned in Numeration? or else have I learned it fully?

Master. I might shew you here who were the first Inventors of this Art, and the reason of all these things that I have taught you, but that I will reserve till ye have learned over all the practise of this Art, lest I should trouble you with over many things at the first.

But yet this you must mark, that there are three kindes of Numbers, one called Digits, another Articles, and the third mixt numbers. Three kinds of numbers.

A Digit is any number under ten, as these: Digits.
1.2.3.4.5.6.7.8.9.

And 10 with all other that may be divided into ten parts just, & nothing remain, are called articles, such as are 10. 20 30.40. 50. &c. 100. 200. &c. 1000. &c. Articles.

And that number is called mixt, that containeth articles, or at the least one article, and a Digit, as 12.16.19.21.38 107.1005.& so forth & for the more ease of understanding & remembrance, mark this. The Digit number is never written with more then one figure, but the article and the mixt number are ever written with more then one figure. And thus they differ, that the article hath evermore this Cypher o in the first place: and the mixt number hath ever there some Digit. Mixt.

Scholar. By these last words I perceive it much better then I did before, and now (I think) I will never misse to know those three asonder.

Master. If you remember now all that I have said, you have learned sufficiently this first kind of Arithmeticke called Numeration. Howbeit I will exhort you now to remember both this that I have

Vse maketh mastery.

said, and all that I shall say, and to exercise your self in the practise of it; for rules without practise, are but a light knowledge, and practise it is that maketh men perfect and prompt in all things.

And as you have learned to gather and expresse the value of a summe propounded and set downe before you, so must you practise to marke, note, and write down with apt figures and in due places, any number onely named, or recited to you, or if your self imagined; as for a proof. How note you, or write downe this summe, five thousand two hundred fifty and seven?

S. This troubleth me now, whether I should begin at the first, or at the last. For reason (me thinketh) should cause me to begin at the first, & yet if I write it as you speake it, I must begin at the last.

Master, When you know your places perfectly, you may begin where you list; but the more ease for your hand is to beginne with the last, that is to say, as I did speak them, yet for the more surety, a while you may begin at the first, repeating my words backwords thus: seven, fifty, two hundred, five thousand: or else sounding them all by their digit or value, as thus: seven, five, two, five; for that way is easiest: But then must you look well whether there be any cypher in your summe, that he may be set in his place: as if the last valuer of your summe (as you speak it) be above 9, then is there a cypher in the first place. And if it be an hundred, or above, then is there two cyphers, one in the first place, and another in the second, and so forth.

But because this thing is such that cannot be set forth without many words; I think best here now

at

at the end of Numeration, to adde a Table easie and ready for the first exercise of it.

Lo this is the Table.

The names of Digits, values certain, or valuers.

The right side or hand.

The denominators of the place or value uncertain.	Nine.	Eight.	Seven.	Six.	Five.	Foure.	Three.	two.	One.	Cyph.	The order of places.
Unites.	9	8	7	6	5	4	3	2	1	0	First.
Tennes.	9	8	7	6	5	4	3	2	1	0	Second.
Hundreds.	9	8	7	6	5	4	3	2	1	0	Third.
Thousands.	9	8	7	6	5	4	3	2	1	0	Fourth.
X. of Thousands.	9	8	7	6	5	4	3	2	1	0	Fifth.
C. of Thousands.	9	8	7	6	5	4	3	2	1	0	Sixt.
Millions.	9	8	7	6	5	4	3	2	1	0	Seventh.
X. of Millions.	9	8	7	6	5	4	3	2	1	0	Eighth.
C. of Millions.	9	8	7	6	5	4	3	2	1	0	Ninth.
M. of Millions.	9	8	7	6	5	4	3	2	1	0	Tenth.
X. M. of Millions.	9	8	7	6	5	4	3	2	1	0	Eleventh.

The left hand or side.

This Table (as ye may see) hath eleven Places, and in each of them are set all the Digits, whose certain value is written on the right hand of the Table, and the value uncertain on the left hand; so that by this Table you may learne both how to

expresse

expresse any Number that you list (if that it exceed not eleven places) that is to say, XC. thousand Millions, and so may you by help of it, value all summes proposed under the said number.

For example: take the summe that I proposed before which was five thousand two hundred fifty and seven. And if you will expresse it, take the first number (as I speak it) which is five M, whose valuer or certain value is v, and his uncertain value, or Denomination is M First, you shall seek at the right hand of the valuer 5. Then seek along under the title of Denomination toward the left hand till you finde thousands, and under it, right at the foote of the Table, is the number of the place, that is in the fourth, wherein you must write your Digit, or valuer 5.

Afterward come to the second part of the number two hundred, whose valuer is 2, and his Denomination C. Seek two at the right hand of the Table, and go along under the Denomination toward the left hand, till you come under C. then look to the foot of the Table, and there you shall see the number of the place, that is to say, the third, wherein you must set your Digit 2.

Then do so by your other two numbers that remain, and you shall finde 5 in the second place for your fifty, and 7 in the first place for your seven. And thus you may do with other numbers.

Scholar. Master, I thank you heartily. I perceive you seek to instruct me most plainly and briefly, and not to hide your knowledge with subtile words, as many do. For this rule is so plain, that I can desire it no plainer. And though it seem somewhat

what long, yet I perceive it to be a sure way.

Master. So it is, and though it be long, yet it is neither too long, neither too plaine for young learners that lack practise: for this Table is in stead of a Teacher to them that lack one. But now I trust I have said enough of Numeration: which after you have well practised, then may you learn forth.

Scholar. Yet I pray you in one thing to tell me your judgement. Why do men reckon the order of the places backward, from the right hand to the left?

Why numbers are written backward.

Master. In that thing all men do agrée that the Chaldees, which first invented this Art, did set these Figures as they set all their Letters, for they write backward, as you terme it, and so do they read. And that may appear in all Hebrew, Chaldee and Arabick Books; for they be not onely written from the right hand to the left and so must be read, but also the right end of the book is the beginning of it, whereas the Greeks, Latins, and all Nations of Europe, doe write and reade from the left hand toward the right: and all their Books beginne at the left side.

Scholar. That reason hath satisfied me.

Master. It neither satisfieth me, neither lyketh me well, because I sée that the Chaldees and Hebrewes doe not so use their own Numbers, as at another time I will declare. But this plain reason may best satisfie you presently, that séeing in pronouncing of Numbers wée kéep the order of our own reading, from the left hand to the right: and again, wée doe ever name the greater numbers

before

before the smaller: It was reason that the lesser places, containing the lesser numbers, should be set on the right hand, and the greater places contain-ing the greater Numbers to procéed toward the left hand.

Scholar. This reason is to me so plain, that it séemeth now against reason to make a doubt of that order. So that now for Numeration I am sa-tisfied: hoping that practise shall make me fully ready and expert in it. And in the mean season I desire to learn the other kinds of Arithmetick.

Master. That is well said: but what should you next learn? can you tell?

Scholar. I remember you said, that Addition was next.

Master. Even so and what that is, must you first know.

Addition.

ADdition is the gathering together and brin-ging of two numbers, or more into one summe. As if I have 106 Books in the Latine tongue, and 136 in the Greeke tongue, and would know how many they bée in all, I must write these two numbers one over another, writing the greatest number highest, so that the first figure of the one being under the first figure of the other, and the second under the second, and so forth in order.

When you have so done, draw under them a right line, then will they stand thus.

$$\begin{array}{r} 160 \\ \underline{136} \end{array}$$

Now begin at the first places toward the right hand allwayes, and put toge-ther

ther the two firſt figures of theſe two numbers, and looke what commeth of them write under them, right under the line. As in ſaying 6 and 0 is 6, write 6 under 6, as thus:

160
136
―――
6

And then go to the ſecond figures, and doe likewiſe: as ſaying 3 and 6 is 9, write 9 under 6 and 3, as here you ſee.

160
136
―――
96

And likewiſe doe you with the figures that be in the third place, ſaying, 1 and 1 be 2, write 2 under them, and then will your whole ſumme appear thus:

160
136
―――
296

So that now you ſee that 160, and 136, doe make in all 296.

Scholar. What? this is very eaſie to doe, me thinketh I can do it even ſince.

There came through Cheapſide two droves of Cattell: in the firſt was 848 ſheep, and in the ſecond was 186 other Beaſts.

Thoſe two ſumms I muſt write as you taught me, thus: Then if I put the two firſt Figures together, ſaying, 6 and 8, they make 14. That muſt I write under 6 and 8, thus:

848
186
―――
14

Maſter. Not ſo: and here you are twice deceived. Firſt in going about to adde together two ſummes of ſundry things, which you ought not to doe except you ſeek onely the number of them, and care not for the things: For the ſumme that ſhould reſult to that Addition, ſhould be a ſumme neither of ſheep nor of other beaſts, but a confuſed ſumme of both. Howbeit ſometimes ye ſhall have ſummes of divers Denominations to be added, of which I will tell you anon:

anon: but first I will shew you where you were deceived in another point, and that was in writing 14, which came of 6 and 8 under 6 and 8, which is unpossible: for how can two figures of two places be written under one figure and one place?

Scholar. Truth it is, but yet I did so understand you.

Master. I said indeed, that you should write that under them that did result of them both together: which saying is alwayes true, if that summe doe not exceed a Digit: But if it be a mixt number, then must you write the Digit of it under your figures, as you have said before: and if it be an Article, then write 0 under them, and in both sorts you shall keep the Article in your minde; and therefore when you have added your second Figures, which occupy the place of tens, you shall put that one thereto, which you kept in your minde; for though it were ten indeed, yet in that place it is but as one, because that every one of that place is tenne, for that it is the place of tens. And in like manner, if you have in the second place so great a number that it amounteth above 9, then write the Digit, & reserve the Article in your minde ever adding it to the next place following, and so of all other places, how many soever you have. And if you have a mixt number when you have added your last Figures, then write the Digit under the last Figures, and the Article in the next place beyond them: so shall your number resulting of Addition, have one place more then the numbers which you shall adde together.

A place.

Scholar. Now do I perceive you, and the reason of this is, (as I understand) because that no one

one place can contain above 9, which is the great-eſt Figure that is, and then all tens or Articles muſt be put to the next place following: for every place (as I may ſee) exceedeth the other place next before him by 10.

Now, if it pleaſe you, I will return to my example of Cattell. But I remember you ſaid I might not adde ſummes of ſundry things together, and that I may ſee by reaſon.

Maſter. Truth it is, if you ſeek the due ſumme of any thing, but if you only ſeek a bare ſumme, and have no reſpect to the thing, then were it better to name the ſumme onely without any thing: as in ſaying 848 wit out naming ſheep or any thing elſe, And likewiſe 186, naming nothing.

Now let me ſee how you can adde thoſe two ſummes.

Scholar. I muſt firſt ſet them ſo that the two firſt figures ſtand one over another, and the other each one over his fellow of the ſame place; then ſhall I draw a line under them both. And ſo likewiſe of other figures, ſetting alwayes the greateſt number higheſt, thus as followeth.

Then muſt I adde 6 to 8, which make 14, that is a mixt number, therefore muſt I take the digit which is 4, and write it under 6, and 8, keeping the Article 1 in my minde thus:

848
186
―――
4

Next that, I doe come to the ſecond figures, adding them together, ſaying, 8 and 4 make 12, to the which I put the one reſerved in my minde, and that maketh 13, of which number I write the Digit 3 under 8 and 4, and keep the Article in my minde, thus:

848
186
―――
34

Then come I to the third figures, ſaying, 1 and

and 8 make 9, and 1 in my mind maketh 10. Sir, shall I write the Cypher under 1 and 8?

Master. Yea.

Scholar. Then of 10 I write the Cypher under 1 and 8, and keep the Article in my mind.

Master. What needeth that, seeing there follow no more figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last Figures I must write the Cypher under them, and the Article in a further place after the Cypher thus:

848
186
1034

Master. So now you see, that of 848, and 186 added together, there amounteth 1034.

Scholar. Now I thinke I am perfect in Addition.

Master. That will I prove by this example. There are two Armies of Souldiers: in the one are 106860, and in the other 9400. How many are there in both Armies say you?

Scholar. First, I set them one over another, beginning with the first number on the right hand, thus:

106800
9400

But the nether number will not match the over number.

Master. That forceth not.

Scholar. Then do I adde 0 to 0, and there amounteth 0, that must I write under the first place thus:

106800
9400
0

Master. Well said.

Scholar. Then likewise in the second place I adde 0 to 0, & there ariseth 0, which I write under the second place thus:

106800
9400
00

Then

Then I come to the third place, saying, 4 and 8 make 12, of which I write the Digit 2, and keep the Article 1 in my mind, thus:

106800
9400
200

Then I adde 9 to 6, which make 15, to that I adde the Article 1 that was in my minde, and it is 16, I write 6 under 6 and 9, and keep 1 in mind, thus:

106800
9400
6200

Master. Why doe you not write both figures, seeing you are come to the last couple of numbers.

Scholar. Nay, reason sheweth me, that I must adde that Article that is in my mind unto the next Figure of the over summe, though there be no more in the nether summe.

Master. That is well considered: then do so.

Scholar. Then say I, 0 in the over summe, and 1 in my minde maketh 1: that write I under 0. Then followeth there yet one more in the over summe, which hath none to be added to it, for there is none in the nether summe, nor yet in my mind: therefore I thinke I must write that even as it is.

Master. Yea.

Scholar. Then doth my whole summe appear thus:

1
106800
9400
116200

Master. If you mark this you have learned perfectly the common Addition of all summes which are of one Denomination: so that ye observe this also, that in Addition you must have two numbers at the least: or else how can you say that you doe adde? And ever let the greatest number bee written highest, for that is the best way; though it be not necessary.

And forget not this, that (if you have many numbers

to adde together) you shall have oftentimes an Article of a greater value then 10, sometimes 20, sometimes 30, sometimes more, yea (peradventure) 100, Therefore as you did with the Article 10, so do with them, reserving them in your minde, and adding to the number next following so many, as their valuer or value certain is: that is to say, 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120, and so forth of other like. So that if the Article be 130, then must you set down the 0 and kéep 10 in minde, to be carried to the next row of Figures or place, if any such happen to come. For your better understanding take this example for all. I would adde these thirteen summes into one, which I set after this manner: then doe I beginne and gather the summe of the first row of figures, which come to 107, (for I take 9 there tenne times, and that is 90) then 9 and 8 is 17, that is in all 107, of which summe I write the 7 under the first row of Figures, and then for that 100 is tenne tennes, I kéepe tenne in mind, which tenne I must adde unto the next row of Figures, which are in the second place: ————————

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
389

which second row of figures (when they are added together with that tenne that I had in my minde) make in all 125, of which summe write the Digit 5 under the second row, and then (for that 120 containeth twelve tennes) I kéep twelve in minde to be added to the third place or row of figures; which being added together, make in all 60, the Cypher 0 I set down under the row of figures in the third place:

And

And the Figure 6 I keep in minde to be added to the row of Figures in the fourth place, which (when they ware added together) make 29. The Figure or Digit 9 I set downe under the fourth place. And because it is my last worke, I set downe the 2 also that I have in my minde to the 9 in the fifth place; so these summes doe make in all 29057.

¶ But (for your more ease in worke) when you have an Addition of so many summes to be added together, you were best part that summe into two or thrée parts, and worke them severally, and so put their Additions together, and this were the best thing you could doe when over many summes fall to be added.

Scholar. This séemeth somewhat hard, by the reason of so many numbers together.

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
389
29057

Howbeit, I thinke (if I do often prove, even with the same example, either by working of it alone, or else by parting it as you said even now) that I shall be able to do so shortly with any other summe.

Master. So shall you. For it is often practice that maketh a man quick and ripe in all things: but because, as well in great summes as in small there may chance to be some errour, I will teach you how you shall prove whether you have done well or no.

Scholar. That were a great help and ease.

Master. Begin first with the highest number, and then to all the other orderly, and adde them together, not having regard to their places, but as though The proof of Addition.

though they were all Unites : and still (as your number encreaseth above 9) cast away 9. Then goe forth, ever casting away 9 as often as it amounteth thereto : and so doe till you have gone over all the numbers that you intended first to adde ; and whatsoever remaineth after such Addition and casting away of 9, write it in some void place by the end of a line, for the better remembrance : and thus is the first place of your worke proved. Then secondly, put together the Figures that result of the Addition under the line, still casting away 9 also. And then that that remaineth write at the other end of the line ; and if those two Figures be like, than have you well done, but if they be unlike, then have you missed. As for example, in this present summe. The first Figure of the over line is 9, let him goe, then 8 and 8 is 16, take away 9, there resteth 7, and adde that 7 to 4 that followeth, and it maketh 11 from which if you take 9, there resteth 2. Then come to the next row, whose first and second numbers are 9, therefore overpasse them both, and take the 5 to the 2 which did remain in the first row, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next unto that, goe to the third line, whose two first numbers you may let passe, because they are nines : then take the two Figures of 2, which (with the other two that remained, in the second row) make 6. Then goe to the fourth row, whose two first numbers let goe, and take the 6 to the 6 that remaineth, and that maketh 12 : take away 9, and there resteth 3, which with the 3 that is next, maketh 6. And so goe through all the other

other numbers, and you shall finde that there remaineth 5, after you have cast away 9, as often as you can finde it: therefore write 5, at the end of the line in a void place thus:

5———— ——

Then gather all the Figures of the Totall summe, which is under the lowest line, and cast away 9 as often as you can find it; as thus, 7 and 5 make 12, take away 9. there resteth 3. to that if you adde the 2 that is last, (for you may omit the 9) then doth it make 5, which 5 you must write at the other end of the line that you made in the void place, thus:

5———— ————5

And then you see that these two Figures be like, whereby you may know that you have done wel', and so you may prove in any other.

Scholar. (If it please you) I will prove in another summe.

Master. With a good will.

Schol. Then will I take one of your former examples, which was this.

First in the highest line 8 and 6 make 14, then 9 taken away, there remains 5. to which I adde the 1 that followeth, and that maketh 6: then come I to the second line, where I finde first 4, which with 6 maketh 10, from that I take 9, and there resteth one, the next Figure is 9, and therefore I let him alone, so finde I 1 remaining, which I set at the end of a line, thus:

```
106800
  9400
______
116200
```

1———— ————

Then I come to the Totall summe, and there I

finde that all the Figures put together make 10, from which I take 9, and there resteth 1 also, which I put at the other end of the line, thus:

1 ——————— 1

And because they be like, I know that I have well added.

Master. So you know now both how to adde two summes or more together, and also how to prove whether you have done well or no: and now I will teach you how to adde summes of divers Denominations together: which thing can never be but when the one Denomination is such that it containeth the other certaine times. And yet you shall adde them to the other, not after this sort (as you did them that were of one Denomination) but after such a sort as I will now shew you, that is to say:

Addition of numbers of divers Denominations.

If you have a summe of divers Denominations, then looke that you set every Denomination by himselfe, with some note or figure of his Denomination, as they are wont to be written. Then write your other summes so under that first, that every one bee set under the other of the same Denominations: As for Example, if your Denominations bee pounds, shillings, and pence, write pounds under pounds, shillings under shillings, and pence under pence: and not shillings under pence, nor pence under pounds.

Scholar. Now that you have spoken it, me thinketh it needeth not to warne me of it, for it were against reason so to confound summes: but yet if you had not spoken of it, peradventure I shou'd have been deceived in it.

Master. If you doe say it is plain, I will speak no

no more of it, but with an example make the matter to appear evidently.

First, one man oweth me 22 l. 6 s. 8 d. another oweth me 5 l. 16 s. 6 d. and another oweth me 4 l. 3 s. I would know what this is all together: Therefore must I first set down my great summe, and then the other, every one under his Denomination agreeing to the greatest summe, as here you see with a line under them.

li.	s.	d.
22	6	8
5	16	6
4	3	0

Then must I begin at the smallest numbers (which must alwayes be set next to the right) and adde them together: and if the summe will make 1 or 2 or 3 of the next Denomination, then must I keep it in my minde till I come to that place, and under that first place must I note the residue (if there remain any of the same Denomination) but if there remaine none, then need I to write under it nothing. And this is all that you must marke in this Addition: for all other things are like to the manner of Addition before mentioned: *Therefore the chiefest point of this Addition is, to know the values of common Coines, and rated summes,* As how many shillings be in a pound, how many pence in a shilling, of which (and of other like things) I will instruct you hereafter in teaching of Reduction: But now I may not disturbe your wit from the thing that we are about.

Therefore let us returne to that former example which I purposed of the Debtors: which summes when I had set orderly, they stand thus with a line under them.

Then to adde them into one ſumme, I muſt begin at the right hand where the ſmalleſt Denomination is, and adde them together, firſt ſaying, 6 and 8 make 14. Now ſeeing theſe 14 are pence, which contain one ſhilling and 2 pence: the 2 pence I ſet down under the line of pence, and the one ſhilling I keep in my minde to carry to the next row being the place of ſhillings.

li.	s.	d.
22	6	8
5	16	6
4	3	0
		2

Then doe I adde the ſhillings together, ſaying, 1 in my minde and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the ſecond place which ſtandeth for 10, make 26, which is 1 pound 6 s. The 6 s, I ſet down under the place of ſhillings, as appeareth in the example. And the 1 pound I keep to carry to the pounds.

li:	s.	d,
22	6	8
5	16	6
4	3	0
	6	2

Then come I to the pounds, adding them all together, ſaying, 1 that I keep and 4 make 5, and 5 make 10, and 2 make 12. The Figure or Digit 2 I ſet downe right under that place or row of pounds where I gather them, and the Article 1 I keep to carry to the next place, ſaying, 1 in my minde & 2 is 3, which 3 I ſet downe directly under the 2. And then appeareth my whole ſumme thus.

li.	s.	d.
22	6	8
5	16	6
4	3	0
32	6	2

And thus muſt you doe with any ſuch like ſummes whatſoever, whether they be money, weight or meaſure, which (if you practiſe divers ſummes) you

you ſhall be well acquainted with the feat of Addition.

But now can you tell how to proue this Addition, or ſuch other like of diuers Denominations, and to try whether you haue well done or no?

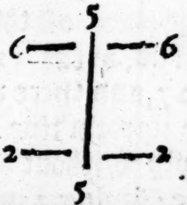
Scholar. I would I could.

Maſter. That ſhall you doe by this means: You muſt make a Croſſe which ſhall haue as many lines as you haue ſundry Denominations in your Addition: As if you haue but two Denominations then you may take it thus: that the over part and nether part may ſerve for one Denomination. And

Proof of Addition of divers Denominations.

if you haue thrée Denominations (as pounds, ſhillings, and pence) then muſt you make thrée lines, thus: The upright line may ſerue for pounds, and the highest thwart line for ſhillings, and the loweſt for pence: as for example, the ſumme which we laſt wrought.

l.	s.	d.
22	6	8
5	16	6
4	3	0



For the proof of which, becauſe it containeth 3 Denominations, I muſt make a croſſe of 3 lines, as in the example before. Then I reckon firſt at the right hand the penee, 6 & 8 make 14, from which I take 12 for the next Denomination, that is to ſay, a ſhilling, and there reſteth 2, which I muſt

write

write at one end of the nether thwart line.

After that I gather the summe of the shillings 3, 16, 6, which maketh 25, to whom I put 1 that I took of the pence and that maketh 26, from those I take 20. the quantity of the next greater Denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds, 4, 5, and 2, which make 11. to them I adde the one that came of shillings, and they make 12, from whence I cast 9, and there resteth 3, that three I joyne to the 2 in the next place, and they make 5, which 5 I set at the Cross also. And thus is my first part of my worke proved.

That done I come to the totall summe under the line, and examine it, beginning at the pence, where I finde but 2, and cannot take 9 from him: therefore I set him at the other end of the nether thwart line: then I come to the shillings, where I finde onely 6, which (because it is lesse then nine) I set it at the other end of the line of the shillings, that is, the overmost thwart line.

Last of all, of the 32 li. I take three times 9, which is 27, and there remaineth 5, which I write under the upright line: either else I may reckon them simply without any respect of their valuation or place: saying, 2 and 3 make 5, which because it is lesse then nine, I set under the upright line as before. Then I consider every number, comparing it to the number that is against it: and because I finde them to be every one like his match, I know that I have well done.

Scholar. This Crosse I perceive doth serve for

theſe 3 Denominations, pounds, ſhillings, pence: but what if I had l. s. d. ob. and qd.

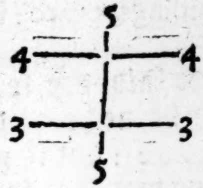
Maſter. Theſe lines, as I have ſaid, doe ſerve for 3 Denominations, ſuch as they be, as here 3 doe ſerve for pounds, ſhillings, and pence: but if you have no pounds in your ſumme, then may they ſerve for ſhillings, pence, and half penies: yea, for d. ob. and q. or in weight for C. q. and l. or in meaſure for Elles, Quarters, and Nailes, if you have no greater Denomination: ſo that you remember that the upright line ſerveth for the greateſt Denomination, and the higheſt thwart line for the next, and the loweſt for the leaſt.

And ſo if you have foure Denominations, you muſt make your croſſe with ſo many lines: And if your ſumme be of more Denominations, make ſo many lines in your croſſe. And thus will I make an end of Addition, ſaving that here (for the better underſtanding of this Rule) I have ſet you down certain examples both of money, weight, and meaſures with their works and proofs.

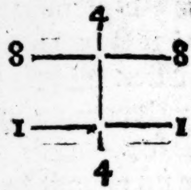
Examples of Addition.

li.	s.	d.	li.	s.	d.
23	10	4	130	17	10
45	6	8	28	6	8
37	2	9	13	13	4
25	13	6	120	0	0
131	13	3	292	17	10

The

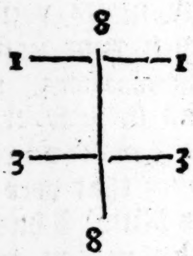


The proofs.



C.	q.	li.
34	1	3
12	2	2
7	3	4
13	0	13
67	2	23

yards	q.	nayles.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3



Subtraction.

Then have I learned the two first kindes of Arithmetick : *now (as I remember) doth follow Subtraction , whose name (me thinketh) doth sound contrary to* Addition.

Subtraction.

Master. So it is indeed : for, as Addition increaseth one grosse summe, by bringing many into one : so contrariwise, Subtraction diminisheth a grosse summe by withdrawing of other from it. *So that*

that Subtraction *or Rebating is nothing else but an* Art *to withdraw and abate one summe from another, that the remainer may appear.*

Scholar. What doe you call the Remainer?

Master. That you may perceive by the name.

Scholar. So me thinketh: but yet it is good to aske the truth of all such things, lest in trusting to mine owne conjecture, I be deceived.

Master. So it is the surest way. And, as I see cause, I will still declare things unto you so plainly, that you shall not need to doubt. Howbeit, if I doe overpasse it sometimes, (as the manner of men is to forget the small knowledge of them to whom they speake) then doe you put me in remembrance your selfe, and that way is surest.

And as for this word that you last asked me, take you this description: The Remainer is a summe left after one Subtraction made, which declareth the excesse or difference of the two other numbers, as if I would abate or subtract 14 out of 18, there should remain 4, which is called the Remainer, and is the difference betweene those two numbers 14 and 18. Remainer.

Scholar. I perceive then what Subtraction is: now resteth to know the order how to work it.

Master. That shall you doe by this meanes. ☞ First, you must consider, that if you should goe about to rebate, you must have two sundry summes proposed: the first, which is your grosse summe, (or summe totall) and it must be set highest: and then the rebatement (or summe to be withdrawn) which must be set under the first, (whether it be in one parcell or in many) and that in such sort, that the

the first figures be one just over another, and so the second and third, and all other following, as you did in Addition: then shall you draw under them a line, and so are your summes duely set to begin your working.

Then begin you at the right hand (as you did in Addition) & withdraw the nether number out of the higher, and if there remain any thing, write that right under them beneath the line: and if there remain nothing (by reason that the two Figures were equall) then write under them a Cypher or nought: And so doe you with all the other Figures, evermore abating the lower out of the higher, and write under them the Remainder still, till you come to the end: And so will there appear under the line what remaineth of your grosse summe after you have deducted the other summe from it, as in this example.

I received of your Father 48 s, of which I have laid out for you 36 s. now would I know what doth remaine. And therefore I set my number thus in order. First, I write the greatest summe, and under him the lesser, so that the Figures at the right side be even one under another, and so the other, thus.

48
36

Then doe I rebate 6 out of 8, and there resteth 2, which I write under them right beneath the line thus.

48
36
2

Then I goe to the second Figures, and do rebate 3 out of 4, where there remaineth 1, which I write under them right, and then the whole sum and operation appeareth thus.

48
36
12

Whereby

Whereby it appeareth, that if I withdraw 36 out of 48 there remaineth 12.

Scholar. Now I will prove in a greater summe, and I will subtract 2367924 out of 3468946, those summes I set in order thus:

3468946
2367924

Then doe I begin at the right side, and deduct 4 out of 6, and there resteth 2, which I write under them. Then go I to the second figures, and withdraw 2 out of 4, and there remaineth 2, which I set under them also, then I take 9 out of 9 and there resteth 0, which I write under them (for you say, that if the Figures be equall, so that nothing doth remain, I must write the Cypher 0 under them.)

Master. It was well remembred: now go forth.

Scholar. Then I come to the fourth place, and draw 7 out of 8, and there remaineth 1. which I write under them also. Then in the fifth place, I take 6 out of 6, & there resteth 0, (for it I write under them the Cypher 0.) Then in the sixth place 3 rebated from 4 there remaineth 1, which I write under them, and likewise in the seventh and last place, 2 taken from 3 there is left 1, which I write under them: so have I done my whole working, and my summes doe appeare thus. Whereby I see, that (if I doe rebate 2367924 out of 3468946) there remaineth 1101022.

3468946
2367924
———
1101022

Master. This is well done. And that you may be sure to perceive fully the Art of Subtraction, let me see how you can subtract 52984732 out of 8250003456.

Scholar.

Scholar. First, I set downe the greatest summe, and after that I will write under it the lesser number, beginning at the right side, and then my Figures will stand thus:

8250003456.
52984732

Note. Then take I 2 from 6 and the rest is 4, which I write under them. Then do I withdraw 3 from 5, and there remaines 2, which I write under them. Then take I 7 out of 4, but that I cannot, what shall I now do?

Master. Marke well what I shall tell you now, how you shall doe in this case, and in all other the like: If any figure of the nether summe be greater then the figure of the summe that is over him (so that it cannot be taken out of the figure over him) then must you put 10 to the over figure, and then consider how much it is, and out of that whole summe withdraw the nether figure, and write the rest under them. Can you remember this?

Scholar. Yes, that I trust I shall. Now then in mine example where I should have taken 7 out of 4, and could not, I put 10 to that 4, which maketh 14, from it I take away 7 and there resteth 7 also, which I write under them.

Master. So have you done well: But now must you marke another thing also: that (whensoever you do so put ten to any Figure of the over number) you must adde one still to the figure or place that followeth next in the nether line: as in the example there followeth 4, to which you must put 1, and make him 5, and then go on as I have taught you.

8250003456
52984732

018724

Scholar.

Scholar. Then shall I say, 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3, but that cannot be; therefore I must put to it also 10, and then it will be 13, from which I take 5, and there resteth 8 to be written under them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the nether line, and that maketh 9, which I should take out of 0 and cannot; therefore I put thereto 10, and that maketh 10, from 10 I take 9, & there remaineth 1, which I write under them.

Thus do I adde 1 likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure above, but I cannot, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written under them.

Then come I to the next figure which is 2, and to him I doe adde 1, which maketh 3 that 3 I cannot take out of nought, therefore of that nought I make 10, and thence doe take 3, so there remaineth 7 to be written under them: likewise doe I put 1 to 5, which make 6, that 6 I cannot take out of 5, therefore I adde 10 to that 5 & make it 15, from which I rebate 6 there remaineth 9, which I write under them. Now have I spent all the nether Figures, and what shall I do more?

8250003456
52984732
8197018724

Master. You should have added one to the next figure following (if there had béen any) because you added 10 to the last figure before of the over line: but being there is no figure following, you

must adde that one to the place following, and then deduct that one from the number above.

Scholar. Then shall I say, because I borrowed 10 to the over 5, I must put 1 in the next place beneath, that is under 2, then must I subtract that 1 from 2, and there resteth 1 to bee written under that in the ninth place. Now I have no more to subtract, for there is not any figure remaining beneath, neither yet any unite to be added, because I borrowed not 10 to the figure last before: and yet is there 8 remaining in the over line, which I thinke (by reason) should be set at the end of the Figures in the lowest row, which is under the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so indeed,

Scholar. But Sir, I beseech you, shall I alwayes when any number so remaineth alone, as this 8 did, write him under the line straight against his own place?

Master. Yea, what else? whether they bee one or many: & this well remembered, you have sufficiently learned Subtraction; Howbeit, because of certain things that might deceive you, if you did not take good heed to your working, I will propose to you another example of many numbers to be subtracted as thus: I received of a friend of mine to keep 2869 Crowns, of which at one time I delivered him again 500, at another time 368, at another time 440, at another time 80, and at another time 64, now would I know how many doe rest behinde.

Therefore first I set down my grosse sum

2869

2869 Crowns received. and underneath it I set all the parcells thus, and under them a double line.

500
368
440 delivered.
80
64

Then first I begin at the first place, and gather together the sum of all those lines (save the overmost) in the first figures, & so I doe with all the figures of the second place, & so forth, as I did in Addition, save that I leave out the highest row of numbers (as the line warneth me) that sum so gathered between the double line, is the sum delivered in all: which summe I doe afterwards subtract out of the highest row of numbers, & the remainder doe I set under the nethermost line: as for example.

2869 *Crownes received.*
500
368
440 *Delivered.*
80
64

1452 *Delivered in all*

1417 *Rest behinde.*

I set the summes as before: then do I gather the first figures of all the places delivered together: where I finde but 4 and 8, that maketh 12, (for three Cyphers increase no sum in Addition, as you learned before,) of the 12 therefore doe I write the Digit 2 between the double line and keep the Article in my minde, till I come to the second place, where I finde 6, 8, 4, 6, that maketh 24, to them I put the Article in my minde, and it is 25, 5 of which I write under the second place, and keepe the Digit 2 in my minde for the third place, where I finde 4, 3, 5, that makes 12, to the which I adde the 2 in my mind, and it maketh 14, thereof I write the 4

under the third place, and because there remains no more Figures to be added, I write the Digit in the fourth place, as you see in the Example, and so it appeareth, I have delivered in all a thousand foure hundred fifty two Crowns.

Then come I to the subtracting of this summe between the lines, for by Addition it is equall to the five parcells over it, therefore I procéede to subtract it from the overmost summe, saying, 2 from 9 remains 7 to be written under them beneath the lowest line. Then in the second place I take 5 from 6 and there resteth 1 to be written under them. Then in the third place, 4 from 8, resteth 4. Last of all in the fourth place, 1 from 2, remaineth 1, And thus I sée that after those five sums are subtracted from 2869, the Remainer is 1417.

Scholar. This I perceive: but is there no shorter way and more spéedy?

An abridgment of the former manner of Subtraction.

Master. Yea, when you are a while exercised in it: for you may (as fast you can gather the numbers together) withdraw them out of the highest sum. But if in quantity those numbers added together, excéed the highest sum, or upper number, then shall you (as before hath béen taught) imagine to borrow 10, 20, or 30 moe, as néed shall require, and put them to the upper number, to help to further the abatement, reserving or restoring the Articles that you borrowed to the next place again: and so still goe forward till you have ended your worke: as for example. In the last summe proposed, I gather first in the first place 4 and 8 that maketh 12, which 12 I should deduct or take out of 9 in the upper number above the line,

line, but I cannot: and therefore I adde unto 9 an Article of 10, and make the upper number 19, from whence I take 12, then there resteth 7, then for the Article 10 I adde to the next place of money delivered; saying, 1 that I bring and 6 make 7, & 8 make 15, & 4 make 19 & 6 make 25, which 25 I would take out of 6 in the upper number, but I cannot, wherefore I adde 2 tens, or 20 unto 6 in the upper number, & that maketh 26, then 25 out of 26 resteth 1; then the tens which I borrowed, or have in minde, I adde to the next row or sum delivered; saying, 2 that I bring, and 4 make 6, and 3 make 9, & 5 make 14; then 14 out of 8 I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to be added, the one that I borrowed, or have in minde, I rebate from 2 in the upper line, & there remaineth 1, which I set down in the remainer line: & so my sum appeareth (as before) to be 1417 Crownes.

Loe thus have you now a shorter way.

Scholar. I like both wayes well: & I perceive both well: yet, as in one the working seemeth somewhat long, so in the other it leaveth very much (mee seemeth) to remembrance, and therefore may cause errour quickly, except a man have a quick and an exercised remembrance. But yet for the sharpning of my wit by your patience (if you will give me leave) I will try what I can do in a like sum, to work it the shortest way: whereupon I would subtract out of 40301964, these three parcels. Therefore I set them first in one order: then I gather the parcels of the first place, which are 8, 2, 1. that is 11,

40301964	*Charge.*
20003428	*Discharge.*
10002432	
10101461	
43	

which

which I should take or deduct out of 4, which is over him, but I cannot: therefore I adde an Article, or one tenne to 4 which maketh 14 then 11 out of 14, there resteth 3 to bée written under the first place betwéen the two lines.

Then come I to the second place, saying, 1 that I borrowed to have in my minde, and 6 make 7, and 3 make 10, and 2 make 12, which I cannot take from 6, therefore I adde 10 to 6, which maketh 16, and then 12 from 16, resteth 4, which I write under the second place betwéen the two lines.

Then come I to the third place saying, 1 that I borrowed or have in mind and 4 make 5, and 4 is 9, and 4 make 13, which I should take out of 9 that is over them but I cannot: therefore I adde 10 to 9 which make 19, then 13 out of 19, rest 6.

Then come I to the fourth place, saying, 1 in minde and 1 is 2, and 2 is 4 and 3 make 7, which because it cannot be taken from 1, I take it from 11, and there resteth 4.

After that I come to the fifth place, where are onely thrée Cyphers, which make nothing, unto which I adde 1 in minde, then should I take that (that is to say) 1 from the Figure over them, which is also a Cypher: therefore I say thus, I cannot take 1 from 0, but 1 from 10 remaineth 9: so must I write 9 under them. Then in the sixth place I finde but 1, and 1 in minde make 2, which I take out of 3 over him, and the remainer is 1: that must be written betwéen the two lines in the sixth place. So I goe to the seventh place where I finde onely Cyphers, and in the grosse

summe

ſumme over them a Cypher alſo: therefore muſt I write the remainer (which is nothing) with a Cypher alſo. Then in the eight and laſt place I gather 1, 1, 2, that maketh 4, which if I take out of that 4 that is over them, there will nothing remaine. And that muſt be noted with a Cypher betwéen the two lines (as I have often ſaid) and ſo have I ended my worke, and the figures ſtand as followeth.

Scholar. But Sir, I remember you taught me that Cyphers ſhould not come in the laſt place, for becauſe they ſerve onely to increaſe the value of other Figures which follow them and ſerve not thoſe figures that goe before them: and now in my Example I have ſet two Cyphers in the two laſt places.

Maſter. I commend you for your remembrance. And truth it is, you ſhould not have ſet them here, but onely becauſe that I would make you plainly to perceive the Art of Subtraction. Therefore ſéeing that you doe now perceive it, whenſoever you would write down a Cypher, looke whether any other figures be yet behind: and if not, then let go the 0 alſo, for it néedeth not to write him in the latter places, where no other figure doth follow, except it be (as I did now ſuffer you) to teach the uſe of Subtraction the plainer.

Therefore your figures muſt ſtand thus when the worke is ended.

40301964	*Charge.*
20003428	
10002432	*Diſcha.*
10101461	
0164643	*Reſt.*

Scholar. Sir, I do thinke with that you taught me before, and by theſe two ſummes that you taught mée

last also, that now I could Subtract any summe.

Master. So may you, if you have marked what I have taught you. But because this thing (as all other) must be learned surely by often practise, I will propound here two Examples to you: wherein if you often exercise your selfe, you shall be ripe and perfect to subtract any other summe lightly; for in them is contained all the observances of whole numbers. And because you shall perceive somewhat both how to doe it, and also whether it be well done when you have proved to doe it; therefore have I written under them both the Remainers.

30606.	*Lent.*	308964.	*Debt.*
10354		103145	
10249	*Paid.*	102597	*Paid.*
163		101024	
20766	*Paid in all.*	02198	*Rest.*
9840	*Rest to pay.*		

Scholar. Sir, I thanke you: but I thinke I might the better doe it, if you did shew mee the working of it.

Master. Yea, but you must prove your selfe to do some things without my aid, or else you shall not be able to doe any more then you are taught: And that were rather to learne by wrote (as they call it) then by reason. And again, there is nothing in these examples, or any other of whole numbers,

but

but I have taught you the rules of them already.

Scholar. Then I truſt by practiſe to attain the uſe of it. And is this all that I ſhall learne of Subtraction ?

Maſter. Yea, ſaving that (as you have ſéen in Addition) there are numbers of divers Denominations, in which the working is not much unlike : yet (without ſome inſtructions be given of it) it might ſéem to a learner more difficult then indéed it is. Therefore I will briefly ſhew you the uſe of it onely by an example or two.

A certain man owed to mee 14 l. 12 s. 8 d.
of which he paid me at one time 4 l. 6 s. 8 d.
at another time 3 *l. at another* 2 l. 3 s. 4 d.
and laſt of all 6 s. 8 d.

Now would I know what remaineth unpaid yet ? therefore I ſet my ſums thus, every one in their due place : As pounds under pounds, ſhillings under ſhillings, pence under pence.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
-	6	8

Scholar. Sir, I pray you why do you write 2 l. for the common ſpéech uſeth rather to ſay, 40 s.

Maſter. We muſt here uſe the Denomination that is greateſt in any ſumme, ſo that we may not write according as wée uſe to ſpeake, ſaying, 16 d. 18 d. or likewiſe 7 groats, 8 groats, 24 s. 40 s. 48 s. and ſuch other ; but we muſt write every Denomination that is in any ſumme by it ſelf. *Note how the penne differeth from the common order of Counters.*

Namely, ſhillings and pounds. So muſt we write for the laſt ſummes now named, 1 s. 4 d. 1 s. 6 d. 2 s. 4 d. 2 s. 8 d. 1 l. 4 s l. 2 l. 8 s. and ſo forth of other like.

Scholar.

Scholar. So that wée may not write in Arithmetick, pence, when the summe amounteth to shillings, nor shillings, when the summe maketh pounds, Now (if it please you) end your example.

Master. When my summes are so set as I shewed, then (according to the rules of Addition) I gather all the particular summes which bée paid mée into one totall summe, directly to be set under them betwéen the two lines, not medling with the 14 l. 12 s. 8 d. as the line warneth mee: therefore must I beginne with the smallest Denomination, saying, 8, 4, 8, is 20, pence, which maketh one shilling and 8 pence, the 8 d. I set downe under the place of pence, and the one shilling I kéep in minde to carry to the next Denomination of shillings: Then come I to the shillings & say one that I bring or have in minde, & 6 is 7, and 3 is 10 & 6 makes 16, which because it containeth not one pound, I set directly under the place of shillings. Then come I to the pounds, whose parcels are 2, 3, 4. that is in all 9, that 9 doe I set down directly under the pounds: And so the totall or whole Addition of all the particulars paid, amounteth to 9 l. 16 s. 8 d.

li.	s.	d.	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8	
4	16	0	*Rest.*

Now for the worke of Subtraction, I must rebate that totall summe of Addition out of the highest number, that is to say from the 14 l. 12 s. 8 d.

There-

Therefore to perform the worke I ſay, 8 d. out of 8 d. remaineth or reſteth nothing, therefore in the place of the reſt or remain, right under the Denomination, I ſet down 0. Then coming to the ſhillings, where I finde 16. which ſhould be taken out of 12, but I cannot: therefore I imagine to borrow 1 out of the next Denomination, that is, of the 14 l. and put that one pound ſo borrowed unto 12 s. that maketh 32 s.

Now 16 s. out of 32 s. reſteth 16 s. which 16 s. I ſet down directly under the place of the reſt.

Laſtly, comming to the pounds, ſaying, one pound in minde that I borrowed, and 9 make 10, then 10 out of 14. there reſteth 4.

So doth my whole reſt or remain appear to be 4 l. 16 s. 0 d.

This I account the eaſieſt way for a young beginner to practiſe, though it be ſomething long.

Scholar. Is there any ſhorter way for this work alſo?

Maſter. Yes, as in this laſt Example I will alſo ſhew you, for you may adde together the particular ſums as they are ſet in order, beginning with the pence, ſaying, 8, 4, 8, make 20 d. which 20 d. you ſhould take out of the 8 d. above the line, but you cannot, therefore ſhall you borrow 1 of the next Denomination, that is to ſay 1 of the ſhillings, and put it to the 8 d. that maketh 20 d.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

now 20, out of 20 d. reſteth 0, which Cypher I ſet

down

down directly under them. Then one shilling that I borrowed or had in mind, and 6 make 7, and 3 make 10, and 6 make 16, the 16 out of 12 I cannot take, therefore of the next Denomination I doe borrow one l. and put it to 12 s. which maketh 32 s. then 16 s. out of 32 s. resteth 16 s.

Lastly I come to the pounds, saying, 1 l. in minde, or that I borrowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remainer or rest appeare as before to be 4 l. 16 s. od.

Scholar. Then doe I perceive very well: and if there be no other thing to bee learned in Subtraction, then may I come to Multiplication, for that you reckoned to be next in order.

Master. Wee have done indeede with the Art of Subtraction, as touching the working.

Proof of Subtraction.

But yet before we go to Multiplication, I will instruct you how to examine your worke, whether it be well done or not. For the performance whereof, if you marke what I sayd in Addition, you may easily perceive what is to be done for the proofe of Subtraction, which is best made by the aide of Addition thus.

Draw under the lowest number (which is your Remainer) a line, and then adde this Remainer and all the other that you did subtract before, together, and write that that amounteth under the lower line: and if the summe that cometh thereof be equall to the highest of the Subtraction, then is the Subtraction well wrought, or else not: As you may see for example in the summes set down before,

fore, & first in summs of one Denomination, whereof one was this.

Where the Number 52984732	8250003456
is subtracted from 8250003456.	— 52984732
& the Remainer is 8197018724	
Now to prove whether it be	8197018724

truly wrought or not, I adde the Remainer and the number Subtracted together, beginning at the right hand; and first I say 4 and 2 is 6 which is set under the line: Example in a sum of one Denomination.

The number given	8250003456
The number to Subtract	52984732
The Remainer	8197018724
The Proofe	8250003456

Then again in the second place I say 2 and 3 is 5, which I write under, next that in the third place, 7 and 7 are 14, of which I write the Digit 4, and keepe the Article 1 in my minde. Then in the fourth place 8 and 4 is 12 and 1 in my minde maketh 13, whereof I write downe the Digit 3, and keep the Article 1 in my minde. Again in the fifth place, 1 and 8 is 9, and 1 in my minde is 10. Whereof I set downe 0 and keepe the 1 in my minde. And so going on to the rest (as it is taught in Addition) when I have made an end, I see that the lowest line of numbers and the highest be alike: wherefore I know that I have well done.

So likewise the Prowfe is to bee made in numbers of divers Denominations: as for Example, in our summe of that kinde which in the first form of working stood thus: (all the particular numbers to be subtracted, being drawne into one)

Where,

Example in a sum of divers Denominations.

Where, in the title of pence, I find 8 & 0: the 8 I set down directly under in that of pence.

Then in the place of shillings I finde 16 and 16 which make 32 shillings, wherein is contained 1 l. and 12 s. the 12 s. I set down directly under them in the due place of shillings, & one pound I keepe.

Then comming to the pounds, I say 1 that I keepe, and 4 is 5, and 9, is 14, which 14 in due order I set downe directly under them as this Figure sheweth. And the whole summe is 14 l. 12 s. 8 d. agreeing with the upper number above. So I find the worke is good, and the Subtraction well wrought.

	li.	s.	d.
	14	12	8
	4	6	8
	3	0	0
	2	3	4
		6	8
Paid in all.	9	16	8
Rest.	4	16	0
Proofe	14	12	8

The same thing is to be done for the latter forme of Subtraction (where the particular summes are not gathered together into one grosse.) For the Remainer and all the particular summes subtracted, being added together, if the summe that commeth thereof bee equall to the highest number above, then is the Subtraction well wrought, or else not.

Example of a proof in the latter forme of Subtraction.

As for example also in the last sums which stood thus,

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

First in the title of pence, I adde 8, 4, 8, that maketh 20 d. which containeth 1 shilling and 8 pence.

The 8 I set down under the lowest line in the row or

title

title of pence, and that 1 shilling I keep to carry of the next Denomination or place of shillings.

Then returning to the shillings, saying: one in minde, or that I keep, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 shillings, which amounteth to one pound, 12 s. the 12 s. I set down under the title of shillings: and 1 pound. I keep or have in mind to carry to the next Denomination or place of pounds. Then come I to the pounds, saying, 1 that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14; then doe I write 14 under the pounds, and so have I ended the Addition: and I see that the lowest line is like unto the uppermost line in number, wherefore I know that I have welldone.

And thus have I taught you the Art of Subtraction, and the means to prove whether it be well wrought or not. Therefore now will I make an end thereof, and will instruct you in Multiplication.

Multi-

Multiplication.

Multiplication what it is.

Multiplication is an operation whereby two sums produce the third: which third sum so many times shall contain the first, as there are Unites in the second. And it serveth in stead of many Additions. As for Example: When I would know how many are 30 times 48, if I should adde 48 thirty times, it would be a long worke. Therefore was Multiplication devised, which shall doe that at once that Addition should do at many times.

Scholar. I perceive the commodity of it partly, but I shall not see the full profit of it till I know the whole use of it. Therefore Sir, I beseech you, teach me the working of it.

Multiplication of Digits.

Master. So I judge it best; but because that great summes cannot be multiplied but by the Multiplication of Digits, therefore I thinke it best to shew you the way of multiplying them. As when I say, 9 times 8, or 8 times 9, &c. And as for the small Digits, under 5 it were but folly to teach any rule, seeing they are so easie that every childe can doe it: but for the Multiplication of the greater Digits, thus shall you do.

First, set your Digits one right over the other, then from the uppermost downwards, and from the nethermost upward, draw straight lines, so that they make a crosse, commonly called Saint

Andrews

Andrews crosse, as you see here: Then looke how many each of them lacketh of 10, and write that against each of them at the end of the lines, & that is called the difference: as if I would know how many are 7 times 8, I must write those Digits thus.

The difference.

Digit difference.
8
X
7

Then doe I looke how much 8 doth differ from 10, and I finde it to be 2: that 2 doe I write at the right hand of 8, at the end of the line thus.

Digit difference.
8 2
X
7

After that I take the difference of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you see in this example.

Digit difference.
8 2
X
7 3

Then doe I draw a line under them, as in Addition, thus.

Last of all, I multiply the two differences, saying 2 times 3 make 6, that must I ever set under the differences, beneath the line: then must I take one of the differences (which I will, for all is like) from the other Digit (not from his owne) as the line of the Crosse warne me, & that is left must I write under the Digits. As in this example, if I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write under the Digits, & then there appeareth the multiplication of 7 times 8 to be 56. And so likewise of any other Digits, if they be above 5, for if they be under 5, then will there difference be

Digits Difference.
8 2
X
7 3
5 6

greater then themselves, so that they cannot be taken out of them. And again, such little sums every childe can multiply, as to say 2 times 3, or 4 times 5, and such like.

Schol. Truth it is. And seeing me seemeth that I understand the multiplying of the greater Digits, I will prove by an example how I can do it. I would know how many are 9 times 6.

Master. It is all one in value to say 9 times 6, or 6 times 9: but yet the order is best to put the lesse sum first, saying, 6 times 9, & so of all other summes.

Scholar. Then would I know how many are 6 times 9: therefore I set the Digits thus, and make the crosse, thus.

9
X
6

Then doe I set their differences from 10 at the right side, the difference of 9, which is 1, against it, and the difference of 6, which is 4, against it also, as in this example.

9	1
X	
6	4

And under them draw a line, Then doe I multiply the differences together, saying: 1 time 4 maketh 4, that 4 doe I write under them thus.

9	1
X	
6	4
	4

Then take I one of the differences from the other Digit, as, 1 from 6, or else 4 from 9, and each wayes there resteth 5, which I doe write under the Digits thus. And so appeareth the multiplication of 6 times 9, to be 54. Thus I see the reason of this manner of multiplication of Digits.

9	1
X	
6	4
5	4

Master

Master. Now might you go straight to the multiplication of great numbers, save that both for your ease and surety in working I will draw you here a Table, whereby shall appear the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seek your first or last Digit in the greater figures; and from it go right forth towards the right hand, till you come under the number of your second Digit, which is in the highest row, and then the number that is in the meeting of the rowes of little squares (which come directly from both your propounded Digits) is the Multiplication that amounteth of thē. As if I would know by this Table the multiplication of 7 times 9, seek first 7 in the greater figures, and then go right forth toward the right hand, till you come under 9 of the highest row in which place where you so come under the

other Digit (as here for example you come under 9) is alwayes contained the off-come or product, which you seek, and that place we terme to be in the common angle, in respect of the two numbers so taken on the outsides: as here in that common angle, where the rowes of little squares directly proceeding from 7 and 9 doe meet, you have 63, which 63 is the summe of the multiplication of 9 by 7.

To multiply greater summes.

Scholar. This is very good and ready. And so may I find the multiplication of any Digits: but now how shall I doe in greater summes?

Multiplier.

Tiems.

Master. *When you would multiply any summe by another, you shall mark that it is the meetest order to set the greatest* number *highest, which is the place of the* number *that must be multiplied: and likewise the lesser* number *under it, for that is the place of the* Multiplier *or* Multiplicator, *that is to say, the* number *by which the* Multiplication *is made, and is in* English *alwayes put before this word, Times: in such speaking as when I say 20 times 70. And the* number *that followeth this word Times, is that which must be* Multiplied.

Therefore when I would multiply one number by another, I must write the greatest highest, and the lesser under it, as in Addition. And under them must I draw a line. As for example, If I would multiply 264 by 29, I must set them thus.

264
29

¶ Of which number thus set downe to be multiplied, may be formed a question, as thus, There are 29 men, and each man hath 264 Lambes. The question is, how many Lambes they have in all.

To

To the performance whereof, I must multiply every figure, of the high row by every figure of the neather row: and that that amounteth I must set under the line, as thus.

First I do multiply 4 by 9, saying, 9 times 4 (or 4 times 9 which is all one) and that maketh 36, as the Table before of Digits doth declare, of that 36, I must write the 6, that is the Digit, under the 9, & the Article 3 I keep in mind to carry to the next place.

264
29
6

Then come I to the second figure of the higher row, which is 6, and say 9 times 6 make 54, and with the 3 in my mind make 57, the 7 I set downe under the 2, and 5 I keep in mind.

264
29
76

After that I come to the next figure, which is 2, and multiply it by 9, and that maketh 18, and with 5 that I have in mind, maketh 23: wherefore because it is the last worke of the Multiplier, I set it down in order as you see:

264
29
2376

And so have I ended the first figure of the Multiplier. Wherefore I give it now a fine dash with my pen.

Then begin I with the next figure, & multiply it into all the higher figures as thus,

264
29
2376

First, 2 times 4 make 8, that 8 do I write under the second place: for evermore the Digit or first figure of the Multiplication that amounteth of the figure of the higher number, must be set under the Multiplier of it, the other in their order toward the left hand.

Scholar. I underſtand you thus that the Digit of the ſumme amounting of the Multiplication of the firſt figure of the higher row, by the firſt figure of the lower row, or Multiplier, muſt be ſet under the firſt place: and that that amounteth of the ſame firſt figure by the ſecond Multiplier, muſt be ſet under the ſecond place, and ſo of the other, if there be more Multipliers.

Maſter. So meane I indeed: and if there amount but a Digit then muſt it be ſet under the Multiplier.

And now to goe forth; I multiply by the ſame 2, the ſecond figure of the higher row, which is 6, ſaying two times 6 make 12, wherefore I write the Digit 2 under the third place, and the Article 1 I keep in mind.

```
264
 29
----
2376
 28
```

Then doe I multiply the laſt figure of the higher ſumme by that ſame 2, ſaying, two times 2 is 4, and with the 1 that I have in minde maketh 5, which I write under the fourth place. And ſo have I ended the whole Multiplication: wherefore I alſo give the 2 a daſh with my pen, thus: and ſo I doe ever as ſoone as I have diſpatched any Digit by which I multiply: and the ſums ſtand thus.

```
264
 29
----
2376
528
```

Then muſt I draw a line under all thoſe ſummes that mount of the multiplication, and muſt adde all them into one ſumme, as in the Example, you may ſee.

```
264
 29
----
2376
528
----
7656
```

Where

Where in the first place I finde but 6, and therefore write I it under the line. Then in the second place 8 and 7 make 15, whereof I write 5, and keepe one in my minde, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656, which amounteth of the Multiplication of 264, by 29, and that is the just number of the Lambes that 29 men had.

Sholar. If there be no more to be observed in it, then can I doe it I suppose, as by this Example I shall prove.

¶ There is a peece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards square there is in all this peece of ground: which numbers I set downe with the greater above, and the lesser under, as you see.

```
1365
 236
----
```

Then doe I multiply 5 by 6, saying, 6 times 5 make 30, of which I write the Cypher in the first place, and the Article 3 I doe keep in minde to carry to the next place.

```
1365
 236
----
   0
```

Then do I by the same 6, multiply the second figure of the higher sum, which is 6, saying, 6 times 6 make 36 & 3 in my mind make 39, of which I write the 9 under the secōd place, & the Article 3 I keep in mind.

```
1365
 236
----
  90
```

Then doe I multiply the third figure, which is 3 by the same 6, and that maketh 18, and 3 in my minde make 21. The 1 I set down, and keep 2 in minde.

```
1365
 236
----
 190
```

Then come I to the last figure of the higher summe, and multiply it by 6, saying, 6 times 1 make 6, and 2 in my minde make 8, that 8 doe I write under the fourth place. And so have I ended the first Multiplier, and dash him sleightly with my Pen.

	1365
	236
	8190

Then begin I with the second Multiplier, and say, first 3 times 5 make 15, of which I set the 5 under the second place, because that the Multiplier is there, and the Article 1 I keep in mind.

Then come I to the second Figure that is 6, and multiply it by 3, which maketh 18, and with one in minde maketh 19, the 9 I set down under the third place, and 1 I keep in mind.

1365
236
8190
95

Then come I to the third figure, which is 3, & multiply it by 3, saying, 3 times 3 make 9, and with 1 in mind, make 10, the Cypher I set under the fourth place, and the Article 1 I keep in mind.

1365
236
8190
095

And then coming to the last figure 1, I multiply it by 3, and it maketh 3, & with the one in mind, it maketh 4, which 4 I set in the fifth place and then have I ended two of the Multipliers, and the summes stand as you may see in the latter end of the page going before, and then I give 3 his dash.

1365
236
8190
4095

Then

Then come I to the third Multiplier, and multiply it into every figure of the higher summe, and first I say, 2 times 5 make 10, of which I set the Cyhper under the Multiplier in the third place, and the Article 1 I keep in minde.

```
 1365
  236
 ----
 8190
4095
   0
```

And so multiplying the second figure 6 by that same 2, there amounteth 12, and 1 in my minde maketh 13, whereof I write the Digit 3 under the fourth place, and the Article 1 I keep in mind:

```
 1365
  236
 ----
 8190
4095
  30
```

Then do I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and the one in minde make 7, which 7 I set down under the fifth place, as appeareth by the example.

```
 1365
  236
 ----
 8190
4095
 730
```

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the sixth place, and then doth the summe stand thus.

```
  1365
   236
  ----
  8190
 4095
2730
```

And so have I ended the whole Multiplication.

But now (as you taught me) to know what this whole summe is, I must adde all those parcels together, and then under the line will appeare, as you may see, the grosse or totall summe is, 322140. Whereby I know there is so many yards square in that piece of ground.

```
  1365
   236
 ------
  8190
 4095
2730
 ------
322140
```

Master.

Master. This is well done.

Scholar. Then me thinketh I could call it well done, when I know, whether I had well done or no.

Master. It is to be proved by 9 as Addition was, but the surest proof is by Division, and therefore I will reserve that proof by Division, till you have learned the Art of Division. And anon I will shew you how it is commonly proved.

¶ *But first, for your further instruction in this exercise of* Multiplication, *I will with one example try your cunning, and so make an end: And the* question *is this. I would know how many daies it is since* the Nativity of our Lord and Saviour Jesus Christ, *unto this yeare* 1630. *Which to performe, you must* multiply *this present yeare* 1630 *by the daies in one whole yeare, which are* 365.

Scholar. Now for that you have given me so much light into the question, you shall see I will handsomely finish the work, for according to your former instructions, I set them down with a line under them thus.

$$\begin{array}{r} 1630 \\ \underline{365} \end{array}$$

Then say I, 5 times 0 is 0, which I set downe under the first place, as here appeareth. Then say I, 5 times 3 make 15, the Digit 5 I set down in the second place under 3, and the Article 1 I keep in minde to bee added to the next Multiplication. Then saying five times 6 make 30, and 1 in minde 31, the 1 I set downe in the third place, and 3 I keepe in minde. Then comming to the last Figure, I say once 5 is 5, and 3 in minde make 8, that 1 doe I set downe under the fourth place: and thus have I ended my first Multiplier, and

and therefore I give it a daſh with my pen.

Then come I to the ſecond Multiplier, which is 6, and do likewiſe multiply it into the upper number, ſaying; 6 times o is o, which I ſet down in the ſecond place, right under his Multiplier: then ſay I, 6 times 3 make 18, the 8 I ſet down under the third place, and 1 I keep in mind.

1630
365
8150
9780

Then ſay I 6 times 6 make 36, and 1 I keep in minde make 37. the Digit 7 I ſet down in the fourth place, and 3 I keep in mind: Then ſay I 6 times 1 is 6, or once 6 is 6, and 3 in minde make 9, which I ſet down next, and ſo have I ended two Multipliers: wherefore I daſh the 6 with my pen.

Then I begin to multiply the third Multiplier into the over number, ſaying, 3 times o is o; the o I ſet down in the third place right under his Multiplier. Then ſay I 3 times 3 make 9, which I ſet down in order next: then ſay I, 3 times 6 is 18, the 8 I ſet down, and 1 I keep. Laſtly, I ſay, once 3, is 3 and 1 I keep is 4, which I ſet down orderly next: And ſo have I ended the Multiplication, & my figures ſtand thus.

	or thus
1630	1630
365	365
8150	815
9780	978
4890	489
594950	594950

Maſter. I commend you for your diligence, the worke is very perfectly done; which parcels if you now adde together into one ſum it will be 594950; which

which is the grosse or totall summe of that Multiplication, and declareth the number of dayes since our Lord and Saviour his incarnation, unto the end of 1630 *yeares, besides* 407 *dayes, and twelve houres for leape yeares.*

Scholar. This is marvellous, me think, that such great matters may so easily be atchieved by this Art, which heretofore I ever thought had been impossible, as infinite sorts of people are of that minde.

Master. Truth it is, that knowledge hath no greater enemy then ignorance, for this is one of the least of ten thousand things that may be done by this Art, as hereafter you shall be able to justifie.

Scholar. The manner of Multiplication I perceive, if there be no more in it.

Master. *Yes, there are other formes and helps for ease and shorter labour of the worke of* Multiplication, *but I will remit them till you have a little tasted* Division, *where also the like help into* Division *may be used: and so therefore under one example for both, will I shew you both ease in* Multiplication, *and also in* Division.

But sith the other formes and workings do nothing differ from these workes in effect, but onely in setting of the numbers, I will overpasse them till a more meete place and time. And now will I instruct you in Division, so that you think your self sufficiently to perceive what I have taught you.

Scholar. Yes Sir, I thank you, but I doe not perceive how to examine my worke, to try whether I have well done or no: therefore as you

you promised me ere-while, I pray you first shew me how I shall prove it.

Master. *That is commonly used by the proofe of 9, as you learned before in* Addition, *saving that it differeth from that forme in divers respects: As for example.*

First, must you make a crosse after this manner.

X

Then must you examine your summe that should be multiplied, and look what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoever remaineth in it after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, and looke what amounteth thereof, if it be under 9, write it at the higher part of the crosse: but if it be above 9, then take thence 9 as often as ye can, and write the rest at the head of the crosse: As for example, put forth of the piece of ground that contained 1365 yards in length & 236 yards in breadth.

Proof of Multiplication.

Therefore first I cast away all the nines from the summe to be multiplyed, saying, 5 and 6 make 11, cast away 9 rest 2: then 3 and 2 makes 5, and 1 is 6, that 6 I write at one side of the crosse thus.

X
6

Then do I examine the Multiplyer which is 236, wherein when

the

the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the crosse.

2 6

Then doe I multiply 6 by 2, and it maketh 12, from which 12 I withdraw 9, then resteth 3, which 3 doe I set at the head of the crosse. Then doe I examine the grosse summe, amounting of the Multiplication, which is 322140, where I finde 9 once, and 3 remaining; that 3 I set at the foot of the crosse, and then I see it to agree with the other 3 at the toppe of the crosse, and so know I that I have done well: for if they two did differ, then were my worke vaine, and the Multiplication false.

3

2 6

3

This is the common proof: but the most certain proofe is by Division of which I will anon instruct you.

A sure proof of Multiplycation.

Schol. Sir, what is the chief use of Multiplication?

Master. The use of it is greater then you can yet understand: howbeit, these plaine commodities it hath, that if you would resolve any great and whole value into many small and lesse proportions, as if you would change pounds into shillings and pence or any other greater or smaller parcels by Multiplication ye shall do it speedily and easily. Also if you should need to adde one sum to it selfe, or to any other often times you shall doe it by Multiplication much more speedily, readily, easily, and surely, then by often and sundry Additions. Take you these commodities grosly shewed for an answer at this time, and hereafter I will more abundantly make you to perceive the use of it.

Division

Division.

Scholar.

WEll Sir, then in Division *I pray you to instruct me. But me thinketh by the name of it, that it should be all one with* Multiplication*: for I call that* Division, *when any thing is parted into diverse and many parts.*

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceive that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but contrarywayes, it bringeth many parcells into few, but yet so, that these few taken together, are equall in value to the other many: for by Division pence are turned into shillings, and shillings into pounds: As for example, of 120 shillings, it maketh 6 pounds, so are 120 turned into 6, which is a smaller number: but then if you consider the Denominators, you shall see that they are such, that one of the latter is equall to 20 of the first, and so in value the summes are one, though in number they doe differ, and the latter summe is the lesser, and so it is alwayes in Division, howbeit, yet in the working the summe is parted by another, and thereof doth it take the name.

Scholar. I thinke I shall better understand the reason of the name when I know the use of the work, therefore now would I gladly learn that.

Division what it is.

Master. *Division is a distributing of a greater summe by the unites of a lesser* : Or, *Division is an Arithmeticall producing of a third number, in respect of two propounded numbers; which third number shall so often contain an unite, as the greater of the two propounded numbers can contain the lesser.* So that as Multiplication did seem to serve in stead of many Additions, so Division may seem to be in place of many Subtractions : Because that third number briefly expresseth how many times the lesser of your two propounded numbers may be subtracted from the greater : as in practise will more largely appeare. Therefore (as you may perceive) unto Division are required three numbers : The first, which should be divided, and that must (generally) bee the greater : and the second, by which the other must bee divided, and that is (generally) the lesser, and is called the Division : And the third, which answereth to the question (How many times?) and therefore is called the Quotient.

A generall rule for placing the figure.

The first must be first written, and the second so set under it, that the last figure of the lower number bee right under the last of the higher, contrariwise to the worke of other kindes of Arithmetick; for in them the two first figures were set ever meet one under the other; but in Division, the last figures must be set meet, except it chance so that the last figure of the Divisor, be greater then the last of the higher number, for then you shall set the last of the Divisor under the last save one of the

higher

higher number, as for example:

If you should divide 365 (which are the summe of the dayes of a year) by 28, which are the dayes of a common Month, then should you set them thus.

365
28

But if you should divide those 365 daies by 52, which is the number of weekes in one yeare, then should you set them thus.

365
52

Likewise if I would divide the same 365 by 4, which is the summe of the quarters of years, then must I set them thus.

365
4

Scholar. Sir this doe I understand, but now how should I doe to divide the one by the other?

Master. You must beginne with the last Figure next the left hand, and see how many times the first Figure of the Divisor may bee taken out of the last Figure of the other Number, and that shall you note within a crooked line toward your right hand: As for example, I would divide 365 by 28, then set I those two summes thus.

365 (
28

And I looke how many times I may finde 2 (which is the last figure of the Divisor) in 3, (which is the last of the number to be divided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I tould you.

Quotient numbers.

Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 over 3, & deface or cancell the 3 & the 2, then will the figures stand thus.

1
~~3~~65 (1
~~2~~8

Then come I to the next figure of the Divisor, and take it likewise so many times out of the figures that be over it, and looke what doth remain, that I must write over them, and cancell them, as in this example.

Therefore now do I take once 8 out of 16, and there remaineth 8, which I must set over the 6, and cancell or crosse out the 16, and the 8 of the Divisor: and then will the figures stand thus. And so I have once wrought.

18	
365	(1
28	

Scholar. So I perceive that you take the nether figure, not only out of the other that is right over him, but out of that with the other also that remaineth before, and are written toward the left hand.

Master. So must you doe: for you must so take the Divisor out of the over number, that there remaine not over it so great a summe as it selfe is: for then were your worke in vaine.

But yet againe here must you mark, that when you seeke how many times the last figure of the Divisor may be found in the number over him, that you looke also whether you may as often finde all the figures following in those that are above them (considering all the remainers, if there be any) if not, take your Quotient lesse by one, and then prove againe, and so still you finde a meet Quotient: and by that meet Quotient must you alwayes multiply your Divisor, and set the product under your Divisor, so that the first figure stand under the first figure of your Divisor, and the second under the second, and so forth; and then subtract that product from the number to be divided that standeth directly

rectly over it, as you have seen me doe.

When you have thus wrought once, then must you begin againe, and write your Divisor anew, neerer toward the right hand by one place, as in this example, you shall set 2 under 8, and 8 under 5, thus.

28
365 (13
288
2

Then (as before) seeke how many times you may take your Divisor out of the number over him now.

Scholar. That may I doe here 4 times.

Master. Truth it is that you may finde 2 foure times in 8: but then marke whether you can finde the figure following so many times in the other that is over him. Can you finde 8 foure times in 5?

Scholar. No, neither yet once.

Master, Therefore take 2 out of 8 once lesse.

Scholar. That is three times.

Mark how to consider this kinde of Remainer.

Master. Well then, 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the five following make 25, in which summe I finde 8 three times also, and therefore I take 3 as a true Quotient, and write it within the crooked line of the Quotient before the 1, thus:

2
~~2~~8
365 (13
288
2

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancell the 8, and write over it the 2 that doth remain, thus:

Then doe I take 8 as many times out of 25, saying, 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1, so then I cancell 25 and 8, and over the 5 set 1 thus:

Or you might (after you find 3 to be a fit (Quotient)

tient) straight way have multiplied the whole Divisor 28, by that at once: which giveth 84, which being set under 28, and duely subtracted from 85, of the number divided, giveth 1, the remainer of the whole division, as before you had. Work which way you list, here you may see also the form.

2
281
365 (13
288
22

And now have I done with the dividing; for I cannot finde my Divisor 28 any more in the over summe.

Scolar. No; except you would part the 1 that remaineth into 28 parts.

Master. That is well said, and so must we doe in such cases, when there remaineth any thing: but I will let that passe now, and will make you perfect in division of whole numbers, and will hereafter teach you particularly of broken numbers called Fractions. Now if you doe perceive the order of division, then doe you divide this summe 136280 by 452.

Scholar. First I set downe the number that should be divided; then doe I set the Divisor under the last figure of the over number. Then will it be thus.

136280
452

Master. Can you take the last of your divisor (which is 4) out of one which is the last of the over number?

Scholar. I had forgotten, because the last of the divisor cannot be taken out of the last of the over number, in so much as it is the greater, therefore must I set the divisor one place more forward toward the right hand, thus.

136280
452

And then must I looke how often I may finde the last figure of the divisor (that is 4) in 13, which I may doe 3 times, therefore doe I say, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doe I make at the right hand of my summes a crooked line, and write before it my Quotient 3, and I cancell 13 and 4, and over the 3 I set the 1 that remaineth, and then the figures stand thus.

1
x36280 (3
452

Then I multiply the same Quotient into every figure of the Divisor, and withdraw the summe that amounteth out of the numbers over them, as first I say, 3 times 5 makes 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write over the 6 that 1 that remaineth, thus.

x1
x36280 (3
452

Then doe I say likewise, 3 times 2 make 6, which I take out of 12 and there resteth 6, therefore I cancell the 12 and the 2 over, and then I write the 6 that remaineth, thus.

x16
x36280 (3
452

Then should I set forward the Divisor into the next place toward the right hand thus.

x16
x36280 (3
4522
45

Master. But you may see that over the 4 is no figure, therefore I must set the divisor yet forwarder by another place.

And marke, whensoever it chanceth so, that you should set forward the divisor, and that it cannot stand here, because there is no number over the

the last place, or if there be any, it is lesser then the last figure of the divisor then must you remove the divisor, yet once again: and because that his first place of removing served not to subtract him so much as once, therefore you shall write in the Quotient a Cypher, and if you should by chance need to do so oft times, for every time write a Cypher in the Quotient. The reason of this will I shew you hereafter.

Scholar. Then must I set my sums thus.

116
136280(30
4522
45

And because I removed the divisor, so that I overskipped one place, I must write a Cypher in the Quotient: and then must I seeke a new Quotient, as in this example I must say, How many times 4 is there in 6? (and sith it can be but once) therefore doe I write 1 in the Quotient: and then say I, 1 time 4 taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 over them, thus.

2
1163
136280(301
45222
455
4

Then say I againe, once 5 out of 28, remaineth 23: I let the 2 stand as it did, and over that 8 I set 3, cancelling the 8 and the 5 under it, thus.

2
1163
136280(301
45222
455
4

Master. You might as well have said, once 5 out of 8, and so remaineth 3, but now go forward.

Scholar. Then once 1 out of 0 cannot bee: What shall I now do?

Master. Borrow of the next number that is behind

hinde (for there is 230) and do as you learned in Subtraction in like case.

Scholar. Then must I borrow 1 out of the 3 comming behinde next, and make that 0 to bée 10 and then take I 2 out of 10, and there resteth 8: and because I borrowed one of the 3, I must cancell the 3, and write 2 over it, then doth the figure stand thus.

```
      10
     22
   xx6z8
 x z6z80(301
   452zz
    455
     4
```

Master. Now have you done, and yet remaineth 228, and your Quotient sheweth you, that if you divide 136280 by 452, you shall finde your Divisor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example (where I divided 365 by 28, the Quotient was 13, and 1 remained: whereby I knew that in a year (which containeth 365 dayes) there are 13 moneths, reckoning 28 dayes (or 4 wéeks) just to a month, and 1 day more.

Scholar. Why then do we call a year but 12 moneths?

Master. Of that at a more convenient time will I fully instruct you: but now it is not convenient to intangle your minds with other things then do directly pertain to your matter. Therefore if you remember what you have heard, you have learned a short manner of Division, which I would have you often to practise, so that you may be perfect in it, and hereafter I will shew you certaine other proper points touching it.

Scholar. Then I pray you tell mée how I shall

examine and try my worke, whether I have done well or no, that though no man be by to tell me, yet I may perceive it my selfe.

Master. Some men, (yea and commonly most) doe try it by the rule of 9, as in all the other kindes, save that their order is; First, they cast away 9 as often as they can out of the Divisor, and that that remaineth they set at one side of a Crosse, as in our first example the Divisor was 28, from which you may take 9 thrée times, and 1 remaineth: which they set by a Crosse, thus.

Proof of Division:

X 1

Then they likewise examine the Quotient, (which in our example is 13) and from thence they cast away 9 as oft as they can, and the remainder they set at the other side of the Crosse, and then they multiply together those two remainers: and to it that amounteth they adde the remainer of the Division, if there were any, from that whole summe they withdraw 9 as oft as they can, and the rest they set at the head of the Crosse, as in our example, the Quotient is 13. from which take 9, and there remaineth onely 4, and therefore must you set 4 at the other side of the Crosse, thus:

4 X 1

Then multiply 4 by 1, and it yéeldeth but 4, thereto adde the remainer of the Division (which was 1) and it will be 5, which summe doth not amount to 9, and therefore must be set wholly at the head of the Crosse, as you sée hear.

5

4 X 1

And this number on the head of the Crosse is the first proof, to which if you find another like in the

number that was divided, then you have done well.

Therefore now shall you likewise examine the whole summe that was divided, and take away 9 as often as you can, and that that remaineth, set at the foot of the Crosse: and if it be equall to that in the head of the Crosse, then have you done well, else not.

As in our example the whole summe was 365, which maketh 14, from that take 9, and there resteth 5, which set at the foot of the Crosse, thus.

4 X 1
5

And you shall see that they agree: therefore have you well done.

Now will I likewise examine our second example, where the Divisor was 452, which maketh 11; from thence I take 9, and the 2 that remaineth I set at the right side of the Crosse, thus.

X 2

Then examine I the Quotient, which was 301, where I finde but onely 4: that I set at the other side of the Crosse, thus:

4 X 2

Then I multiply 4 by 2, and it maketh 8: to that I adde the remainer of the Division (which was 228, and it maketh 12) and they two make 20, wherein I find twice 9, and 2 remaining: that 2 must I set at the head of the Crosse, thus:

2
4 X 2

Then I examine the whole number to be divided, which was 136280, where I finde twice 9 and 2 remaining, which I set at the foot of the Crosse, thus:

2
4 X 2
2

And because it doth agree with the figure at the head of the Crosse, I know that the

Division

Division was well wrought.

The Proof of Division more certain by Multiplication.

Master. This is the common proof: Howbeit, the more certain working is by the contrary kind: as to prove Division by Multiplication, thus:

Multiply the quotient by the Divisor, and if the summe that amounteth be equall to the summe that should be divided, then have you well divided: else not.

Howbeit, this must you marke, that if there remained any thing after the Division, that must you adde to the summe that amounteth of the Multiplication. As in our first Example our Quotient was 13, and the Divisor was 28: Now multiply the one by the other, and the summe will be 364: to that if you adde the 1 that remained after the Division; then will it be 365, which was the sum that should be divided: and therefore I know that I have well done.

Scholar. Now will I prove the same in the second example, whose Divisor was 452, and the Quotient 301: these doe I multiply together, and there amounteth 136052: to which if I adde the 228 that remained, then will it be 136280. which was the whole summe to be divided: and therefore I perceive that I have well done.

Master. This is the surest way to examine Division by Multiplication: and contrariwise, the surest proof of Multiplication is by Division.

And therefore (according to my promise) now will I shew you how you may prove Multiplication by Division.

Proof of Multiplication by Division.

When you have ended Multiplication, and would know whether you have well done or not, set the

grosse

grosse summe that amounteth of the Multiplication overmost, and divide it by the Multiplier: and if the Quotient be the same number that should be multiplied, then have you well wrought, else not, as in that example where we multiplied 264 by 29, the grosse summe was 7656.

Now if you will know whether that Multiplication be true, you shall divide that 7656 by the multiplier 29, and you shall perceive that the Quotient will be 264, and that is a token that you have well wrought.

Scholar. By your patience I will prove that, and first set downe the grosse summe and the multiplier, not after the rule of Multiplication, but after the rule of Division, for now that number is become the Divisor, that was before the Multiplier: I should set them therefore thus.

7656
29

Then shall I seek how many times 2 in 7, that may be three times, & one remaineth: but then may not 9 be found so often in 16? therefore must I take a lesser Quotient, that is to say 2: then say I, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then do I cancell 7 and 2, and over 7 I write 3, and in the Quotient I set 2: so the figures stand thus.

3
7656 (2
29

Then say I further, two times 9 make 18, which I abate out of 36, and there resteth 18: then cancell I 3, and over him set 1, and likewise I cancell 6 and 9, and over them I set 8: so that thus stand the Figures,

1
38
7656 (2
29

Then I set forward the Divisor by one place,

and

and seeke a new Quotient, that is to say, how many times 2 are in 18, which I finde to be 9 times: but then can I not finde 9 so many times in 5: but therefore I take a lesser Quotient, as to say 8: but yet that is too great: for if I take 8 times 2 out of 18, there remaineth but 2, & I cannot finde 8 times 9 in 25: therefore yet I take a lesse Quotient, that is 7, which is also too great; for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seek a lesser quotient, as to say 6, then say I 6 times 2 make 12, that I take out of 18, and there remaineth 6, so I cancell 18, and the 2, and write 6 over 8, thus:

```
 16
 38
7656  (26
 299
  2
```

Then say I forth, 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus:

```
  1
 16
 381
7656  (26
 299
  2
```

Then must I set forth the Divisor again, and seek a new quotient, which will be 4: for though I may find 2 in a 11, 5 times, and 1 remain, yet I cannot find 9 so often in 6, therefore I set the figures thus:

```
  1
 381
7656  (264
 2999
 22
```

And the 4 in the quotient I multiply into the figures of the Divisor, saying foure times 2 makes 8, which I take out of 11, and there rests 3, therefore I cancell the 11, and the 2, and set 3 over the first place of 11, thus:

```
  1
 263
 381
7656  (264
 2999
 22
```

And then doe I say forth, 4 times 9 maketh

9 maketh 36, which I take from 36, and there remaineth nothing, so that the Quotient of this Division also (where 7656 is divided by 29) is 264: Which doth declare, that if 264 be multiplyed by 29, the summe will be 7656. And thus I perceive now how both Multiplication is proved by Division, and Division also by Multiplication.

Master. Now have I ended the five common kindes of Arithmetick: For (as touching Mediation, Duplation, Triplation, and such other) they are no severall kindes of Arithmetick, but are contained under the other. For Mediation is contained under Division, and is nothing else but dividing by 2: & so are Duplation and Triplation contained under Multiplication: for Duplation is nothing else but multiplying by 2, and Triplation is multiplying by 3, of which I will onely propose an example, for the Rules you have heard already.

If you would mediate, or divide into 2, this summe 4531010, you shall set 2 for the Divisor, and worke as you learned before, as thus: An example of Mediation.

4531010 (
2

Then I finde 2 in 4 two times, therefore my Quotient must be 2: so I cancell 4 and 2, and remove the Divisor forward thus, as the work requireth, and as before in Division hath been declared.

4531010 (2265505
2222222

Which mediation or division by 2 being finished, you shall have for your Quotient 2265505, which is the halfe of 4531010, as you may try by Duplation; for double that Quotient, or multiply it by 2, and the same number will amount.

I

I will no longer tarry about these, seeing they are but members of the other kindes. But here now (according to my promise) I will teach you certain easie formes both of Multiplication and of Division. And first of Multiplication.

Easie formes of Multiplication.

If you would therefore multiply any summe by 10, you shall need to do no more but adde a Cypher before his first place; as for example, 36 multiplied by 10, make 360.

Likewise if you would multiply any summe by 100, put two Cyphers at his beginning. So if you would multiply any summe by 1000, adde three Cyphers to the beginning of it.

Scholar. This doe I well perceive, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmetick also: and as to our matter now, look, as I have told you, that you both remember it, and also often practise it.

And now you have learned how to multiply easily by 10, 100 1000: and of like manner may you do with any other of like sort.

But now if you will multiply by 20, 30, 40, and so forth, or by 200, 3000, and such like, where there is one Cypher in the first place, or many orderly in the first places, you shall take away those Cyphers, and multiply the summe onely by the other figure or figures (if they be many) and then at the beginning of the summe that amounteth, you shall set so many Cyphers as you take away.

Example of 2873, which I would multiply by 300. First, I omit the 2 Cyphers from the Multiplier,

plier, and I multiply the summe by thrée onely that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted or toke away, and then is it 861900. And that is the summe that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures beside the Cyphers, I must onely take away the Cyphers, and multiply by the other figures, as I learned before: As if I would multiply 93648 by 25000, I should take away the thrée Cyphers, and multiply the same by 25, & then at the beginning of that totall sum should I adde the 3 Cyphers again.

Master. Even so: but if it chance the number that should bée multiplied, or both the summes, as well the number that should be multiplied as the mnltiplier, to have Cyphers in their first places, evermore omit the Cyphers and work by the rest. But remember to restore as many Cyphers to the amounting summe as you bated before. As in this example: 30200 shall bée multiplyed by 206, I shall onely take away two Cyphers from the greater number, and then multiply 302 by 206, and afterward adde the two Cyphers again. But if I would multiply the same 30200 by 2060 I shall not onely take away the two Cyphers from the number that should be multiplied, but also I may take away the one Cypher from the Multiplier, and then must I adde thrée Cyphers to the summe that amounteth: but take héed that you take away no Cypher that commeth after any signifying figure, as in the last example, you must not take away that in the fourth place of the higher number,

neither

neither that in the third place of the multiplier ; howbeit yet thus you may doe : If one Cypher or more come in the last of your summes, you may multiply the other figures, and overskip them : but so that you give every figure his due place : as thus, I will multiply 3026 by 2004, therefore I set them thus. And thus doe I multiply them. First 4 times 6 make 24, I set 4 under the first place, and keepe the two still in my minde. Then say I againe, 4 times 2 make 8, and the 2 that is in my minde maketh 10, I set downe the Cypher 0, and keepe the Article 1 in my minde : Then 4 times 0 is 0, and the 1 in my mind maketh 1, I set downe the figure 1, and say again, 4 times 3 is 12, I set downe 2, and keeping the 1 still in my mind (having no more places of the upper number to multiply it withall) I put it downe next 2 in the fifth place.

3026
2004
12104
52

But now when I come to the next place (being a Cypher 0) I let it go, because it multiplieth nothing, and likewise the second Cypher.

But then when I come to the 2, & multiply it into the 6 of the over number, you must take heed (according as I taught you in Multiplication) that the first number amounting of the multiplication be set right under the multiplier, and the other orderly toward the left hand, according as you may see in this example, which being finished, with the addition thereof gathered together, will stand as in this Example sheweth.

3026
2004
12104
6052
6064104

Which

Which is indeed wrought ſo much the ſooner and ſhorter by overskipping of the two Cyphers: which otherwiſe, (if the ſame example were wrought at length) it would have had two workings more, as by the ſame example here alſo ſet down doth appear.

```
   3026
   2004
 -------
  12104
  0000
 0000
6052
 -------
6064104
```

Scholar. Sir I thanke you, for I ſee great eaſe in this way of Multiplication: and (if you can ſhew me ſuch like in Diviſion) you ſhall greatly further me.

Maſter. Yes, I will teach you ſome eaſie wayes in Diviſion alſo, and firſt this: If you would divide any ſumme by 10, you ſhall onely with your pen make a ſquare line betweene the firſt figure of your ſumme and the ſecond, and then have you done: for the whole number that followeth the line, ſtandeth for the Quotient, and the figure that is before the line, is the remainer: As for example, 3648 divided by 10. Where 364 is the Quotient, and betokeneth that ſo many times are 10, in 3648, and the 8 after the line is the remainer, which cannot be divided into 10, but by breaking it into Fractions, wherewith I will not meddle yet.

Eaſy forms of diviſion

364 | 8

And ſo likewiſe if you would divide any ſumme by 100, with your Pen you ſhall cut away the two firſt figures, and if you would divide by 1000, you muſt cut away the three firſt figures, and ſo of any other diviſor, whoſe laſt figure is 1, and the other Cyphers; looke how many Cyphers the diviſor hath; and ſo many figures at the beginning ſhall

you cut away with the square line, & they stand alwayes for the remainer, because they are lesse then the divisor, and cannot be divided by it, and the other figures that are behinde the line stand for the Quotient.

But now if your divisor have any other figure in his last place then 1, and in all his other places have Cyphers, looke how many Cyphers they be, cut away so many of the first figures of the number that should be divided, and divide the rest that followeth the line by that figure that is in the last place, as if it were the whole divisor.

Example of 64284, which I would divide by 300, here must I cut away the two first figures, (for so many cyphers my divisor hath) & must divide the rest by 3, which is the figure in the last place of the divisor. First, therefore I part away the two first figures, and the summe standeth thus:

642	84	(2
3	00	

Then do I divide 642 by 3, and the Quotient is 214: for in 6 I finde twice 3, and in 4 once, & 1 remaining, which 1 with the 2 next before, doth make 12, wherein I finde 3 foure times: And this is a ready way to turn shillings into pounds: for sith one pound doth contain 20 shillings, I must divide the whole number of shillings by 20. Therefore easily do I see that my divisor hath one cypher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then I do mediate or divide by the 2 other figures or summe that followeth.

Scholar. I will put an example.

If you would divide 64287 shillings, by 20: that

that is to say, If I would turn so many shillings into pounds, I must cut away the first figure, that is 7, and divide the rest, that is 6428 by 2, so shall the Quotient bée 3214, whereby I know that 64287 shillings make 3214 pounds, and 7 shillings remaining.

Master. Now prove by Multiplication whether you have well done or no.

Scholar. The Quotient is 3214, which I do multiply, by the divisor 2, and it doth amount to 6428.

Master. Hereby you may perceive not onely that you have well done, but also how by Division you may turn shillings easily into pounds: and contrariwise by Multiplication you may turn pounds into shillings.

But here shall you see amongst divers men divers forms of such division: but if you marke what I have told you, you shall perceive easily all the wayes. For some men doe not cut away so many of the first figures of the summe that they would divide, as there are Cyphers in the first place of the divisor: but they set all their Cyphers orderly under the first places of the Number that they would divide: and then with the other figure or figures, (if there be many) they divide the rest of their summe.

Example. If they would divide 725931 by 3400, they doe set their summs thus:

7259	31
34	00

And then do they divide orderly till they come to the Cyphers: for there they stay and end their worke, as in this example.

They séek how often 3 may be found in 7, which

is two times, and 1 remaining: therefore they set 2 in the quotient, & cancell 3 & 7, & over 7 they set the 1 that remaineth thus:

```
1
725931 (2
34   00
```

Then doe I goe forth, saying, two times 4 maketh 8, which they take out of 12, & there remaineth 4 thus:

```
14
725931 (2
34   00
```

Then remove they the divisor forward, & seeke how often 3 may bee found in 4, which is but once, and 1 remaineth, then set they 1 in the Quotient & cancell 3 & 4, & over them they set that 1, thus:

```
1
14
725931 (21
344  00
3
```

Then take they once 4 out of 15, & there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth 1: so they cancell the 4 and 5, and set 1 over them, thus:

```
1
141
725931 (21
 344  00
 3
```

Then set they forth the divisor again, and seek how many times 3 are in 11, which they finde three times, and 2 remaining: so they set 3 in the Quotient, and cancell 11 & 3, & over them set 2, thus:

```
12
141
725931 (213
344400
 33
```

Then doe they multiply 4 by 3, which maketh 12, that withdraw they out of 29 & there resteth 17, of which the 7 must bee set over the 9, and the 1 over the 2 thus:

```
1
12
1417
725931 (213
344400
 33
```

And now are the two Cyphers next ensuing, so that the divisor can no more be set forward, & therefore is the division ended, & the remainer is 1731. Now the Quotient which is 213, doth declare that if you divide

divide 725931 by 3400, you shall find it therein 213 times, & there remaineth 1731: so shall you find it, if you work as I taught you, by cutting away the two first figures, because of the two Cyphers. But this must you mark (as you may perceive by this last example) that if there be left any other Remainer in the sum that was behind the square line that the Remainer must be set to the latter end of the first remainer, which was cut away with the square line: as if you would divide 725931 by 3400, after the form that I taught you, then would your summes appear, thus:

Note.

```
  1
 12
1417
725931 (213
3444
 33
```

So that 17 which remaineth after the line, must be set to the 31 (that was cut away with the line) in higher places, as you see here, where that 17 with the 31, do make 1731.

Scholar. Sir, is there no other form of division in practise but this?

M. Yes verily, there are other forms in practise, but because I love brevity, I will declare onely one, which I first learned of, and is practised by that worthy Mathematician, my ancient & especiall loving friend, Master Henry Bridges wherein not any one figure is defaced or cancelled. As if I should divide 72 by 6, first place them thus. 6)72

Then if you please you may write the divisor in a loose paper that it may more easily without cancelling or defacing of the worke be applied to, and removed from the dividend at pleasure; then apply your divisor 6 to 7, the first figure of the

Write the Divisor in a loose paper, to remove at pleasure.

dividend, and inquire how oft it may be had in 7, and seeing 6 is but once in 7, set 1 in the Quotient line, thus: 6)72(1

Then multiply the divisor 6, by the quotient 1, and set the Product 6 under 7, thus: 6)72(1
6

Then draw a line under 6, and subduct 6 out of 7, setting the remainer 1 under 6, thus: 6)72(1
6
1

Then bring down the next figure of the dividend, & set it with the Remainer 1 under the line, thus: 6)72(1

And bring the moveable divisor 6 under the 2, and as before enquire how oft 6 is in 12, and finding it to bee twice in 12, set 2 in the Quotient, thus: 6
12
6)72(12
6

And multiply 6 by that new Quotient 2, setting the Product 12 under the other 12, and subducting it out of the upper number, there resteth nothing. 12
12
0

And since the unites of this Product do stand under the unites of the dividend, the division is ended: otherwise you should proceed as before, bringing down the next figure; removing the divisor, dividing, multiplying, subducting, &c.

Schol. This is very easie; but if there be greater numbers propounded, is the operation the same?

Master. If the numbers be never so great, the worke is the same without any difference, as shall appear by this example.

Divide 7890 by 33. 33)7890(2

First set them thus: then bring the divisor under 78, and see how oft it is there found, which is twice, and therefore set 2 in the Quotient, by which multiply

multiply the divisor 33, and set the Product 66 under 78, and subduct it out of it thus.

Then bring the next figure 9 down, and set it with the Remainer 12, it maketh 129, and removing the divisor 33 thereto, enquire how often 33 is contained in 129, & I finde it but thrice, (though at the first it made a shew of more) therefore set 3 in the Quotient, and multiplying 33 by 3, set the product under 129, subducting that product out of the number above, and proceed as before.

33)7890(239 $\frac{3}{33}$
66
129
99
300
297
3

Then shall you finde the Divisor 9 times in the Remainer, therefore seeing 9 in the Quotient, multiply, and subduct as before, and at the last you shall find onely 3 remaining, which must be set above a line after the Quotient, and the Divisor under, as above appeareth.

Scholar. Is there no more difficulty in the whole Rule?

Master. Not any, although your Number be never so great, as before I have said.

¶ And here will I make an end of Division, (saving that I doe request you to exercise your selfe well herein by many summes, till you have attained some expertnesse therein.)

For the reasons & conclusions thereof are so many, and so available for all sorts of men whatsoever; that if I should speak of the infinite uses thereof, I should rather lack words then matter. And therefore recommending it to your judgement hereafter, upon your further travell into the Art, I will here end

this Treatise, representing unto you one example, or simple question of Division and Multiplication, in stead of many, which is this.

A question of shooting in Ordinance.

There are foure brasse Peeces: The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed against the battery of a Hold and there is allowed by the Master Gunner, 700 pounds of powder to be spent by these foure Peeces, in this assault. The question is twofold. The first how many shot each Peece shall justly make about with this 700 pounds of powder? And lastly, how many pounds of powder ought justly to be allowed to each Peece for his true proportion?

Scholar. Why Sir, you make me smile, to beare mee in hand, that these two demands may be simply resolved by Multiplication and Division.

Master. Truely that they may, and that you may by and by worke your selfe with a little labour: First adde together their quantities of powder, that is 9 pounds, 5 pounds, 4 pounds, and 2 pounds, all which make 20: Divide the 700 pounds of powder by that 20, and your Quotient giveth 35, as here appeareth, which sheweth for most certainty that they shall make just 35 shootes about.

x
700
202(35
x

Scholar. Sir, all this have I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To try the truth of the same, multiply the first peece that spends 9 pounds by 35, and you shall see his allowance, which is 315 pounds of powder. Multiply also the second peece that

spends

spends 5 pounds by 35, & you shall finde 175 pounds his allowance: then 4 by 35, and you shall finde 140 pounds his allowance. Lastly, multiply 2 by 35 and you shall finde 70 pounds his allowance. All which four particular sums you shall adde together by Addition, as here appeareth, and it maketh just 700 pounds, and so is the question truely absolved.

315
175
140
70
700

Scholar. Truely Sir, these excellent conclusions doe wonderfully more and more make me in love with the Art.

Master. It is an Art, that the further you travell, the more you thirst to goe on forward. Such a Fountaine, that the more you draw, the more it springs: and to speake absolutely, in a word (excepting the study of Divinity, which is the salvation of our Soules) there is no study in the world comparable to this, for delight in wonderfull and godly exercise; for the skill hereof is well known immediately to have flowed from the wisdome of God into the heart of man, whom he hath created the chiefe image and instrument of his praise & glory.

Scholar. The desire of knowledge doth greatly incourage me to be studious herein: and therefore I pray you cease not to instruct me further in the use hereof.

Master. With a good will. And now therefore for the further use of these two latter, that is, Multiplication and Division, I will briefly shew you the feat of Reduction.

Reduction.

Reduction.

Reduction what it is.

REduction is by which all summes of grosse Denomination may be turned into summes of more subtile Denomination. And contrariwise all summes of subtile Denomination may be brought to summes of grosser Denomination.

Grosse denomination.

Scholar. What call you grosse Denomination, and subtile Denomination?

Master. That I call a grosse Denomination, which doth containe under it many other subtiler or smaller: as a pound (in respect to shillings) is a grosse Denomination: for it is greater then shillings, and containeth many of them. And shillings (in comparison to pounds) are a subtile Denomination, for because they are lesser then pounds, & many of them are contained in one of the other: and so likewise of other things: whatsoever thing is compared to other, if it be greater and containeth many of them, it is a grosse Denomination, but if it be lesser (so that many of them are in the other) then are they called the subtile Denominations: whereby you may perceive that one Denomination may be called a grosse Denomination, and also a subtile (that is to say a great, and small) in diverse comparisons. For shillings compared to pounds, are a subtile or small Denomination: but compared to pence they are a grosse, or great Denomination.

Subtile denomination.

Scholar, Now I understand the name, I pray you teach me the use.

Master. The use is easily learned, if you remember

member what you have learned before. For if you will reduce any summe of a grosse Denomination into a summe of a smaller or subtiler Denomination, you must consider how many of that subtiler Denomination do make one of the grosser Denomination, and by that number or numerator doe ye multiply the summe. As if you would reduce 30 pounds into shillings, you must consider that in a pound are included 20 shillings, therefore multiply the one 20 by the other 20, and there will amount 400, whereby you may know that in 20 pounds are contained 400 shillings. Likewise, if you would reduce 30 shillings into pence, considering that in a shilling are 12 pence, you must multiply 30 by 12, and it will be 360, whereby you may finde that in 30 shillings are contained 360 pence. And thus may you reduce any grosse Denomination into a more subtiler, by Multiplication, if you know how many of the lesser doe make the greater: of which thing I will anon give you a brief Table for the most accustomed kinds of Money, Weights, Measures, and Time, and suchlike: whereby you may know how often each subtile Denomination is contained in the grosser, when you shall need it for the foresaid kinde of Reduction. And also the same shall serve you, if you would reduce any summe of a subtiler Denomination, into a summe of a greater Denomination. For in such Reduction you must consider (as in the other form) how many of the smaller doe make the greater: and by that number you must divide the other summe, and the Quotient will declare how many of the greater Denomination are comprehended in that summe: as

To reduce grosse denomination to subtile.

as for example; If you would know how many shillings are contained in 3240 d. consider that 12 pence doe make 1 s. you must divide that 3240 by 12, and your Quotient will be 270, whereby you may know that so many shillings are in 3240 d. But if you would know further how many pounds are in these 270 shillings seeing that every pound containeth 20 shillings: divide that 270 by 20, and it will be 13 and 10 remaining, whereby you may know, that in 3240 d. (or 270 shillings) are 13 pounds and 10 shillings. For evermore the Remainer must be named by the name, or Denomination of that summe that was divided, which in this place were shillings. And thus may you do with any other kinds of Denominations.

¶ Wherefore, to the intent you may have certain light or knowledge in most common Coynes, Weights and Measures, (which is the chief and principallest thing in trafficke to be known) I have in each Reduction, as they come in order, set downe certain instructions incident thereunto. And first I have hereunto added this Table, wherein is comprehended, not onely our currant and common coynes, but also the most part of the usuall coynes of Christendome, with their just weights and value currant in the Realme of England, intending at the latter end of my Addition to this Book, to write of the ordinary Money used in divers places, and their common values currant for traffick, with the manner of their exchanges from place to place, &c.

A

A Table of the names, and now valuation of the most usuall Gold-coyns throughout Christendom, with their several weight of pence and Grains : and what they are worth of currant English money, this present year 1630.

The names & titles of the Golde.	The weight in Pence.	Graines.	The value in Shil.	Pence.
Great Soverain,	10	0	33	0
Double Sover. K. H.	8	1	22	0
Double Sov. of Q. E.	7	7	22	0
Royall.	4	23	16	6
Half Royall.	2	1d.	8	3
Old Noble.	4	6	14	8
Half Noble.	2	3	7	4
Angell.	3	8	11	0
Half Angell .	1	16	5	6
Salute.	2	5	6	11 ob.
2 parts of Salute.	1	11	4	7
George Noble.	3	0	9	9 ob.
Half George Noble.	1	11	4	[illegible]
First Crown K. H.	2	9	6	11 ob.
Base Crown K. H.	2	0	5	6
Sover. K. H. best,	2	14	11	8 ob. q
Soverain K. H.	4	0	11	0
Edward Sover:	3	15 d.	11	0
Elizabeth Sover.	3	15 d.	11	0
Elizabeth Crown.	1	9	5	6
Half Crown.	0	19	2	9
Vnite.	0	12	22	0
Double Crown.	3	6	11	0
Britain Crown.	1	1	5	6
Thistle Crown.	1	7	4	4 ob. q

Half

The names & titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	Pence.
Half Crown.	0	19½.	2	9
Crosse Dagger.	3	6 d.	11	0
Half Crosse Dagger.	1	15	5	6
Rose Royall	0	21	33	0
Spur Royall.	4	10d.	16	0
The Angell.	2	23 d.	11	0
Half Angell.	1	11 d.	5	6

All the severall pieces of gold heretofore mentioned, are set down according to their valuation by the Kings Majesties Proclamation for Gold, Dated the 23 of November, 1611.

A table of forain Gold coyn, according to their ancient valuation and severall weight in Pence and Grains.

The names & titles of the Gold.	The weight in Pence.	Graines.	The value in Shil.	Pence.
Unicorn of Scot.	2	10	6	0
Scottish Crown.	2	5	6	0
French Noble.	4	16	13	4
All sorts of French Crowns.	2	5	6	0
Flanders Riders.	2	6	6	6
Gelders Riders.	2	2	3	6
Philips Royall.	2	10	10	0
Philips Corwne.	2	5	5	0
Collen Gilden.	2	2	4	8
New And. Gild.	2	2	5	0

Flanders

Flanders Noble.	4	10	12	0
Half Flan. Noble.	2	6	6	0
Flan. Angel best.	3	6	2	0
Flan. Royallorke.	3	10	10	0
Carolus Gilden.	0	12	3	6
Flanders Royall.	2	6	5	0
Saxon Gilden.	2	2	4	8
Flanders Crowne.	2	5	6	0
Philips Gilden.	2	3	4	2
Half Phil. Gilden	1	1	2	1
Golden Lion.	2	16	7	8
3 parts of goldē Liō	0	21	2	5
2/3 parts of gol. Lion	1	19	4	11
Davids Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old under Gilden.	2	3	4	10
Crusa. long crosse.	2	6	6	0
Crusa. short crosse.	2	6	6	2
Milreys.	4	20	1	4
Half Milreys.	2	10	6	8
Portague 1 ounce.	2	16	68	0
Golden Castile.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungary Ducket.	2	7	6	4
Double Pistolet.	4	9	11	8
Single Pistolet.	2	4 d.	5	10
Ducket of Floren.	2	5	6	4
Double Ducket	4	11	13	0
Single Ducket.	2	6	6	6
Double duc. of Rōe	4	13	12	8

It is to be understood (gentle Reader) that whereas in these Tables, the weight is called by the name of a penny

penny, it is not ment a penny of silver money, but a penny of Gold-smiths weight, which containeth 24 Barley-corns. Concerning which see Troy weight in folio 133.

So if a man have not the weight wherewith to weigh any peece of gold, he may do it with barley-corns, being dry, and as it is said, folio 133.

The prices of Gold which the bringers in of forrain Gold shall receive at the Mint, according to the Kings Majesties Proclamation Dated the 14 of *May, Anno* 1612.

FOr an ounce of *French* crowns being 22 Karacts fine.———	3 li.	6 s.	
For every ounce of *Spanish* Pistolets, being 21 Karacts, 3 grains and a half fine.———	3 li.	6 s.	
For Duckets of *Spaine*, being 23 Karacts, 1 grain fine at least the ounce.———	3 li.	8 s.	8 d.
For Milreas Crusado long crosse. Crusado short crosse, the ounce.———	3 li.	6 s.	2 d.
For *Hungary* Duckets being 23 Karacts, 1 grain fine at least the ounce.———	3 li.	9 s.	2 d.
For the Checkeen of *Venice*, being 23 Karacts, 1 graine fine at least the ounce.———	3 li.	10 s.	
For *Barbary* Gold being 23 Karacts & di. grain fine, at the least the ounce.	3 li.	9 s.	

¶ *And if the said Barbary Gold be of lesse fineness, abatement to be made according to the rate.*

For

For Sultaines being 23 Karects, 1 Graine fine at least the ounce.———	3 li. 8s. 8d.
For all other Gold, being 22 Karects fine the ounce.———————	3 li. 6 s.

¶ *And being finer, a greater price according to that rate, and being courser, a less, so that the bringer in supply the lesse fine with the more fine, in such sort, that in the totall it makes good the same rate of 22 Karects fine.*

The Price of Silver, which the bringers in of forain Silver shall receive at the Mint, according to the Kings Majesties aforesaid *Proclamation*.

FOr the ounce of *Spanish* Silver money of *Sevill*.————	5 s.
For the ounce of *Mexico* money.———————	4 s. 10 d.
For Ingots of Silver, being 11 ounces, 2 d. weight fine according to the Standard of *England*, the ounce.——	5. s.

¶ *And for other Silver of more finenesse, a better price according to that rate, and for courser a lesse: so that the bringer in supply the lesse fine with the more fine, in such sort, that in the totall it makes good the said rate of* 11 *ounces*, 2 *pence weight fine, according to the Standard of England.*

Of Silver Coynes currant in this Realme.

The *Edward* crown of 5 s.
The *Edward* half-crown of 2 s. 6 d.
The *Edward* shilling, half shilling, and the three pence.
Philip and *Maries* shilling, and half shilling.
The *Mary* groat, and *Mary* two pence.
Queen *Elizabeths* shilling 9 d. 6 d. 4 d. 3 d. 2 d. 1 d. three farthings, and half penny.

HEre would I now expresse the values of sundry other Coynes of divers Countries, but for

three causes I now refraine. The first and chiefest is because they are not currant by the statutes of this Realme. Another cause is, by reason they are so uncertaine, that they be never long at one rate. And againe, they are so different in so many places, that it were matter enough for a great Book to speak sufficiently of them all. Howbeit, because you shall not be altogether ignorant of them, I will shew you the values of some that are most in use, and first of France.

French Coynes. The most common Money are Deniers, Soulx, and Frankes: 12 Deniers make 1 shilling, 20 Soulx make 1 Frank: so that as you see these three kindes are like in the rate to pence, shillings and pounds with us; but that this is the difference, that their Denier is but the ninth part of our penny, & so their Soulx (commonly called Souses) goe 9 to our shilling, and 9 of their Franks to an English pound of money. So that three of their Franks make a Noble. And by those three you may practise how to reduce French money into English money, according as I have set forth here following.

2160	Deniers make	240 d. or 20 s.
3240		360 d. or 30 s.
8352		928 d. or 3 li. 17 s. 4. d.

2160 Soulx make 240 shillings. And so of other in like rate. As for the rest of their Coynes I omit them till hereafter that you have some understanding in broken numbers.

Flanders Coynes. But now as for the Coynes of Flanders, they be so changeable, that you must know them from time to time, else you cannot reduce them into our money certainly: but yet that you have an example

ample of their money to exercise you withall, you shall take those that be most common: as Stivers both single and double, Groates Flemish, Carolus and Gyldens. A Flemish Groat is a little above 3 farthings English. A single Stiver is 1d. ob. q. half farthing. The double Stiver Carolus is 4 d. ob. halfe farthing. Then there is also the Carolus Gylden which is worth 10 Stivers. And the Flemish Noble is worth 3 Carolus Gyldens, and 12 Stivers.

So that if you would convert Flemish money or any other kinde of money whatsoever it be, justly into Sterling, you must reduce it first into the smallest part of English money that is in that Coyne. As for example: If I would reduce 368 double Stivers into English money (considering that a double Stiver containeth 3d. farthing) you shall first looke how many farthings be in the double Stiver, and you shall finde them 13, therefore multiply the summe of the Stivers by 13, and then have you their value in farthings, which is 4784. Now if you divide that by 4, then there will appear the number of pence: but better it were to divide it by 48 (for so many farthings are in one shilling) and then will the Quotient declare the summe of 4 li. 19 s. 8 d.

Likewise if you would reduce any summe of single Stivers into English money, you must multiply the summe first by 13, and then have you reduced them into a certaine summe, that is to wit, half-farthings, which summe if you divide by 8, then will amount the summe of pence: or if you divide it by 96, the summe of shillings will appear.

But marke this in all Division: when ye do reduce to bring one Denomination into another, if there be any Remainer after the Division, that must be named by the Denomination of the grosse sum that was divided. As for example, I would bring 254 farthings into pence, therefore I divide that 254 by 4 (for so many farthings make a penny) and the Quotient is 63, which is the summe of the pence, and then remaineth yet 2, which are farthings still, as one may prove by dividing. And this must be marked in all Division, namely, when it is done for Reduction.

Note well.

¶ Touching Danks Mony, they have their Soulx, whereof 20 is a Liver which is 2 shillings sterling. They have also their Grash whereof 80 make a Gilden, which is 4 shillings sterling. They have also Dollors, & their common or old Dollar is 35 Grash, New Dollors they have which be divers, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Author, touching Danks Money.

Danks Money.

Concerning Spanish Money, whereof the most common are Cornadoes, Marveides, Marveide, 4 Marveides make a Ryall, and 11 Ryalls make one Ducket, so the Ducket containeth 374 Marveides, which is about 5 shillings 10 pence sterling. Therefore if you would convert 124 li. 5 s. sterling into Duckets, consider that pence is the last value or Denomination named in this question: therefore reduce 124 li 5 s into pence, and it maketh 29820 pence: which if you divide by pence that a Ducket is worth, (which is 70) you shall have for your Quotient 426 Duckets your desire.

Spanish Money.

In

In Venice they have Bettes, Souldyes, Lieure 5 Bettes make an English penny, 60 Bettes a shilling, which is 2 Souldyes, and 20 Souldyes a Lieure of Venice which is a pound sterling. Venice Money.

Thus much have I said of Money: Now will I shew you in the like sort the distinction of weights. Weights.

After a Statute made anno 11 *H.* 7. *there ought to be but one sort of weight, as* 24 *Barley-cornes dry and taken out of the midst of the Ear, do make a penny weight,* 20 *of these penny weights make an Ounce;* 12 *Ounces a pound of Troy weight, by which is weighed Bread, Gold, Silver, Pearle, Silke, and such like.* But commonly there is used another weight called Haberdupoise; in which 16 Ounces make a pound. Therefore when you would reduce Ounces into Pounds, you must consider whether your weight be Troy weight or Haberdupoise: and if it be Troy weight you must divide your Ounces by 12, to bring them to Pounds, but if it be Harberdupiose, you must divide them by 16. Now againe, there be greater weights, which are called a hundred, halfe a huudred, and a quarterne, and also a halfe quarterne, &c. Troy weight. A penny weight. An ounce A pound Troy. Haberdupoise weight. A hundred weight.

Scholar. Why? is there may be reckoned 20 Pound, 40 Pound, 200 Pound, and such innumerable.

Master. All these are numbers of weight, but they have not common weights made to their rate as the other have. And again, these that I did name are not just in number as they séem by their name: for an hundred is not just 100, but is 112 pound. And so the half hundred is 56, the quarterne 28, and the half quarter 14.

And those be the common weights used in most things that are sold by weight.

Wooll Weights. Todde Stone.

Howbeit there are in some things other names, as in Wool 28 pound is not called a quarterne: but a Todde: and 14 pound is not named half a quartern, but a Stone, and the 7 pound half a Stone. Other names because they differ in many places, and agrée in few, I let them passe.

Sack of Wooll.

But a sack of Wooll by the Statutes is limited to be 26 Stone.

Cheese weights.

¶ Now in Cheese, though it be sold by the hundred, and by the Stone in some places, yet the very weights of it are Cloves, and Weyes. So that a Clove containeth 8 pound, and a Weye 32 Cloves, which is 256 pound, that is 12 score and 16 pound, and so much weigheth the Weye of Suffolk Cheese, and the like is or should be the Barrel of Suffolk Butter.

The Weye of Essex Cheese containeth six score and sixteen pound: and so much is also the Barrel of Essex Butter.

Moreover this weight is used by the Apothecaries in their Physicall composition, and mixture in medicine, wherein the least is a grain,

The Apothecaries weights.

And		make		thus charactered
	20 *Graines*		*A Scruple*	℈
	3 *Scruples*		*A Drachme or Dragme*	ʒ
	8 *Dragmes*		*An Ounce*	℥
	16 *Ounces*		*A Pound.*	lb.

Measures for liquor A Pint. Gallon.

Now of weights are made other measures both for grain and liquor. For a pound in Troy weight maketh a pint in measure, so that 8 pound or 8 pints doe make a gallon: half a gallon is named a pottle,

and

and half a pottle is called a quart, which contain-eth two pints: Now above a gallon the next mea-sure is a Firkin: then the Tertian a Kilderkin, or half a Barrell, and a Barrel: And by these measures are sold commonly Ale, Beere, Wine, and Oyle, Butter and Soap, Salmon, Herrings, and Eeles. Pottle. Quart. Ferkin. Tertian. Kilderkin. Barrell.

But as these be unlike things, so the measures of their vessels doe differ, for the measures of them all are as followeth.

Ale measures.

Of Ale	*The firkin*	*containeth*	8	*Gallons.*
	The kilderkin		16	
	The barrell		32	

Beer measures.

Of Beer	*The firkin*	*containeth*	9	*Gallons.*
	The kelderk.		18	
	The barrell		36	

Sope measures, both Firkin, Kilderkin, and Barrel should be equall to Ale measure. Sope measures.

Moreover the Statutes do limit the weight of every of those three vessels being empty.

A barrell	*to weigh empty*	26	*Pounds.*
Halfe a barrell		13	
A firkin		$6\frac{1}{2}$	

Herrings also sold by the same measures that Ale and Sope be sold by. Herrings.

Herrings are sold by the tale, 120 to the hundred, ten thousand to the Last.

Salmon and Eeles have a greater measure.

Salmon & Eeles.

Salmon & Eeles	*The butte*	*holdeth*	84	*Gallons.*
	The barrell		42	
	Half barrell		21	
	The firkin		$10\frac{1}{2}$	

Howbeit, some Statutes did limite Eele vessels equall with Herring vessels.

Wine meaſures. Now as for wine veſſels they are ſeldome ſmaller then Hogſheads which are of 63 Gallons: Every Hogſhead, is two Barrels: yet there are many other wine veſſels, but of them all ſée this Table; and marke the meaſures one by another.

Of wine and oyle	*The runlet*	*holdeth*	$18\frac{1}{2}$	*Gallons.*
	The barrell		$31\frac{1}{2}$	
	The hogſhead		63	
	The tertian		84	
	The pipe		126	
	The tunne		252	

But you ſhall marke that there bée other kinds

Tertians. of Tertians: for there bée Tertians, (that is to ſay) Thirds of pipes, of hogſheads, and of barrels, as well of other things as of wine.

Alſo Malmſeyes, and Sacke, &c. the half Tun is not called a Pipe, but rather a Butte.

A Butte. And thus much have I thought méet to tell you at this time.

Scholar. And is that allwayes true?

Maſter. I have tould you how it ſhould be, but how it is, I may not ſay: how they doe differ daily from their juſt meaſure, that Gaugiers can tell you better then I. But I will let this paſſe now, and ſpeak briefly of the other meaſure.

And as of weights there did ſpring the liquid meaſures (whereof I ſpake laſt) ſo of the ſame

Drie meaſures. ſpringeth dry meaſures, as Pecks, Buſhels, Quarters, and ſuch like, whereby are meaſured corne and like grains, alſo Salt, Lime, Coals, and other like. And this is the order and quantity of them.

A pecke is the meaſure of two Gallons.

A

A bushell	containeth	*foure pecks.*	A Bushell
A quarter		*eight bushels.*	A Quarter.
A wey		*six quarters.*	A Wey.

These are the common names and measures, but in divers places there be divers sorts.

The Bushell in many places is two bushels, but then is that bushell there called a strike: and in some places half a quarter is called a Cornock. But those diversities are too many to tell you briefly them all, and again, sith they are against the Law and statutes, I count them unmeet to be used. Strike. Cornock

But now remaineth yet another kinde of measures, whereby men mete length, breadth and thicknesse, and those are, an Inch, a Foot, & such other: whose names and quantities this Table sheweth. Measure to mete length, breadth, & thicknesse.

3 *Grains of barley in length make an inch.* An Inch.

12 *Inches*	*make*	*a foot.*	Foot.
3 *foot*		*a yard.*	Yard.
3 *foot and 9 Inches*	*make*	*an ell.*	Elle.
5 *Yards and a halfe*		*a pearch.*	Pearch.

1 *Pearch in breadth, and 40 in length, doe make a rodde of land, which some call a rood, some a yard land, some a farthendele.*

2 *Farthendels*	*make*	*halfe an acre of ground.*	
4 *Farthendels*		*an acre.*	Acre.

¶ *More, 40 rods in length do make a furlong, 8 furlongs make an English mile, which containeth 320 Perches.*

So that an English mile, grounded upon the Statute, is in length 1760 yards, 5280 foot, and 63360 Inches.

Somewhat greater then the Italian mile of 1000 paces, and 5 foot to a pace.

Here

Here might I tell you many things else touching measures, and also how to reduce strange measures to our measures, but because it cannot be well done without the knowledge of Fractions, which as yet you have not learned, I will let them passe till another time, that I have taught you the knowledge of broken numbers.

The parts of time.

Scholar. But yet Sir of the parts of time, I pray you tell me some what.

A day. An houre, Weeke, Monet Yeare.

Master. You know that a naturall day hath 24 houres, and every houre hath 60 minutes. It needeth not to tell you, that 7 dayes make a weeke, and 4 weekes make a common moneth, and 13 months make a yeare, lacking one day and certain houres and minutes: but of that I shall instruct you hereafter.

Here will I make an end of Reduction *for this time, which though it be counted no kinde severall of* Arithmetick, *you see it is no lesse needfull to be known, or easier to be done, then any of the other.*

Scholar. Marry sir, it seemeth unto me much harder then any other sort, for it requireth the knowledge of so many things: but now Sir when you see time I am ready to learne forth, as much of Reduction as you have taught me, I remember; but and if I doe at anie time forget, I shall have recourse to the Tables which you set forth for me.

Master. So doe you: for it will not be remembred without exercise. But in as much as you understand so much as we have intreated of, I will now instruct you in Progression.

Progressi-

Progression.

ALthough untill this day the most part of Writers have defined Progression as a compendious kinde of Addition, yet truely it is not so: for Progression as the very nature of the word doth informe any man) is a going forward and preceeding in numbers, and that regularly and orderly, whose place is aptly chosen to be very neere, or rather next after the exposition of the foure principall parts of Arithmeticke: for in it after a most easie manner, are al the foure former parts exercised and practised: and not onely Addition, as customably is done. Which custome hath been the cause, why it hath so specially been named a kinde of Addition, and defined to be a quick and brief Addition of divers summes, proceeding by some certain and reasonable order, What Progression is.

You shall also understand there are infinite kinds of Progressions, but for you (as yet) two are sufficent to be exercised in, of which the one I call Arithmeticall and the other Geometricall.

Arithmeticall progression is a rehearsing or placeing downe of many numbers, number after number, in such sort that between every two next numbers rehearsed or placed downe the difference, diversity, or excesse, be equall and alike. Arithmeticall Progression.

Scholar. Sir I thank you for that you have both opened unto me what Progression is truly, and also why it is here placed.

But I pray you with an example make plaine your definition.

Master.

Master. Examples cannot want, seeing all reasonable creatures naturally use the order of one kind of Arithmeticall progression (which therefore is also named naturall) whensoever they distinctly do count or number any multitude by one, saying, 1, 2, 3, 4, 5, 6, whereby the proceeding from number to number, and every one surmounting and exceeding his fellow next before by a like quantity (which here is 1) declareth the same to be Arithmeticall progression. And for the more plainnesse, I set it down in this manner.

The comon excesse. 1 1 1 1 1

The progression. 1 2 3 4 5 6

Scholar. This is most evident. And I think that I am able to tell you now of any progression Arithmeticall propounded, what is that common excesse or difference whereby it proceedeth, if this order be kept in it.

Master. What say you of 3, 6, 9, 12, 15;

Scholar. They exceed each other by 3: And that may I set down in such evident order, as you did your example of naturall progression, in this wise.

The common excesse. 3 3 3 3

The progression. 3 6 9 12 15

Master. And doe you not also now perceive, that the whole Table of Multiplication may be made by the other of progression Arithmeticall? either

either if you will begin at the first number of any of them on the left hand, and so proceed right over-thwart: or any of the first number of the upper row, and go directly downward.

Scholr. I pray you let me consider the thing a little, and I will answer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9.	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceive it now very well, for the common excesse or difference betweene any two next, is continuallie as much as the first number of everie row, either from the left hand overthwart taken, or from anie of the uppermost overthwart rowes downward.

Master. Now then, if of any such progression, you would speedily know the totall summe much quicklier then by common Rules of Addition: first tell how many numbers there are (which numbers here we call places or parcels) and if they be odde, write their summe downe by it self: as in this example, 2, 4, 6, 8, 10, 12, 14. where the numbers are 7, as you may see: therefore set downe 7 in a place alone,

To kuow the totall summe of an Arithmeticall progression.

alone, then adde together the first number and the last, as in this Example: Adde 2 to 14 and that maketh 16, take half of it, and multiply by the 7 which you noted for the number of the same places, and the summe that amounteth, is the summe of all those figures added together: as in this example 8 multiplied by 7 maketh 56, and that is the summe of all those figures.

Scholar. That will I work by another example. I would know how much this summe is, 5, 8, 11, 14, 17, 20, 23, 26, 29, I tell the places and there are 9, that I note. Then I put the first number 5, & the last 29, together, and they make 34. I take half of it, that is 17, and multiply by 9, and it maketh 153. That you say is the summe of all the numbers.

Master. So shall you find it if you try it.

Scholar. How shall I try it?

Master. By your common Addition, for if you adde all the parcels together, you shall see the same summe amount, if you did work well. And that manner of Addition trieth all kinds of summing any progression.

Scholar. Then can I summe any progression, if the number of the parcels be odde. But what if they bee even, as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Master. When the number of the parcels is even, then note that also as you did before, and likewise adde the first summe to the last, and by the half of the number of the places, doe you multiply it: as in our example, the parcels are 8, that I note, then adding the first summe to the last

last, there amounteth 9, that doe I multiply by half of the parcels, that is, by 4, and it maketh 36, which is the summe of the parcels.

But if you will take one Rule for these both, doe thus: Multiply the half of the one by the other whole, and the summe will amount all one. For sometime it chanceth that the number of the parcels be odde, so their half cannot be taken: and that some time it chanceth the Addition of the first number and the last, doe bring forth an odde number, so that halfe of it cannot be taken: but they will never be both odde. A generall rule.

Scholar. Then I perceive this, if there be no more belonging to it.

Master. As accustomably it hath béen taught, this hath been the chiefe and onely exercise in Progression used. But that you may perceive how divers waies and to how great profit so simple a thing (as this Arithmeticall Progression is) may be considered and used, I will here propound you six Propositions: of which foure of them were invented by a friend of mine, and never before this published: and the two first were never to my knowledge written of but by three men.

Scholar. This doth greatly encourage me to be attentive unto your words, séeing I shall not onely be instructed at your hands in the common knowne Rules of this excellent Art, but besides that so abundantly in other new Rules enformed, as my very entrance shall seeme to passe a great many mens further studie, and longer continuance. Therefore Sir, I beséech you let me know your six Propositions.

Master.

Master. These they are.

1 *To know the last* number *without proceeding by continuall* Addition, *till you come unto it, so that the common excesse, the first* number, *and the* number *of the places be known.*

2 *The first* number *of the* Progression *and the last being known, with the common excesse to finde the* number *of the places.*

3 *The excesse being given, and the* first *or* last, *to know the quantity of any middle* number, *whose place is given from the* first *or* last.

4 *The* totall summe *being given, and the* first *and* last, *to finde out the* number *of the places.*

5 *The* totall summe *of any* Arithmeticall Progression *being given, and the* first *and* last, *to find out the common excesse.*

The totall summe *being given, and the mutuall excesse with the* number *of the places, to give the first or last* number *of the same* Progression.

Many moe considerations could I propound you in these *Arithmeticall Progressions*, but these are sufficient, to give you occasion to think, that Rules of knowledge and Arts are infinitely capable of enlargment.

S. Happy were I if I did but well understand that which is already invented and written: And yet in my simple fantasie, these things offer themselves (in manner) to be studied for about Progression, therefore I pray you to proceed to the Rules answering to these propositions.

Master. I will orderly for every of these 6 propositions give you Rules, and with every one an example, unlesse the plainnesse and easinesse need no further exemplifying.

For

For the Solution of the firſt multiply the exceſſe by a number *leſſe by* 1 *then the* number *of the places, and the off-come adde to the firſt* number, *ſo you ſhall have the laſt* number, *which is ſought for.* 1 Propoſition.

As for example. If there were ſeven places in a progreſſion Arithmeticall, whoſe continuall encreaſe or mutuall exceſſe were 4, and the firſt number were 5, and I would know what the laſt and ſeventh number is: I multiply 6 which is one leſſe then 7, (the number of the places) by 4, thereof commeth 24, which I adde to 5, that maketh 29, and that is the laſt number which I deſire to know. And this you may ſtraightway prove by continuall proceeding from 5, till the ſeventh place, encreaſing every one by 4, as thus.

5, 9, 13, 17, 21, 25, 29.

Loe here the laſt, being alſo the ſeventh, is 29.

Scholar. I perceive already one good property in this Rule, which in all workes is to be deſired: that is, it will eaſe one from great labour, if a progreſſion were propounded of a hundred or two hundred places, or moe: And alſo it is very eaſie to work, and moſt neceſſary for the totall ſumme finding, in a very long progreſſion.

M. It is true, & therefore now let me ſee if you can anſwer me this queſtion by this propoſition.

A Merchant buyeth 50 pounds of Spices, & agreeth to pay for the firſt pound 4 pence, for the ſecond 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The queſtion is, how much he muſt pay for the laſt pound, and then how much the 50 pound commeth to?

Scholar. According to the proposition, I multiply 49 (which is lesse by one then the number of the places) by the excesse, which is 3: to the product 147, I adde the first number which is 4, it maketh 151 peace, the price of the last pound. Now I adde 4, the price of the first pound, to 151 the price of the last pound, it maketh 155, which I multiply by half the number of the places, which is 25 the Product, 3875 pence is the totall sum or price of the 50 pounds of Spices, as appeareth.

49 places 1 lesse	151 last
3 excesse	4 first
147	155
4 first	25 halfe places
151 the last	775
	310
	3875 totall summe which amounteth to
	li s d
	16——2——11

Master. It is truly wrought.

Scholar. Then I intreate you to procéed to your second proposition.

2 Proposition. Master. *The second* Rule *is this. From the last subtract the first, the* remainder *divide by the common excesse, to the* Quotient *adde 1, and you have the* number *of the places, which you would know, as in this* Progression.

6. 11. 16. 21. 26. 31.

If I know onely *6* and 31, and that they encrease by 5, then according to the Rule, from 31 I subtract

subtract 6, there remaineth 25, which 25 I divide by 5 (the common excesse) the Quotient commeth forth 5, to which I adde 1 that maketh 6: and so many are the places as you see.

Scholar. This Rule is so easie, that I were much to blame, if I could not remember it.

Master. *The third* Proposition *may alwayes thus be solved.* *Multiply the* excesse *by a* number *lesse by one, then the distance of the place is from the* first, *or the* last number given: *the offcome adde to the* first, *if the* distance *be reckoned from the* first, *& the* first *also known; or subtract from the* last, *if the distance be from the* last counted, *and the* last given *also, and that which commeth forth, either in that* Addition *to the* first *or* Subtractiō from *the* last *is the* number sought. As for Example, *I propound you this* Progression. 3 Proposition.

8 15 22 29 36 43 50 57

And for the apt considering the manner of this question, I will note over every place his distance from the first, and under every place his distance inclusively from the last, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the excesse whereby this Progression standeth, be known to be 7, and the first numbers given, being 8, I would know what number standeth under 4, that is to say, in the fourth place. I multiply 7 by 3 (which is lesse by 1 then the number of the place proposed) that yeeldeth 21, to which I adde 8, the first number, so commeth 29: which I say to belong to the fourth place, as you see in the example: or if in the

third place from the laſt, you would know what number in this ex·mp·e ſhould ſtand, the laſt number being known to be 57, and the common exceſſe 7, then by 2 which is leſſe by 1 then the place propounded, I multiply 7, that giveth 14, which appertaineth to the third place incluſively reckoned from the laſt, and ſo my example giveth you.

Scholar. I perceive right good uſe of this rule: for if I had forgotten what the firſt number were and remember ſtill but the laſt, the common exceſſe, and the number of the places, then might I come by the knowledge of my firſt number again.

And me thinketh, that it differeth not much from the firſt Propoſition, ſaving that which you make here a middle number, there was made the laſt: and alſo in this point it differeth, that in it the laſt was only ſought, and no conſideration had in numbring the places from the laſt, as here I marke in your numbers noted under your Progreſſion.

Maſter. And thinke you not, the middle numbers of progreſſion ſtanding off a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continuall encreaſing from the firſt by the common exceſſe, or abating from the laſt continually the common exceſſe as the very ſmall Numbers in a ſhorter Progreſſion would doe?

Scholar. Yes ſir, that I think right well and therefore I am glad of this new framed propoſition, and the manner of the working of it.

4 Propoſition. Maſter. *The rule of the fourth is this. Adde the firſt & the laſt together, and by the off-come divide the totall*

totall ſumme. *Double the* Quotient, *and that will be the* number *of the places.*

Scholar. Then if in a progreſſion, whoſe ſumme were 207, and the firſt number 12, and the laſt 57, if I add 57 and 12 together, that maketh 69, and by it I divide 207, the Quotient will be 3, which I double; and ſo I have 6, and ſo many muſt be the number of the places that this progreſſion ſtandeth on.

Maſter. Whether it be ſo or no, how will you trie?

Scholar. Half 6, which is 3, being multiplied by 69, muſt make 207, the totall ſumme; if 6 be the number of the places. For ſo the whole worke of your Rule in ſumming any Arithmeticall progreſſion did enforme me. I will then multiply 69 by 3, thus. It commeth forth juſtly.

	69
	3
	207

Maſter. I muſt much herein commend your promptneſſe both in memory and in well applying your Rule: although in manifeſt words it did contain no ſuch matter.

Scholar. Sir, I pray you heare me frame one example or more.

Maſter. I am well pleaſed ſo that ye be ſhort, for you make me longer here then willingly I would have béen: but I cannot perceive how I could have omitted any thing as yet, without your great lack thereof.

Scholar. If I had received 85 pounds of certaine men, but of how many I have forgotten, yet I remember that the firſt gave me 7 pound, and the laſt 27 pound, and every payment after other did riſe by

A queſtion of money.

by a like summe. And the man for whom I received this money, conditioned with me, that of every payment I should have twelve pence for my labour: now unlesse I can by Art finde the truth of this case, I am like to lose the most part of my reward.

Master. I perceive you can handsomely frame an example, which should concern your own gaine: I pray you let me see how you would do justice in this point.

Scholar. I adde the first and the last together, that maketh 34: by which I divide 85, thus:

1	
27	17
85 (2	
34	34

Why how now? Sir, here is a remnant of 17, in which 34 cannot be had: so that now I am in the briers for doubling of my Quotient, and farewell then both my Justice, and a good lump of my gains.

Master. Ye are never the farther from the matter, though it fall into a Fraction. For you shall understand that the Fraction which of any such work proceedeth, is every half of one such, as the units of the Quotient before are. And that you may try, if you double that which so remaineth, for then it will be equall to your divisor, as if ye double 17, (the remnant) it maketh 34, and your divisor also was 34, this noteth the remainder to be halfe of one.

Scholar. Now I am glad of this hard Example: For with it I have a generall rule for the Fraction that may hap in this work. So that the Quotient being 2 & a halfe, I double that, it maketh 5, therefore should my gain be 5 shillings. And to be sure (by your leave) I will trie it, for I will multiply halfe

halfe of 34 (which is the first and last number joyned together) by 5, thus.

	17
	5
	85

It is most true (I see) that I should leese nothing by the former working.

Master. *The fifth proposition hath this rule appertaining unto it: By the fourth rule finde the* number *of the places, that being done, from the last subtract the first, and the residue divide by a* number *lesse by 1 then the* number *of the places, and the* quotient *will shew the excess, which is sought for.* 5 Proposition.

An example hereof shall be this. If ye had disbursed 685 pounds to a certain number of men, you neither can tell how many they were, or how much the ones money exceeded his next before, but you are sure that the excesse was equall between every two next: and also you remember that the first had 19, and the last 118 pounds, how would you find the number of the name and the excesse continually observed in the succession of their payments? Example.

Scholar. Your Rule doth plainely bid, first to finde the number of the places, which I will do according to the fourth rule: I adde 19, and 118 together, thus.

	118
	19
	137

By this 137, divide 685 thus.

Seeing there is no Fraction but a whole number, being 5, I double that, and then must the number of the places bee 10. Now from the last I subtract the first, as 19 from 118, thus: And so remaineth 99.

x3	
685	(5
x37	

	118
	19
	99

This 99 I divide by a number lesse by one then the number of the places, and seeing the places were 10, I divide

99 by 9; thus: 99

The quotient is 1, and so was 99 (11

the excesse, if I have followed your rule right.

Master. You have wrought every part of this question both well in order, and truly in the practise of your Rules.

Scholar. I will then set it down also formably, so that the number of the places, the excesse, and the totall summe may straight appear, as your first example stood.

The common excesse.

The common excesse.	11	11	11	11	11	11	11	11	11	11
The progression.	19	30	41	52	63	74	85	96	107	118

That the places be 10, and that from the first to the last, the common excesse is 11, I perceive most evidently: but whether the totall sum be 685, I have not yet proved, which I will now doe: I adde 19 & 118 together, that maketh 137: I multiply that by halfe the number of the places, thus. 137 5

All things agrée most exactly, so that I am perfect enough in these Rules if I forget them not again. 685

Master. Use maketh all things perfect.

6 Proposition.

Your sixth rule is this. By the number of the places divide the totall summe, double the quotient, and that will be the first and last joyned in one summe. Then by a number lesse by 1, then the number of the places, multiply the excesse, that off come subtract from the first doubled quotient, and the halfe of the residue is the first number. The last number you may diversly find out, as by the first of our 6 Rules*, or by subtracting this first number from the* summe *which here contained both the first and last ioyntly, (or thirdly) by continuall adding the excesse,*

Scholar.

Scholar. I pray you make this somewhat more plain with an example.

Master. If every moneth in the year (counting them now as 13) you gained clearly 40 shillings more then you did the moneth next going before, and at the years end you find the whole gain 5720 shillings, but ye remember not how much either the gaine of the first moneth or the last was, by this Rule it may be tried out. Example of graine.

Scholar. So that here ye seeme to apply the 13 moneths to thirteen places, the 40 shillings every one more then the other next before it to be the common excesse, and 5720 shillings to the totall summe.

Master. It is true: by 13 then I divide 5720 in this manner.

x
x5
5720 (440
x333
xx

I double this Quotient, so have I 880 for the first and the last summe joyned together; by 12 which is lesse by one then the number of the places; I multiply 40 (the common excesse) so commeth 480.

40
12
80
4
480

This 480 I subtract from 880, so remaineth 400: half whereof is the first number which we desired to know: that is 200. And as for the last number, I can give you itt three wayes, As by the first of my six rules I multiply the excesse, by a number lesse by 1 then the number of the places, as 40 by 12 that giveth 480, which I adde to the first, being 200, so shall the last be 680.

The same summe commeth forth if ye subtract 200 from 880.

And thirdly, If I begin at 200, and so proceed, encreasing by 40, I shall at the thirteenth place

place have 680, as thus:

200 240 280 320 360 400 440
480 520 560 600 640 680

Scholar. I thanke you most heartily for these six Rules. Now if it be your pleasure I would heare and learn somewhat of progression Geometricall.

Master. There are yet very many Rules and propositions which fall into this Arithmeticall progression.

And for the use and practise of them, I will propose unto you certain pleasant an necessary Questions of Arithmeticall progression, and to the performance of their workings, such necessary rules and documents, as are requisite for the better understanding of them, or any such like.

A question of Velvet.

A certain Mercer *sold 20 yards of Velvet to be paid in 12 weekes by* Arithmeticall proportion; *that is to wit, to receive the first week 6* shillings, *the second week* 12 shillings, *the third week* 18, *and so forth, increasing the* number *of weeks by* 6 shillings, *till the twelfth and last week were expired.* The question is how many pounds he had for 20 yards of Velvet.

To the performance of this question, and such other the like, I set forth the 12 payments in such sort, as for example, here appeareth.

6
12
18
24
30
36
42
48
54
60
66
72

Then touching the adding together of these

theſe ſummes, without the aid of Addition, according to the rules I taught you in Progreſſion Arithmeticall. I note the number of the places, which are 12, then adding the laſt number of the Progreſſion, which is 72, and the firſt number together make 78; and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 23li. 8s. And ſo much hath the Mercer for his 20 yards of Velvet, which is nigh about 23 ſhillings 5 pence a yard.

Scholar. I underſtand this work very well, but is there any proofe for the juſtifying hereof, as you have of other workes?

Maſter. The worke of it ſelfe (being ſo perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantity, is all that is demanded for juſtifying of the ſame: yet notwithſtanding, because your requeſt is reaſonable, I will propoſe an example for the proofe hereof.

A certaine man is bound to pay for 20 yards of Velvet, the ſumme *of* 23 pound 8 ſhillings, *and it is to be paid weekly in* 12 *weeks or termes by* Arithmeticall Progreſſion. The queſtion is therefore to know with what number the ſame Progreſſion is to be begun and continued in ſuch equall proportion Arithmeticall, that in 12 weeks the ſame may be juſtly accompliſhed. The proof of the laſt queſtion.

For the reſolution whereof, and of all ſuch other like, reduce 23 pound 8 ſhillings all into ſhillings, which maketh 468 ſhillings. A generall rule.

Then adde 1 unto 12, the number of the termes, it

it maketh 13 which 13 you shall multiply by half the number of the termes which is 6, it maketh 78; then divide 468 by 78, and you shall finde 6, in the Quotient, which is the true number, that shall begin and continue the said Progression. That is to say, the first week 6 shillings, the second 12 shillings, and the third weeke 6 shillings more, which is 18 shillings, & so every weeke as they rise 6 shillings more then the weeke before as is manifest in the question aforesaid.

Aquestion of a Farm. *A Farm is to be sold to be paid by the weekes in a year; the first weeke to pay 4 shillings, the second weeke 8 shillings, the third weeke 12 shillings, and so forth, increasing each number by 4, till the number of 52 (which are the number of weekes in a yeare expired.)* The question is, what the price of the Farm commeth to?

Schol. I doubt not, but by that you have already taught me, to end this question very well; wherefore I set forth the Progression with his excesse 52 times.

Master. Nay stay a while: And here for your further ease, (to abridge you of great labour that appeareth to fall out in this question, and so may doe in any other the like) If a question were propofed of 100 or 200 places, or moe, and that this question, or any other the like can be ended, unlesse you may know absolutely what the last number of the Progression of the 52 place is, (or ought to be) I will give you a generall rule how to know the last number of any Progression Arithmeticall, as well as if you had ordinarily proceeded by continuall Addition, till you had come to the last worke which is this.

Mul-

Multiply the excesse by a number lesse by one then the number of the places, and thereto put the first number of the Progression, and you shall have your desire. A generall rule.

Scholar. This Rule is well worth the noting: for if I understand you aright, I consider that my excesse is 4, which I multiply by 51, which is one lesse then the number of the places, and it maketh 204, whereunto I adde the first number of the Progression, which is 4, and then it is 208, which you say is or should be the last number of the Progression.

Master. This is a most approved truth, if there were never so many places.

Schol. This Rule is so easie, that I were much to blame, if I doe not remember it. For by the benefit hereof, I have such an ease and light into this excellent Art, that my first entrance doth seem to passe a great many mens further study, and longer continuance.

Master. Many moe Considerations could I propound you in these *Arithmeticall Progressions*; but these are sufficient for a taste, to give you occasion to think that *Rules* of Knowledge and *Arts*, are infinitely capable of enlargment.

Scholar. Happy were I, if I did but well understand that which is already invented and written. But these things in my simple fantasie, offer themselves to be greatly beneficiall unto the aide of Progression. Therefore now I will goe forward with your Question.

Now considering that the 52 and last place is 208, I adde thereunto the first numbers of the Progression

gression, which is 4, it maketh 212, which I multiply by half the number of the places, which is 26, and it amounteth to 5512 shillings: And so much is the totall summe or addition of this progression: which maketh 275 pounds 12 shillings, as appeareth here by my Tables.

Master. I like well your labour, and commend you for your diligence: I will here propose one example more, and therewithall for this time will end progression Arithmeticall.

A question of Holland. *A certain man brought* 20 Ells *of* Holland, *to be paid in* 17 *weekes, or termes, by* progression Arithmeticall. *And the first week to pay* 1 shilling 8 pence, *the second week* 3 shillings 4 pence, *the third weeke* 5 shillings, *the fourth weeke* 6 shillings 8 pence, *and so forth, each weeke succeeding* 20 pence *more then the weeke before.* The question is, what the summe of his 20 Ells cometh to.

Scholar. Because here is mention made both of shillings and pence, I feare there is some harder matter contained herein, then in the other before: therefore I pray you worke it your selfe, and I will diligently mark your labour.

Master. There is no more to be done in this, then in the other before; but because your request is so reasonable, be attentive unto me.

First, by the generall Rules, I seek to finde out the last Number of the 17 place, what this progression ought to be. Therefore here in my Tables multiplying the excesse 20 by 16, which is one lesse then the number of the termes or places, and it commeth to be 320; and thereunto adding the first number of the progression, which is 20

pence:

pence, all is 340 pence, or 28 ſhillings 4 pence, for ſo much ought the laſt number of the payments to be.

Then finally, to know what the whole 17 places amount unto, I adde the firſt number of the progreſſion and the laſt together, which make 360. Now becauſe 17 is an odde number, whoſe halfe cannot be taken, I take the halfe of 360, which is 180; and multiplying 180 by 17, commeth to 3060 pence, which maketh as you ſée by Diviſion 12 pound 15 ſhillings. And ſo much is the buyer to pay for his 20 Elles of Holland. Which 3060 pence if you divide by 20, the number of Elles that was bought, you ſhall find 12 ſhillings 9 pence, and ſo much payed hée for an Elle one with another.

The Proof.

A certaine man doth owe 12 pound 15 ſhillings, *to be paid in* 27 weeks *or* termes *by* Arithmeticall progreſſion. *The queſtion is, to know with what* number *he ſhall begin and continue the* progreſſion, *in ſuch equall proportion, as the ſame may be truly paid and ſatisfied in* 17 weeks. A queſtion of debt.

The Anſwer.

First I reduce 12 pounds 15 ſhillings, all into pence, which as you ſée here in my Tables, make 3060 pence, that I let ſtand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I ſhould multiply by halfe the number of the weekes or termes,

which

which is 8 $\frac{1}{2}$ which 8 $\frac{1}{2}$ multiplied by 18 cannot well be done, unlesse you were acquainted with Fractions or broken Numbers, therefore you shall let that passe & multiply 17 by the half of 18, which is 9 (for that is all one with the multiplication of 8 $\frac{1}{2}$) & the multiplication of 9 into 17 maketh as you see 153, with which number you shall divide the 3060 pence beforesaid, and the Quotient bringeth forth 20 pence, which is the first number or payment to beginne the progression withall: and so each week succeeding to rise 20 pence more then the weeke before, and thereby in 17 weeks shall 12 pounds 15 shillings be payed: as before was sufficiently declared. Thus much for progression Arithmeticall.

Scholar. Certainly Sir, I know not how to render you condigne thanks for these benefits shewed me, which me thinketh are so easie, delightfull, and pleasant, that I count my selfe happy to be in your company.

Master. I am glad you delight so well herein, which is an Art of wounderfull dexterity to all sorts of men, of what degree or profession soever they be. And now will I proceed to progression Geometricall, wherein I will be more briefe, both because I have been so long in this part of Arithmeticall progression, and also for that it would require the knowledge of Roots and surd numbers. (whereof ye have learned nothing) if I should frame the like propositions in them as I have done in these. Therefore I will onely teach you to practice about it, and so end the considerations and works of these progressions.

Progression

Progreſſion Geometricall *is when the numbers increaſe by a like proportion, that is, if the ſecond number containe the firſt, 2, 3, or 4 times and ſo forth, then the third containeth the ſecond ſo many times alſo: and ſo the fourth the third, and the fifth the fourth; wherefore I ſet theſe 3 examples.*

Progreſſion Geometricall.

3	6	12	24	48
1	3	9	27	81
2	10		50	250

Here in the firſt Example you ſee that every number containeth the other (that goeth next before him) two times; and in the ſecond example three times, and in the third example five times. Now if you will know how to finde eaſily the ſumme of any ſuch number, doe thus: Conſider by what numbers they be multiplied, whether by 2, 3, 4, 5, or any other, and by the ſame number multiply the laſt ſumme in the Progreſſion.

Scholar. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, which I have framed by proceeding from 2, and continually multiply by 4.

To finde the totall ſumme in any Geometricall Progreſſion.

Maſter. Then muſt I multiply the laſt ſumme (which is 2048) by 4 alſo, and it will be 8192, Now muſt I abate from this ſumme the firſt number of the progreſſion, which here is 2, then reſteth 8190, which ſumme I muſt divide by 1 leſſe then was the Number that I multiplyed by. Seeing then I multiply by 4. I muſt divide by 3, ſo dividing 8190 by 3, the Quotient will bee 2730, which is the ſumme of all the progreſſion. And now to prove whether you can doe the ſame, I give you theſe Numbers to adde by this Rule 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tell by what number

this progression doth increase.

Master. In any such doubt doe thus: Divide the second number by the first, and the Quotient will shew you the number that engendreth the progression.

Scholar. Then is that number in this example 5, for so many times is 3 in 15.

Master. So is it. Now work as I taught.

Scholar. The last number is 46875, which I multiply by 5, & it yeeldeth 234375, from which I abate the last number of the progression, that is 3, and there resteth 234372, which I divide by 4, for that is one lesse then 5, and the Quotient is 58593, which is the whole summe of the progression.

Master. If you remember well this, you have learned the Art of progression both Arithmeticall, and also Geometricall, which you may prove either by subtracting of each number alone from the sum, and so will there nothing remain: or else by adding together of all the parcels, for so will the same summe amount.

A question of Satten.

A Mercer hath 12 *yards of Satten, which he valueth at* 16 *shillings the yard, and selleth the same* 12 *yards to another man to be paid as followeth: That is to wit, for the first yard to have one shilling, for the second yard two shillings, for the third yard foure shillings, for the fourth yard eight shillings, &c. doubling each number following, till the twelfth and last yard.* The question is, who hath made the better bargain of the buyer or seller.

First you may set down 12, the number of the yards as you see here in this Example. And against each number the number of shillings due to be paid, as

as the order of Duplation or Multiplication by two teacheth.

Then resorting to the adding up or summing of this Progression, where I consider that the increase of this summe proceeded by the Multiplication of 2, and therefore after I have drawn a line under the 12, I worke and multiply the last summe by 2 also, and it yeeldeth 4096: from whence I abate the first number of the progression, which is 1, & then resteth 4095: which I should divide by one lesse then I did multiply by, but seeing it is 1, I need not to divide it: for 1, (as I have said before) doth neither multiply nor divide, therefore I take that summe 4095 for the whole summe of the shillings, which by Reduction amounteth to 304 pounds 15 shillings, and so much hath the Mercer for his twelve yards of Satten: which is 17 pound, 1 shilling, 3 pence a yard. But I think you will buy none so deare.

s	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	

Schol. No Sir, by the grace of God this yeare.

Master. *Then what say you to this* question? *If I sold unto you an horse having* 4 *shoes, and in every shoe* 6 *nailes, with this condition, that you shall pay for the first nayl one* ob. *for the second nayl two* ob. *for the third nayl foure* ob. *and so forth, doubling untill the end of all the nayls.* Now I aske you, how much would the price of the horse come unto?

A question of an horse.

 Scholar.

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24
16777216	

Scholar. Firſt, to know the number of the nayls, I muſt multiply 6 by 4, & it maketh 24. Then will I doe thus. I will write the number of the nayls every one in order from 1 to 24, and againſt each number of the nayls the ſumme of halfe pence duly, as the order of Duplation or Multiplication by 2 teacheth, and as in the next figure following appeareth.

Then do I reſort to the Rule of ſumming up the progreſſion, where I conſider that the increaſe of this ſumme proceedeth by the multiplication of 2, as the laſt Example did. And therefore multiplying the laſt ſumme by 2 alſo, and it yeeldeth 16777216, from which I abate the firſt Number which is 1, and then reſteth 16777215, which I ſhould divide by one leſſe then I did multiply: but ſeeing that it is 1, I need not to divide it, for 1 (as you have before ſaid) doth neither multiply nor divide, therefore I doe take the number, 16777215 for the whole ſumme of the halfe pence, which by Reduction I find to bee 699050 ſhillings and 7 pence halfepenie, that is 34952 pounds 10 ſhillings 7 pence, ob.

Maſter. That is well done, but I thinke you will buy no horſe of the price.

Scholar.

Scholar. No Sir, if I be wise.

Master. Well then, answer mée to this Question.

A Lord delivered to a Bricklayer a certain number of loads of Bricke, whereof he willed him to make twelve walles, of such sort, that the first wall should receive two thirdells of the whole number, and the second two thirdells of that which was left; and so every other two thirdells of that that remained: and so did the Bricklayer: and when the 12 walls were made, there remained one load of Brick.

Now I aske you, how many load went to each wall, and how many load was in the whole?

Scholar. Why Sir, it is impossible for mée to tell.

Master. Nay, it is very easie if you marke it well. Marke well that I said, that every wall should receive two thirdels of the summe that was left. Now take away two thirdels from any summe, & you must néedes grant that that which remaineth, is one thirdel of the summe last before, Example of 9. from which if you take 2 thirdels, there will remain 3, which is 1 thirdle of 9. Likewise from thrée bate two thirdels, and there remain 1.

Scholar. This is true, and now I perceive the least wall had but two load of brick.

Master. And by the same reason may you know how many load every wall had, according as this figure following doth shew, and likewise what the whole summe of brickes was, for if you make 12 summes, multiplying by 3, still from the last remainer, as you may sée here on the left side of the Table, there will appeare all the remainers of the whole wall: and if you multiply the last of those 12 summes by 2 also, then will that be the summe of

the loads which was delivered to the Bricklayer.

The remainer after every wall.			Loads due to each wall.
1	12	2	
3	11	6	
9	10	18	
27	9	54	
81	8	162	
243	7	486	
729	6	1458	
2187	5	4374	
6561	4	13122	
19683	3	39366	
59049	2	118098	
177147	1	354294	

Summe of the loads delivered 531440

Again if you double every Remainer, as you may see at the right side of this Table, those numbers will shew the summe of loads that went to each wall, whereby you may perceive that each wall was three times so great as the next lesse.

Scholar. Lo now it appeareth easie enough. Now surely I see that Arithmetick is a right excellent Art.

Master. You will say so when you know more of the use of it: For this is nothing in comparison to other points that may be wrought by it.

Scholar. Then I beseech you cease not to instruct mee farther in this wonderfull cunning.

The

The Golden Rule or Rule, of Proportion direct, called *the Rule of Three.*

Master.

The Rule of Proportion.

BY *order of the Science (as Men have taught it) there should follow next the* Extraction of Roots of Number, *which because it is somewhat hard for you yet, I will let it passe for a while, and will teach you the feat* of the Rule of Proportion, *which for his execellency is* called the Golden Rule. Whose use is, by three numbers known to find out any other unknown; which you desire to know, as thus.

The Golden Rule.

Question of boarding.

If you pay for your board for three moneths sixteen shillings, *how much shall you pay for* eight moneths ?

To know this and all such like questions you shall consider which two of your numbers be of one Denomination, and set those two one over the other, so that the undermost be it that the question is of: as in my question 3 and 8 be both of one Denomination, for they both be months; and because 8 is the number that the question is asked of, I set the one over the other, and 8 undermost thus, with such a crooked draught of lines.

3
8

Then doe I set the other number which is 16, against 3 at the right side of the line, thus.

3 16
8

Note.

And now to know my question, this must I do: I must multiply the lowermost on the left side, by that on the right side & the sum that amounteth, I

must divide by the highest on the left side; or in plainer words, thus, I shall multiply the number of which the question is asked (which is called the third number) by the number of another Denomination (which is called the second) & the sum that amounteth, must I divide by the summe of like Denomination (which is called the first) Then for the knowledge of this question, I multiply 8 into 16, and there amounteth 128, which I divide by 3, and it yeeldeth 42 shillings, and 2 shillings remaineth, which I turne into pence, and they bee 24 pence, of which third part is 8 pence, so the third part of 128 shillings, is 42 shillings, 8 pence, which summe I write at the right hand of the figure against 8 thus.

The third number.
The second number.
The first number.

3 — 16 shillings
8 — 42 shil. 8 pence.

Hereby I know that if three moneths boarding, doe come to 16 shillings, that 8 moneths boarding will come to 42 shillings 8 pence, and likewise of any other like question.

But here must you marke, that the first number [illegible] the third be of one Denomination, and also the [illegible]d and the fourth, for which you seeke: or else be [illegible]ch Denominations, that you in working may [illegible] them into one; As if a man should aske me [illegible]stion.

Question of expence

[illegible]ue weeks journeying cost me 14 French [illegible] 6 shillings the peece, how many pounds is [illegible] yeare? Here you see no two numbers of [illegible]mination, but yet in working you may [illegible] into like Denomination: as thus; turn [illegible]e into 52 weekes and the fourth summe [illegible]ch Crowns, by the order of the working.

Then

Then to know this question multiply the third summe 52, by the second 14, and the summe will bée 728: that divide by your first number 12, and the Quotient will be 60 Crownes, and 8 Crownes remaining: which if you turn into shillings, they will be 48 shillings, which if you divide by your first number 12, the Quotient will be 4, which signifieth 4 shillings; put those 60 French Crownes, which make 18 pounds with the 4 shillings for the summe that answereth to the question, and it is the just expences of a yeare: And the work will be thus.

W		C
12	Z	14
52		60 $^{a}_{s}$

And take this evermore for a generall rule touching this whole art, that the doubtfull or unknown number that you would be resolved of, shall alwayes be set in the third place. Note also the first number and the third must ever be of of one nature and denomination, or else must in working be brought to like Denomination, & then of necessity must the other number be in the second place. A generall Rule.

Remember also that the place of the first number is highest on the left side, & the place of the second, right against it on the right side; the place of the third number is under the first, as by those examples you have seen.

Scholar. This I trust I can do.

Master. But and if the question be asked thus: *In 8 weeks I spend 40 shillings, how long will 105 shillings serve me?* Here you sée that 8 weeks answers himselfe and saith 40 shillings.

But how long time 105 shillings will serve you know not. Therefore you shall set 105 in the third place, according as I told you even now. And the first

first place must allwayes be of the same nature or Denomination that the third is of, which here is 40. Then must 8 needs be that other: Now multiply 105 by 8 and it will be 840, which if you divide by 40, it will yeelde 21, which is the fourth number, and sheweth how many weeks 105 shillings will serve if you spend 40 shillings in 8 weeks.

The figure of this question is this: as if you should say: if 40 shillings serve for 8 weeks: 105 will serve for 21 weekes.

Shillings.		Weeks.
40	Z	8
105		21

Other diversities there bee of working by this Rule, but I had rather that you would learne this one well, then at the beginning to trouble your minde with many formes of working, sith this way can doe as much as all the other, and hereafter you shall learne the other more conveniently.

Note. ¶ And for your further aide and instruction, to make you better acquainted with this Golden Rule, I have here proposed sixe questions, and their answers, which I thinke most convenient and meet to preferre the desirous to perfect understanding. The first foure are all branches of one question sprung out of the best tree (for a young learner to taste of) that groweth in this *Ground* of *Arts*: for that no manner of question in the Rule of three whatsoever it be, can be proposed, but it must be comprehended under the reason or stile of one of these foure.

The

The Questions.

If 15 *Elles of Cloth cost* 7 pound 10 shillings, *what comes* 27 *Elles to at that rate?* Answer; 13 pounds 10 shillings.

If 27 *Elles cost* 13 pound 10 shillings, *what are* 15 *Ells worth?* Answer; 7 pound 10 shillings.

If 27 *Elles cost* 13 pound 10 shillings: *how many Ells shall I have for* 7 pound 10 shillings? Answer; 15 *Elles*.

If I sell 15 *Elles for* 7 pound 10 shillings: *how many Elles are to be delivered for* 13 pounds 10 shillings? Answer; 27 *Ells*.

If 8 *pound of any thing cost* 16 shillings 6 pence: *what money is to be received for* 49 pound? Answer: 5 pound, 1 shilling 0 $\frac{2}{4}$.

If 4 pound *of any thing cost* 7 pence: *what money will* 8765 *pound of that commodity cost?* Answer; 63 pound, 18 s shillings, 2 d $\frac{3}{4}$.

Of all which questions, I omit the work of purpose, that you shall whet your wit thereby at convenient leisure, to clime each branch, and gather the fruit of them, and doe minde now, b[illegible] we make an end of this Rule, to give you s[illegible] structions of the backer Rule of three, whose o[illegible]er is quite contrary to this that you have learned.

Scholar. I thanke you heartily for the six Questions, which I will (God willing) practise at convenient times; I pray you proceed therefore to the Backer or Reverse Rule.

The

The Golden Rule, or Rule of Proportion Backward, or Reverse.

Master.

Note this well.

IN the former evermore look how much the third number is greater then the first, so much the fourth number is greater then the second. And contrariwise, look how much the first summe is greater then the third (if it doe chance so) so much is the second summe greater then the fourth.

The backer or reverse rule of three

But in this Rule, there is a contrary order, as this: That the greater the third summe is above the first, the lesser the fourth summe is beneath the second: and this Rule therefore you may call the Backer or Reverse Rule, as in example.

A question of cloth.

If I have bought 30 yards of Cloth of two yards breadth, and would have Canvas of three yards broad to line it withall, how many yards should I need?

Scholar. Why there is none so broad.

Master. I doe not care for that, I doe put this Example onely for your easie understanding: for if I should put the Example in other measures, it would be harder to understand. But now to the matter: If you would know this question, set your numbers as you did before: but you shall multiply now the first number by the second, and that ariseth thereof, you shall divide by the third: which thing if you doe here, I mean if you multiply 30 by 2 it will

will be 60: which summe if you divide by 3 there will appeare 20. whereby I know, that if 30 yards of cloth of two yards broad, should bée lined with Canvas of three yards broad 20 yards of Canvas would suffice, as this figure sheweth.

Breadth.	*Length.*
2	30
3	20

And now because ye found fault with my Example, how say you, perceive you this?

Scholar. Yes Sir, I suppose.

Master. *Then answer me to this question: how many Elles of Canvas of Ell breadth, will serve to line twenty yards of Say, of three quarters broad?*

Scholar. In good faith Sir I cannot tell, for I know not how to bring the sums to like Denominations.

Master. Then will I tell you: sith there is mention here of quarters, & again every one of the measures both Ells and yards may be parted into quarters, part them so both in the breadth & length, and then put forth the question by quarters.

Scholar. Then shall I say thus. How many quarters of Canvas of 5 quarters broad will line 80 quarters of 3 quarters broad?

Master. Now answer to the question.

Scholar. First, I will set them downe in their forme thus: for 5 is joyned with the question and is therefore the third number, then is 3 the number of the same Denomination, I meane because they be both referred to breadth. Now I multiply 80 by 3, & it is 240, which I divide by 5, & it yeeldeth 48.

Breadth.	*Length.*
3	80
5	

Then

Then ſay I that 48 quarters of 5 quarters broad will ſuffice to line 80 quarters of three quarters broad.

Maſter. Turne the quarters again into Ells and yards.

S. Then I ſay, that 9 Ells and three quarters of a yard of ellbroad, will ſerve to line 20 yards of three quarters broad, as this figure ſheweth.

Breadth.	*Length.*
3	80
5	48

¶ Maſter. *Now what ſay you to this queſtion? I lent my friend* 400 pound *for* 7 *moneths, how much money ought he to lend mee againe for* 12 *moneths to recompence my courteſie ſhewed him? can you anſwer to this?*

Scholar. Yes Sir, I ſuppoſe, for I will ſet downe my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I divide by 12, and it yeeldeth 233 pound, and there is 4 pound remaining of my Diviſion, what ſhall I do therewith?

Months.	*Pounds.*
7	400
12	

Maſter. Turne the ſame 4 pound into ſhillings, and then divide it by 12 as you did before.

Scholar. Well Sir, it ſhall be done: ſo have I 6 ſhillings for my Quotient, and yet remaineth 8 ſhillings upon my Diviſion.

Maſter. You muſt alſo reduce that 8 ſhillings into pence, which maketh *96*, and divide that alſo by your Diviſor.

Scholar. So have I done, and I finde 8 pence for my Quotient, and nothing is left.

Maſter.

Master. This must you alwaies doe when any thing remaineth upon your Division, whether it be money, weight, measure, or any kinde of thing whatsoever. This Rule is so profitable for all estates of man, that for this Rule onely (if there were no more but it) all men were bound highly to esteem Arithmetick.

By this rule may a Captain in war, worke manie things, as Master Digges in his Stratiocos doth declare. Onely now in this my simple addition, for a taste and incouragement, I will inlarge the Author with a question or two more, wishing you & every my Countreymen or Gentlemen whatsoever; that by nature be any thing given to Military affaires, to be familiar and acquainted with this Excellent Art, the which he shall finde not onely at the Sea, but also in the Campe and Field-service, aboundantly to aid him, either in fortification, paying of Souldiers wages, charges of Ordnance, Powder, Shot, Munitions, and Instruments whatsoever as for example.

If it should chance a Captain which hath 40000 *souldiers to be inclosed with his enemy, that he could have no fresh purveyance of victuals, and that the victuals he had would serve that Army but onely three moneths, how many men should he dismisse to make the victuall to suffice the residue* 8 *moneths?* Question of an Armie.

Scholar. As you taught me, I set the numbers thus, saying, If three moneths suffice 40000, to how many will 8 suffice.

Moneths.		*Men.*
3	Z	40000
8		

To know this; I multiply the first number 3 into

the

the ſecond 40000, and it yieldeth 120000, which ſumme I divide by 8, and there will be in the quotient 15000, which if I doe ſubtract from 40000, the remainder will declare that hee muſt diſmiſſe 25000 as this figure ſheweth.

Months.	*Men.*
3	40000
8	15000

A queſtion of a Fort.

Maſter. *Now anſwer mee to this queſtion: If 136 maſons in a moneth bee able to build a Fort to preſerve the ſouldiers from the Enemy, and ſuch expedition requireth that I would have the ſame finiſhed in eight dayes: how many workemen ſay you is there to be appointed?*

Scholar. As you taught mee, I ſet the numbers thus, ſaying: If 28 dayes require 136 Maſons, what number of men by the like proportion will 8 dayes require?

28	136
8	

To know I this multiply the firſt number 28 into 136, and it yieldeth me 3808: which I divide by 8, and my Quotient is 476: which is the juſt number of Maſons that ſhall ſupply this worke. And now me thinke theſe queſtions are very eaſie.

Maſter. Truly if you take delectation herein, you ſhall finde this Art not onely eaſie, but wonderfull pleaſant and profitable. Now therefore one queſtion more I will propoſe, and ſo leave off this Rule in whole numbers untill we come to the uſe of it in broken numbers: for had you the underſtanding of broken numbers perfectly, not onely in this Rule, but in all other, the queſtion that in the ſight or appearance ſeemeth to bee 100 times harder to reſolve, may thereby bee wrought as ſoon or ſooner then this.

Scholar.

Scholar. Your words doe greatly incourage me to bee studious to attain whole numbers: but might I once againe to be a Practitioner in broken numbers, I should thinke my selfe happy.

Master, *What say you then to this question? If 48 Joyners in two dayes make 200 light horsmen staves (esteeming they worke but 12 houres a day) and such need requireth that 384 Joyners are set to the finishing of those 200 staves; in what time, say you, will they make them up?*

Scholar. I see here that I must turn my 2 dayes into houres. And so doing, I set my numbers thus:

48 Z 24
384

Saying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded upon an old Proverb, *many hands make light work.*

I multiply 48, into 24 and it amounteth to 1152, which I divide by 384, and my quotient is 3 houres, which is my desire.

I take this for a note worthy the marking, either in the Rule of Three, forward, or backward, when the two numbers are multiplied together, the product is of the same nature and determination that the second number is of. Note.

The double Rule of Proportion direct.

Master.

The double Rule.

WEll, sith you perceive now the use of this rule, I will shew other which insue of the same and first the *double Rule*, which is so called because there is in it double working, by which thing onely it differeth from this.

Scholar. Then by an example I shall understand it well enough.

Question of carriage

Master. *So shall you, and let this be the example: If the carriage of* 100 *weight (that is* 112 *pound)* 30 *miles doe cost* 12 *pence, how much will the carriage of* 500 *weight cost being carried* 100 *miles?*

Scholar. I pray you shew me the working of it.

Master. You must make two workings of it: the first thus: If C weight cost 12 pence, how much will five hundred weight cost? Set your figures thus:

C Weight.		*Pence.*
1	Z	12
5		

And multiply 5 by 12, and thereof amounteth 60, which if you divide by one, the Quotient will be still 60, that is the price of 500 weight for 30 miles.

Then begin the second worke, saying: If 30 miles cost 60 pence, how much will 100 miles cost? Set your figures thus.

Miles.		*Pence.*
30	Z	60
100		

Then

Then multiply 100 by 60 whereof amounteth 6000, which being divided by 30, will yield 200 pence. Then you must say, that so many pence shall cost the carriage of 5000 pound weight 100 miles, after the rate of 12 pence for the 100 carried 30 miles.

Scholar. Now I perceive it also.

¶ Master. These and such other like questions of the double Rule of Three, are to be answered much sooner, at one onely working by the Rule of Proportion composed of five numbers, which anon I will shew you, and then when you have the use thereof, you may use it which way you think good.

Scholar. Sir, I thanke you much for your courtesie, And I long now till this Rule bee ended, that I may see how I may behave my self with that new Rule of five numbers: for that I have ever since you taught me hitherto in the Golden Rule, both forward and backeward, wrought but with three numbers onely.

Master. But yet awhile we will go on forward with this Rule of Three, therefore answer to this question.

Thirty bushels of wheat sowed, yielded in one yeare 360, how many will 80 bushels yielde in 7 yeares? I meane sowing every yeare of those seven, still fourescore bushels? Question of sowing.

Scholar. First I say, that if 30 bushels will yield 360 in one yeare, then 80 bushels will yield 960 in one yeare. Then for the second worke, I say, If one yeare yield 960, then 7 yeares will yield 6720; as these two figures doe shew.

Seed. *Encreaſe.* *Yeare.* *Encreaſe.*

30 Z 360 1 Z 960

80 Z 960 7 Z 6720

Queſtion of Corn.

But now Sir if I ſet forth 30 buſhels of Corne to another man for 7 yeares, agreeing ſo that hee ſhall ſow every yeare the whole increaſe of the Corn, and I at the end of theſe 7 yeares to have the halfe of the whole increaſe: I would know how many buſhels will there amount to my part, ſuppoſing the increaſe to be after the rate of the laſt queſtion, for 30 buſhels in one yeare to yield 360?

Maſter. In ſuch a queſtion you muſt have ſo many ſeverall workings as there be yeares: As for Example, in the firſt yeare thirty buſhels yield 360, then to know the yielding of the ſecond yeare, I muſt ſay, If thirty yield 360, how many yieldeth 360? Worke by your Rule, and you ſhall finde 4320, Then ſay for the third yeare, If thirty yield 360, How many will 4320, yield? you ſhall have 51840, and ſo every yeare multiplying the whole increaſe by 360, and dividing it by 30, the increaſe of the next yeare will amount, as theſe 7 figures following do orderly declare: where I have ſet 7 letters for the 7 yeeres, of which the firſt is ſet without Art, becauſe that is the increaſe which you doe preſuppoſe: and the laſt number of each other doth ſhew the increaſe of that yeare that it ſtandeth for, which the letters doe declare, ſo that the increaſe of the ſeventh yeare is 10 7495 4240 buſhels: how many quarters that is, and alſo how many waies, you may by Reduction ſoon finde.

a

30 —— 360

b

30 Z 360
360 Z 4320

c

30 Z 360
4320 Z 51840

d

30 Z 360
51840 Z 622080

e

30 Z 360
622080 Z 7464960

f

30 Z 360
7464960 Z 89579520

g

30 Z 360
89579520 Z 1074954240

Now with one question more I will prove you. *If sixe Mowers doe mow 45 Acres in 5 dayes, how many mowers will mow 300 Acres in 6 dayes* — Question of mowing

Scholar. If 45 Acres require 6 Mowers, then 300 Acres require 40. Now again, if 5 daies require 40 mowers, then 6 daies need but 33 mowers.

Master. Why doe you not make mention of the 2 that remaineth in the last Division? for the last part of the question is wrought by the Backer Rule, where the first number 5 is multiplied into the second, that is 40, whereof amounteth 200, which if you divide by the third number 6, the Quotient will be 33, as you said: but then will there remain 2, which cannot well be divided into 6 parts: howbeit you may understand by the 6 part of 2, the third part of one mans worke, which you must put

to the 33; or else you must say that 33 Workmen will end all the 300 Acres in 6 dayes save 2 mens worke for one day, or two dayes worke for one man. But such broken numbers called Fractions, you shall hereafter better perceive, when I shall wholly instruct you of them.

Master. *Yet one question more of field matters I will propose, and so I will make an end of this double Rule of Three.*

Scholar. With all my heart Sir I thanke you, and I will dispatch it as soon as I can, because I would faine see the order of the next Rule of 5 numbers.

Question of entrenchings.

Master. *If a Captain over a band of men did set 300 Pioners a worke, which in eight houres did cast a trench of 200 Rods: I demand how many labourers will be able with a like trench in three houres, to intrench a camp of 3400 Rods.*

Scholar. I thinke I am now in the Backehouse ditch: for I know not well which way to go about it. And besides that, truly I think I shall never come to preferment that way my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and service into the favour of your Prince, for the avail of your Countrey.

Example for Navigation: Sir *Francis Drake*, a man greatly honoured for his knowledge, was not the tallest man, and yet hath made as great an adventure for the honour of his Prince and Countrey, as ever Englishman did.

Scholar. Sir, I thanke you for your good incouragement. My minde, though I be little,

is as desirous of knowledge, as any other: I have pondred now a little of it, and thus I set forth the work.

	Rod.		Men.
	200	Z	300
	3400		

Saying, If 200 Rod require 300 men, what shall 3400 rods require? I multiply 3400 by 300, and it yieldeth 1020000, which I divide by 200, and my quotient is 5100 men.

Then must I say for the second worke, if in 8 houres 5100 men, be able to discharge it, how many shall performe the same in 3 houres? Now if I would worke by the Golden Rule of Proportion forward, I should finde a lesse number of men: because 3 houres is lesse then 8 houres: but because reason teacheth me, that the lesser the time is, wherein the trench must be made, the more Labourers I ought to have, thereupon I use now the Backer Rule, as in example. And I have in my Quotient 13600. So many Pioners must I have to intrench the Campe in 3 houres.

Master. You have answered the question very artificially: And truely I commend you for your diligence and apt understanding: and now according to my promise, I will (in whole numbers) give you a little taste of the Rule of Proportion, compounded in 5 numbers.

The Rule of Proportion, composed of 5 Numbers.

The first part of the Rule of Proportion compound, direct.

THe Rule of proportion composed, is distinct for most needfull questions into severall parts or workings: And there belongeth unto it alwayes five numbers, whereof in this Rule being the first par, tthe second number & the fifth, are alwayes of one nature & like denomination, which Rule is to be wrought thus: you must multiply the first number by the second, & that shall be your divisor: Then again, multiply the other 3 numbers, the one by the other, and their Product shall be your dividend.

And now according to my promise, we will first work the question of weight and carriage, which I delivered you in the double Rule of Three, to be absolved by this Rule, which was this.

If the carriage of 1 C. weight, 30 miles cost 12 pence, what will the carriage of 5 C. weight stand me in, being carried 100 miles?

C. weight.	*Miles.*	*Pence.*	*C. weight.*	*Miles.*
1	30	12	5	100

Now marke well how these five numbers stand: Then multiply the first number by the second, as 30 by 1. which maketh but 30, that number keep for your Divisor. Then multiply the other 3 numbers, the one into the other, that is to wit, 12 by 5, which maketh 60: Lastly 60 by 100, which as you

you see here in our Tables, ariseth to 6000, which 6000 you shall divide by the Product of the two first numbers, which here is 30. And you see there is found 200 pence, which is the dutie that you ought to pay for the carriage of 500 weight 100 miles, after the rate of 12 pence a hundred, and agreeth with the conclusion of the double Rule of Three.

Scholar. Sir I thank you, it is even so.

Master. *Yet note this in a generality in this Rule, looke what nature of denomination your middle number is of (which here are pence) and of the like denomination or nature is alwayes your quotient.* Note.

Scholar. Well now and if it please you, by your patience, I will see how I can end the question next following of 30 Bushels of wheat sowed, which in one year yeeldeth 360, how many then will 80 Bushels yeeld in 7 year following every year of those 7 still 80 bushels, and according to your reasons I set my numbers thus.

Bush.	*Year.*	*Bush.*	*Bush.*	*Year.*
30 —	1 —	360 —	80 —	7
		80		
		28800		
		7		
		201600		

There I multiply 30 by 1, and it maketh 30 my Divisor; then multiplying the other 3 numbers the one into the other, as here appeareth in my Tables, they make 201600, which I divide by 30: & my Quotient is 6720 bushels, my desire; for so much also it came to at two workings by the Rule of Three.

Master.

Master. Yet one question more I will propound unto you, and so leave this Rule, till it please God hereafter, that I may make you worke it in broken numbers.

Question of Interest. *What comes the interest of* 258 pound, *for* 5 *moneths to, after the rate of* 8 pound, *taken in the* 100 pound *for* 12 *moneths?*

Scholar. Sir, this is yet within the compasse of some reasonable usance. Therefore to minister equitie in this case, I will see how I can worke the same, which I set downe thus, praying you if I have not done well, to shew me mine errour.

li. *moneths*. li. li. *mon.*
100---12--8--258--5.

Master. Proceed, you have done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yeeldeth 1200, and the three other numbers multiplied together produce 10320, which I divide by 1200: and my Quotient is eight pounds. Then according as you have taught mee heretofore, I turne the 720 pound that I left, into shillings: and dividing it by the first number my Quotient is 12 shillings. So I answer, that the loane of 258 pounds for 5 moneths, after the rate of 8 pound in the 100 pound for a yeare, comes to 8 pound 12 shillings.

Master. You say true, I commend your diligence: now behold the manner of the second part of this rule.

The

The backer Rule, or the second part of the Rule of Proportion compound.

Master.

IN *the second part of this Rule of Proportion composed, the third number is like unto the first. And the Rule is to bee wrought thus: you shall now, contrary to the last Rule, multiply the third number and the fourth together, and that Product shall be your Divisor. Then multiply the fifth by the second, and the Product thereof by the first: and that is the number that shall be divided.* For example I propound this question for a proofe of my last question of Interest.

A Merchant hath received 8 pound 12 shillings, *for interest of certain money for* 5 *moneths tearme, which he received after the rate of eight* pound *in the* 100 *for a yeare. The question is now, how much money was delivered to raise this interest.* The proofe of the last question.

Behold therefore the manner,	li.	*moneths.*	li.	*moneths.*	li.	s.
	100 —	12 —	8 —	5 —	8-	12

how the question is set forth.

Scholar. Sir I perceive it very well: and according to the doctrine which you prescribed for the working thereof: if please you now it is set downe, I thinke I can follow the worke.

Master.

Master. Nay, stay a while, and before you worke, marke well how I deliver a reason for the perfect understanding of this Rule, which is thus: Note. *If 8 pound in 12 moneths doe yeeld mee 100 pound, to take 8 pound 12 shillings for five moneths, must needes yeelde a great deale more.*

So upon the knowledge that I have in this Art, The first part of this Rule is answerable to the Rule of Three forward: and this latter part accordeth to the Rule of Three backward.

Scholar. Sir, I yield you most heartie thanks for these your last instructions, they have given me great light into these two rules, whereby I may the better by deliberation conceive how to use them hereafter when occasion shall require.

Note. Master. You say well, goe to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turn all your monie as you work into shillings, for your more ease in working.

Scholar. If it please you to behold me a little, I will quicklie end it: for I have but my first, my second, and my last number to be multiplied together for my Dividend: And my third into my fourth for my Divisor.

Ii.

li. *Moneths.* *Moneths.* li. s.

100 —— —— 12 8 —— 5 —— 8 —— 12
20 20 20

2000 160 172
12 5

4000 800
2000

24000
172

48000 4128000(
168000 8 00
24000
—— 4128000

Which 4128000 I divide by 800, and my Quotient is 5160 shillings, which in pounds yieldeth 258 my desire.

Master. I will here for this time in whole numbers end this Rule, and I will instruct you in the Rules of fellowship. You may at your convenient leasure for your exercise worke the same by the Rule of Three at twice. And for your aide and incouragement therein, I set down here a proffer how to apply it.

A

Moneths. li. s.
5 8 — 12
12 $412\frac{4}{5}$.

B

pounds. li.
100
$412\frac{4}{5}$ 258 li.

The

The Rule of Fellowship.

The Rule of Fellowship without time.

BUt now will I shew you of the Rule of Fellowship or Company, which hath sundry operations according to the divers number of the Company. This Rule is sometime without difference of time, and sometimes there is in it difference of time. First I will speak of that without difference of time, of which let this be an example.

A question of company.

Foure Merchants of one Company made a bank of money diversly: for the first laid in 30 pound, *the second* 50 pound, *the third* 60 pound, *and the fourth* 100 pound, *which stock they occupy so long, till it was increased to* 3000 pound. *Now I demand of you what should each receive at the parting of this money.*

Scholar. I perceive that this Rule is like the other, but yet there is a difference which I perceive not.

Master. Then will I shew it to you: First by Addition, you shall bring all the particular summes of the Merchants into one summe, which shall be the first summe in your working by the Golden Rule, and the whole summe of the gaines by that stocke shall be the second summe. Now for the third summe you shall set the portion of each man one after another, and then worke by the Golden Rule, and the fourth summe will shew you each mens gaines: as in example.

30
50
60
100
240

The parcels of the foure Merchants make in one summe 240 pounds: set that in the first place, the gaines in the second, and the first mans portion of stocke in the third place, thus,

240 Z 3000
30

Now multiply the second by the third, and it will be 90000, which you shall divide by 240, and there will appeare 375 pounds, thus:

And that is the gains for the first man.

240 Z 3000
30 375

Now for the second man set the 50 pound that hee brought, in the third place, and work as before: and his part will be 625 pound: as this figure sheweth.

240 Z 3000
50 625

Likewise for the third man, set his money which was 60 pounds, and his part of gains will be 750 pounds, as here appeareth

240 Z 3000
60 750

And so for the fourth man; if you set his summe which is 100 pound, his gaines will be 1250 pound, as the worke will declare.

240 Z 3000
100 1250

Scholar. This I perceive: but is there any way to examine whether I have well done or no?

Master. For the triall hereof, adde together all their foure portions, and if their addition make the whole summe of their gaines, then is the work well done. *Note this common proofe*

Scholar. That will I trie by and by; the foure parcels are those, which added together make

3000,

3000, which is the just summe 375
of money that they gained, 625
whereby I know the work is 750
well done. 1250
3000

Master. Well, now another example will I put to you not of gains, but of losse: for one reason serveth for both.

A question of losse.

If three Merchants in one ship, and of one fellowship, had bought Merchantdise, so that the first had laid out 200 *pound, the second* 300 *pound, the third* 500 *pound, and it chanced by tempest that they did cast over board into the Sea Merchantdise of the value of* 100 *pound, how much should each man bear in this losse?*

Scholar. If I shall do in this as you did in the other question, then must I joyne their 3 portions together, 200, 300, 500, which maketh 1000. Then say I, If 1000 lose 100 then shall 200 lose 20, and 300 shall lose 30, and 500 shall lose 50, as by the three figures it doth appear plain.

1000 Z 100
200 Z 20

1000 Z 100
300 Z 30

1000 Z 100
500 Z 50

Master. Well, sith now you have done these, I will propound a question of more importance, which shall make you not onely the abler to understand this Rule, but also it will greatly aid you in the next Rule of fellowship with time, if such need be that your money be of divers Denominations.

For

For this may not bee forgotten in all such questions: If the number be of divers kindes, you must by reduction bring it into one kinde, that is to say, to the least value that is named in the question. And likewise shall you doe, if the time be of divers kindes, as some yeares, some moneths, weeks, and dayes, you shall make all moneths, weeks, or dayes, according as the least name of time in the question is, as for example.

First in diversity of money. Three companions bought 2000 sheep, and paid for them 241 pound 13 shillings 4 pence, of which summe one payd 101 pound, 10 shillings. The second 82 pound 17 shillings 10 pence. And the third paid 57 pound, 5 shillings 6 pence: How many sheepe must each of them have? Answer. *The first shall have 840. The second 686. And the third 474. And that must you worke thus:* — A question of sheepe.

First, considering that your money is of divers Denominations, you shall (by reduction) bring it all into the smallest Denomination which is in it, that is to say, pence; and so will the Totall summe be 58000 pence. — Solution.

Now if you turne each mans money into pence also, the first mans summe will be 24360 pence: The second mans money will bee 19894 pence; and the third mans money will be 13746 pence.

Now to know how manie sheep everie man shall have, let the whole summe of money, that is, 58000 pence be set in the first place, and in the second place set the number of sheep, and then orderly in the third place set each mans money, and then multiplying the third and the second summes together, and dividing that that amounteth by the first,

there will appeare the number of ſheep that each man ought to have: as theſe 3 figures doe ſhew.

a

58000 2000
24360 840

b

58000 2000
19894 686

c

58000 2000
13746 474

S. Why doe you ſet the money in the firſt place, ſeeing in the queſtion you ſay 2000 ſheep coſt 58000 pence, and not thus: 58000 coſt 2000 ſheep?

Maſter. You remember I taught you at the beginning of the Golden Rule, that the firſt and the third numbers muſt be of one name, and of like things: & evermoꝛe the number that the queſtion is asked of, muſt bée ſet in the third place.

Now is the queſtion plainly this: If foure men bought 2000 ſheep for 58000 pence, how many ſheep ſhall each man have?

But ſéeing in this queſtion, there ought moꝛe reſpect to be had to the ſumme of money, then to the ſumme of the perſons; (foꝛ in the ſumme of money is their pꝛopoꝛtion toward the ſheep, and not in the number of perſons.)

If 58000 pence bought 2000 ſheepe, how many did 24360 buy? Againe, how many did 19894 pence buy? And how many bought 13746 pence?

Scholar. I perceive it reaſonable, and ſo will I doe in all queſtions.

Note. M. Even ſo. But foꝛ eaſineſſe of the worke marke this: Whenſoever the firſt & ſecond mumbers

have

haue Cyphers in the first places, you may both in the Multiplication and in the Division leaue out those cyphers, so that you leaue out like manie out of both summes, as in this question, the first number 58000 hath 3 cyphers, and so hath the second, that is 2000: therefore cast away their cyphers, and so will the first number bee 58, and the second 2. set them in their places, and worke according to the Rule, and you shall perceiue that will bee all one, sauing that this is the shorter and easier way, as these three figures doe shew.

a

58 Z 2
14360 Z 840

b

58 Z 2
19894 Z 686

c

58 Z 2
13746 Z 474.

And this you see is both easier, and also the more certaine way to know the answer to this question.

Scholar. Truth it is as you say: But Sir me seemeth I might aske a further question here, not onely how many sheepe each man should haue, but also what euerie sheepe cost.

Master. That question doth not onelie belong to this Rule, but may also be discussed by Division, especially if the questions number bee one onely, as thus: Divide the totall summe 58000 pence by 2000 (or 58 by 2) omitting the cyphers, and the Quotient will be 29 pence, that is 2 shillings 5 pence. Howbeit, by this Rule you may doe it, and best when the number of the question doth exceede 1; as if I

ſhould aske this queſtion, 2000 ſheep coſt 58000 pence, how much doe 20 coſt? Then ſhall I ſet my figures as before.

2000 Z 58000
20

And doing after the Rule, there will amount 580 pence, that is, 2 pound 8 ſhillings 4 pence, the price of one ſcore: but if you will uſe that eaſie way that I did teach you now, you may change the firſt and ſecond number thus.

2 Z 58
20

Thus doe you perceive the uſe of the Rule without time.

Scholar. All this I underſtand very well: I pray you now inſtruct mée in the Rule of Fellowſhip with Time.

The Rule of Fellowſhip *with Time.*

Maſter.

The Rule of Fellowſhip with time.

TO the intent you may as well perceive the ſame Rule with diverſity of time, I propoſe this example.

Foure Merchants made a common ſtocke, which at the years end was increaſed to 35145 pound. *Now to know what ſhall be each mans portion of gain, you muſt know each mans ſtocke, and time of continuance.*

The firſt man of theſe foure laid in 669 l. which he

he did take from the stock again at the end of 10 moneths. The second man laid in 810 pound, for 8 moneths. The third man laid in 900 pound, for seven moneths. And the fourth laid in 1040, for 12 moneths. Question of a Bank

This question shall you examine as you did the other before, saving that whereas in the third place of the Figure you did set each mans summe alone, here you shall set the same being multiplied by the number of their time: and likewise in the first place of the figure you shall set the number which amounteth of their whole summes so multiplied by their time, and added into one whole summe, as thus. Note. A generall Rule.

The first mans summe is 669 pounds, which I multiply by 10 (that was the number of his time) and it maketh 6690. The second mans summe 810 pound, multiplied by 8 (which was his time) maketh 6480. The third mans summe 900 pound, multiplied by 7 (for that was his time) yeeldeth 6300. The fourth mans summe was 1040 pound, and his time 12: multiplie the one by the other, and it will be 12480.

The foure summes thus multiplied by their time, must be set orderly in the third place of the figure, and in the first place must be set the whole summe of all foure, which is 31950; and the gain must be in the second plcce, which is 35145. Now to end the question, I say first, If 31950 did get 35145, what did 6690 get? Answer. 7359 pounds, as by this figure apeareth.

31950	Z	35145
6690		7359

Likewise, the second man had to his part 7128 pound, the third must have 6930 pounds, and the fourth man shall have for his part 13728 pounds, as these figures doe partly declare.

	b			c	
31950	Z	35145	31950	Z	35145
6480		7128	6300		6930

	d	
31950	Z	35145
12480		13728

Another proofe.

Scholar. This I like very well: but what proof is there of this work?

Master. The same that I taught you for the other: howbeit, there is used both for this work and the other also, this manner of proofe, to adde all the portions together, and it may agree to the whole summe, then seemeth your worke well done: but this is no sure proofe.

S. Yet will I prove in this example: the 4 parcels are these, which if I adde together, there will amount 35145, and that was the whole summe, whereby I perceive the work is well done.

7359
7128
6930
13728
35145

Master. If it fall out otherwise, be sure it is not well.

Scholar. Then do I understand this worke also very well: But what have I now to learn?

M. There are many other excellent parts behinde, of which I will not as now make mention, because that without the knowledge of Fractions they cannot bee duely taught, and much lesse understood. Therefore will I propose to you two or three questions

questions more (that thereby you may better perceive the use of this Rule and all other the like) and so make an end for this Time.

¶ *Three Partners by some ill adventure sustained the losse of* 160 pound, *whereof the first laid into the common stocke* 200 pound, *for ten Moneths. The second laid in* 350 pound, *and the third* 100 pound, *but for how long the two latter, is unknown: But braking off their Partnership, the first found himselfe a looser* 80 pound, *the second* 56 pound, *and the third* 24 pound. *The question is, for how long time was the money of the two latter in company.* A question of losse.

For the solution hereof, and of such other like, you must also multiply the first mans 200 pound, that hee put into the stocke by his time of continuance, which was ten moneths; and it maketh 2000: wherefore now I affirme, if his money that lost 80 pound multiplied by his time make 2000: what shall his money make that lost 56 pound, and his that lost 24 pound, which two numbers I commit to the triall of the Rule of Three at two workings, thus;

If 80 give 2000, what giveth 56? And again, if 80 give 2000, what giveth 24?

80	2000		80	2000
56	1400		24	600

To conclude, if you now divide 1400, the second mans portion, by 350, which was his stock that he laid into company, you shall find in your quotient 4 moneths, and for so long time did the second man put his money into the common stock.

Lastly, if you divide the third mans new laying in, which was 600 by 100, which was his stock that

that he put into the company, the quotient declareth his time of continuance, which was six moneths. And thus is the question resolved.

Scholar. Sir, I have attentively beheld your working, and the more we travell herein, the more me thinke I am in love with this excellent Art.

Master. Then what say you to this Question?

A question of Canons

There is in a Cathedrall Church 20 Canons, and 30 Vicars, those may spend by year 2600 pound, but every Canon must have to his part 5 times so much as every Vicar hath: how much is every mans portion say you?

Scholar. I pray you make the answer your self also, so shall I perceive best the means to answer to such other like.

Master. In this Question, you must doe as in those beforesaid, that have diversitie of time, for here is diversity of portions. Therefore shall you multiply the number of the persons by their difference of portions: (as you did in the other by time:) Then must you multiply the 20, (which is the number of Canons (by 5.) for that is the number of their portion) so will it be 100. Then 30, (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put these two summes together, and they make 130. Then say thus; If 130 spend 2600 pounds, what may 100 spend? The Rule sheweth 2000 pounds.

Again for Vicars if 130 spend 2600 pound, what may 30 spend? Answer 600 pound, as these figures shew.

130	Z	600	130	Z	3600
100		2000	30		600

But

But if every Canon should have so often times 4 pound as the Vicar should have 3 pound, then should I multiply 20 by 4, (that were 80) & 30 by 3, (that were 90) and then both were 170. Then should the figures be set as followeth.

	li.	s.	d.			li.	s.	d.
170 Z 2600					170 Z 2600			
80 Z	1223,	-- 10,	7		90 Z	1376,	-- 9,	5

But this sort is too hard for you, by reason of the fractions, therefore I will let it rest to that place.

And by this rule you see what the 20 Canons may spend; which summe if you divide by 20, you shall see each Canons proportion: and so of the Vicars, if you divide their summe by 30, the quotient will declare every Vicars portion.

The second Dialogue.

The accounting by Counters.

Master.

N*ow that you have learned* Arithmetick *with the Pen, you shall see the same Art in* Counters *: which feat doth not onely serve for them that cannot write and read, but also for them that can do both; but have not at some time their pen or tables ready with them.*

This sort is in two formes commonly, The one by lines, and the other without lines. In that

that

that hath lines, the lines doe stand for the order of places: & in that that hath no lines, there must be set in their stead so many Counters as shall need for each line one; and they shall supply the stead of lines.

Scholar. By examples I should better perceive your meaning.

Master. For example of the lines; loe here you see six lines, which stand for six places, so that the nethermost stands for the first place, & the next above is for the second, & so upward till you come to the highest, which is the sixth line, and standeth for the sixth place.

——100000——
——10000——
——1000——
——100——
——10——
——1——

Numeration by Counters.

Now what is the value of every place or line, you may perceive by the figure which I have set on them, which is according as you learned before in Numeration of figures by the Pen: for the first place is the place of unites or ones, and every counter set in that line, betokeneth but one: and the second line is the place of 10, for every counter there standeth for 10: the third line the place of hundreds, the fourth of thousands, and so forth.

Scholar. Sir, I doe perceive that the same order is here of lines, as was in the other figures by places, so that you shall not need longer to stand about Numeration, except there be any other difference.

Master.

Maſter. If you do underſtand it, then how will you ſet 1543?

Scholar. Thus as I ſuppoſe.

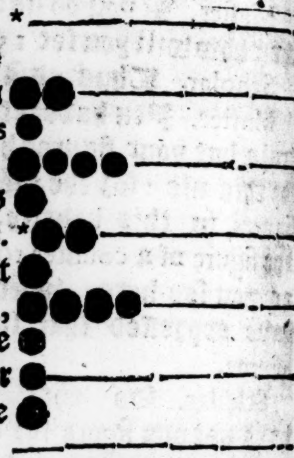
Maſter. You have ſet the places truly, but your figures be not méet for this uſe: for the méeteſt figures in this behalfe, is the figure of a counter round, as you ſee here, where I have expreſſed that ſame ſumme.

Scholar. So that you have not one figure for 2, nor 3, nor 4, & ſo forth, but as many digits as you have, ſo many counters you ſet in the loweſt line, and for every 10 you ſet one in the ſecond line, and ſo of other. But I know not by what reaſon you ſet that one Counter for 500 betwéen two lines.

Maſter. You ſhall remember this, that whenſoever you néed to ſet down 5, 50, or 500, or 5000, or ſet forth any number whoſe numerator is 5, you ſhall ſet one counter for it in the next place above the line that it hath his denomination of: As in this example of that 500, becauſe the numerator is 5, it muſt be ſet in a void ſpace, and becauſe the denomination is a hundred, I know that the place is the void place next above hundreds, that is to ſay, above the third line.

And further you ſhall marke, that in all working by this ſort, if you ſhall ſet downe any

summe between 4 and 10, for the first part of that number you shall set down 5, & then so many counters more, as there rest numbers above 5. And this is true both of digits & articles. And for example, I will set down this summe 297965, which summe if you marke well, you need none other examples for to learne the numeration of this forme.



But this shall you marke, that as you did in other kindes of Arithmetick, set a prick in the places of thousands, in this work you shall set a Starre, as you see before. *

Scholar. Then I perceive Numeration: But, I pray you, how shall I do in this Art, to adde two summes or more together?

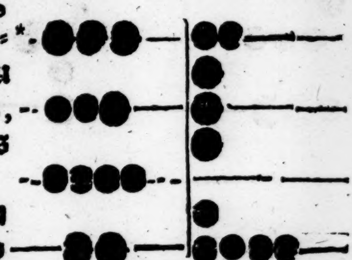
Addition.

Master.

The easiest way in this is to adde but two summes at once together: Howbeit you may adde more, as I will tell you anon.

Therefore when you will adde two summes, you shall first set down

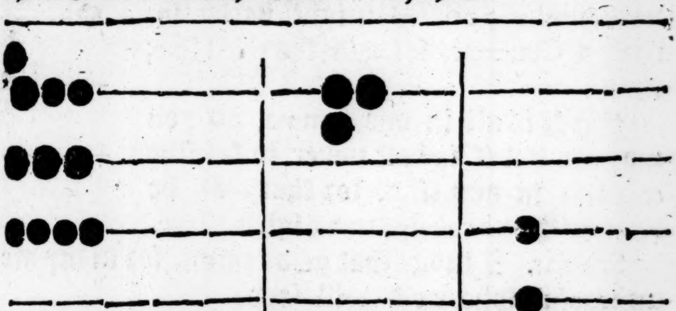
one of them, it forceth not which, and then by it draw a line croſſe the other lines. And afterward ſet downe the other ſum, ſo that the line may be be-tweene them: as if you would adde 2659 to 8342, you ſet your ſummes as you ſee here.



Addition of two ſummes.

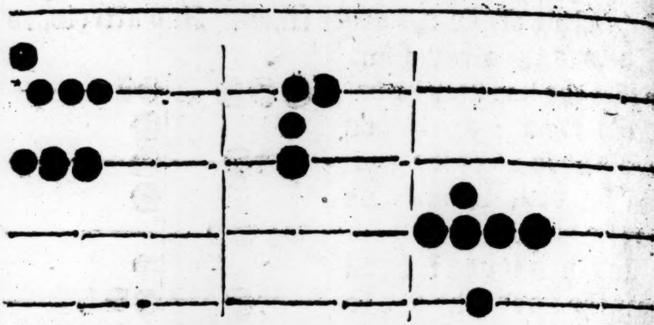
And then if you liſt you may then adde the one to the other in the ſame place: or elſe you may adde them both together in a new place: which way, becauſe it is moſt plaine, I will ſhew you firſt.

Therefore will I beginne at the Unites, which in the firſt ſumme is but 2, and in the ſecond ſumme 9, that maketh 11: Thoſe do I take up, and for them I ſet 11 in the new room, thus:

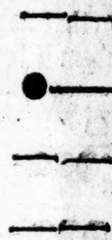


Then doe I take up all the Articles under a hundred, which in the firſt ſumme are 40, and in the ſecond ſumme 50, that maketh 90: or you may ſay better, that in the firſt ſumme there are foure Articles of 10, and the ſecond ſumme 5, which maketh 9, but then take heed that you ſet them in their right lines, ſee here.

Where



Where I have taken away 40 from the firſt ſumme, and 50, from the ſecond, and in their ſtead I have ſet 90 in the third room, which I have ſet plainly, that you might well perceive: howbeit ſéeing that 90 with the 10 that was in the third room, already, doth make 100, I might better for thoſe 6 Counters, ſet 1 in the third line, thus:

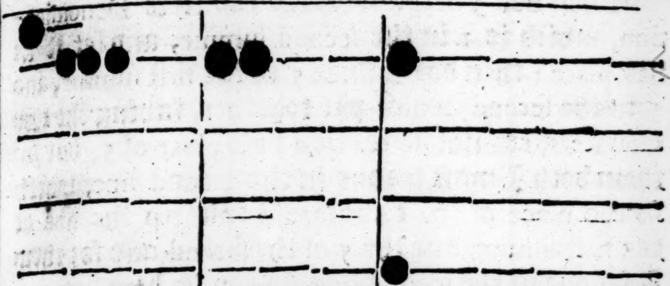


For it is all in one ſumme, as you may ſée, but it is beſt never to ſet five counters in any line, for that may be done with one counter in a higher place.

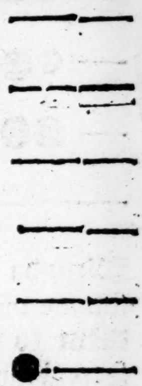
Scholar. I judge that good reaſon, for many are unnéedfull where one will ſerve.

Maſter. Well, then will I adde forth of hundreds: I finde 3 in the firſt ſumme, and 6 in the ſecond, which maketh 6000, them doe I take up, and ſet in the third room, where is 100 already, to which I put 900, and it will be 1000: therefore I ſet one counter in the fourth line for them all, as you ſée here.

Then



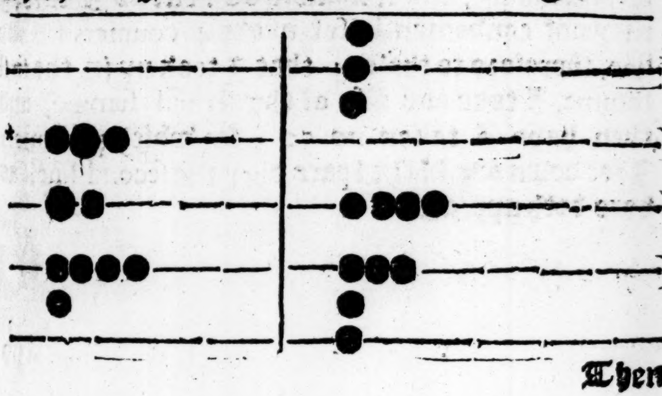
Then adde I the thousands together, which in the first summe are 8000, and in the second 2000 that maketh 10000, them doe I take up for those two places, and for them I set one counter in the fifth line, and then it appeareth as you see to bee 11001, for so many doth amount of the Addition of 8342 to 2659.



To adde summes together,

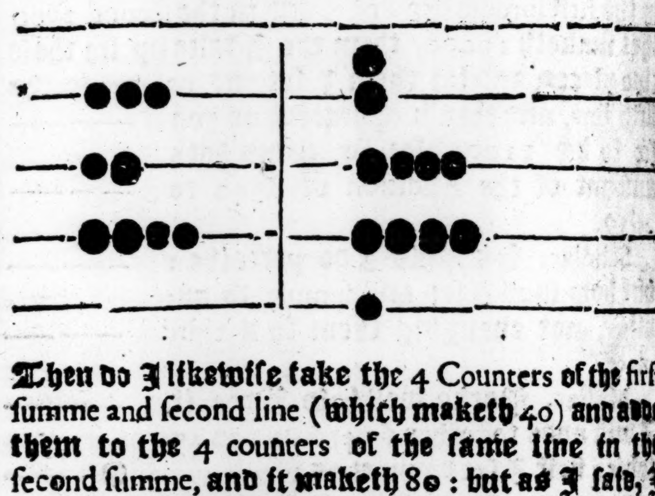
Scholar. Sir, this I do perceive: but how shall I set one summe to another, not changing them to a third place?

Master. Marke well how I doe it. I will adde together 65436 and 3245, which first I set down thus:



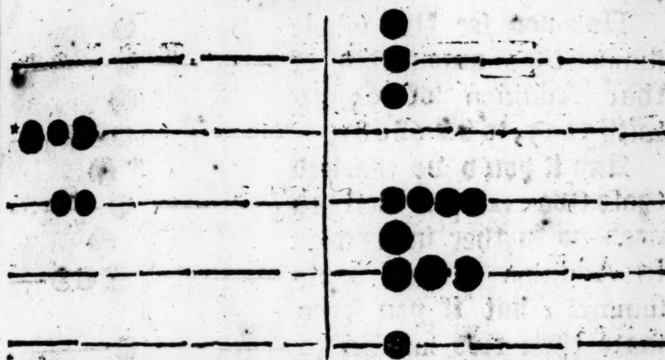
Then

Then doe I begin with the smallest Denomination, which is 1 in the second summe, and set it in his place: then doe I finde 5 in the first summe, and 5 in the second, which put together, saving the two Counters, cannot be set in a void place of 5, but for them both I must set one in the second line, which is the place of 10, therefore I take up the five of the first summe and the 5 of the second, and for them I set one in the second line, as you see here.



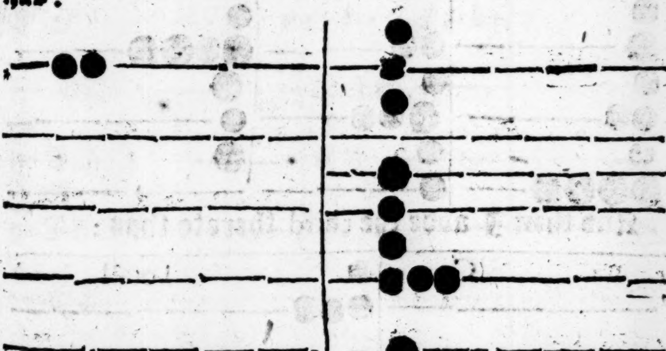
Then do I likewise take the 4 Counters of the first summe and second line (which maketh 40) and adde them to the 4 counters of the same line in the second summe, and it maketh 80: but as I said, I may not conveniently set above 4 counters in one line, therefore to those 4 that I took up in the first summe, I take one also of the second summe, and then have I taken up 50: for which 5 counters I set down one in the space over the second line, as here doth appear.

And

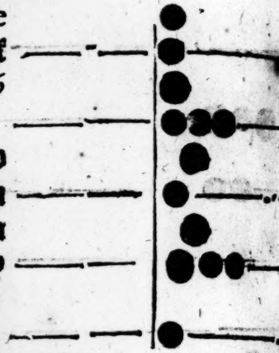


And then is there 80, as well with those 4 counters, as if you had set down the other 4 also.

Now do I take the 200 in the first summe, and adde them to the 400 in the second summe, and it maketh 600, therefore I take up the two counters in the first summe, and thrée of them in the second summe, and for them 5, I set 1 in the space above, thus:



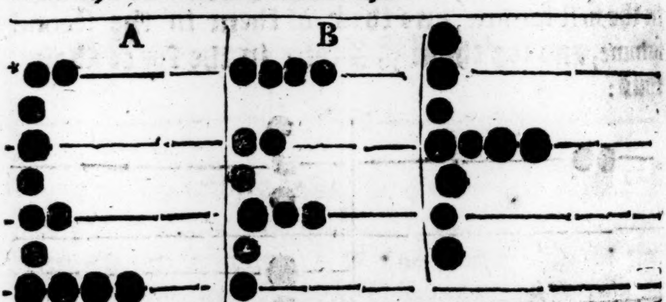
Then take I the 3000 in the first summe unto which there are none in the second summe agréeing. therefore I doe onely remove those thrée counters from the first summe into the second, as here doth appear.



And you see the whole summe that amounteth of that Addition of 65436 with 3245, to bee 68681.

And if you have marked those two examples well you need no further instruction in Addition of two onely summes: but if you have more then two summes to adde, you may adde them thus:

First adde two of them, and then adde the third and fourth, or more, if there be so many: As if I would adde 2679, with 4286, and 1391. First I adde the two first summes thus:



And then I adde the third thereto thus:

And

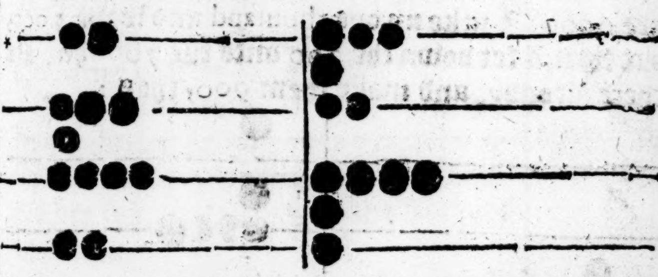
And so of more if you have them.

Scholar. Now I think it best that you passe forth to Subtraction, except there be any way to examine this manner of Addition, then I thinke that were good to be knowen next,

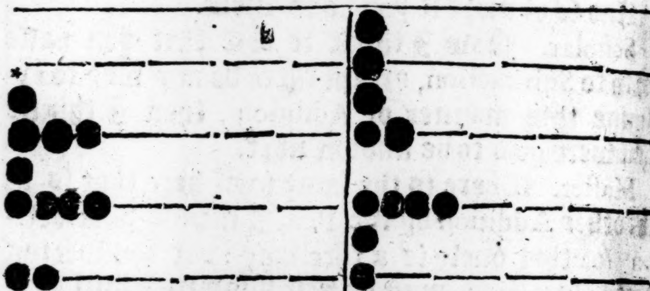
Master. There is the same proof here that is in the other Addition by the Pen, I meane Subtraction; for that onely is a sure way: but considering that Subtraction, must be first knowen, I will first teach you the Art of Subtraction, and that by this Example.

Subtraction.

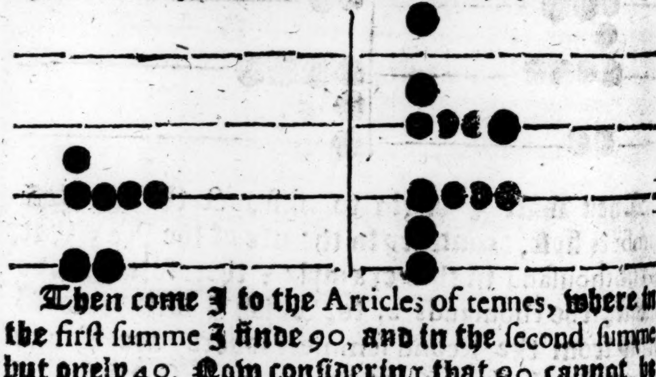
I Would subtract 2892 out of 8746. These summes must I set downe as I did in Addition: but here it is best to set the lesser number first, thus:



Then shall I begin to subtract the greatest numbers first, (contrary to the use of the Pen) that is the thousand in this example: therefore I finde among the thousands 2, for which I withdraw so many from the second summe, (where are 8) and so remaineth there 6, as this example sheweth.



Then doe I likewise with the hundreds, of which in the first summe I finde 8, and in the second summe but 7, out of which I cannot take 2, therefore this must I doe: I must look how much my sume differeth from 10, which I find here to be 2, then must I abate for my summe of 800, one thousand, and set down the excesse of hundreds, that is to say 2, for so much as 1000 is more then I should take up: therefore from the first summe I take that 800, and from the second summe (which are 6000) I take up one thousand, and leave 5000, but then I set down the 200 unto the 700 that are there already, and make them 900, thus:



Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 cannot be

abated

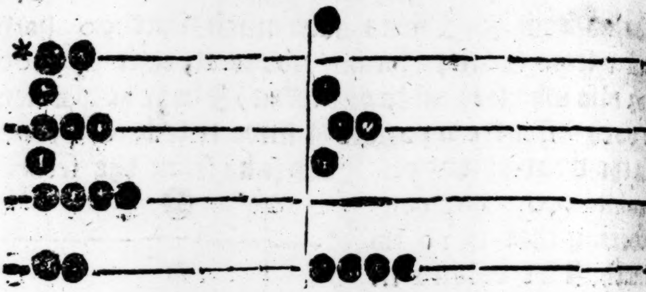
abated from 40, I looke how much that 90 doth differ from the next summe above it, that is, 100 (or else which is all to one effect) I looke how much 9 doth differ from 10, and I finde it to be 1 : then in the stead of that 90, I doe take from the second summe 100 : but considering that is 10 too much, I set down 1 in the next line beneath for it, as you see here.

Having that here I have set 1 Counter in the space instead of 5 in the next line. And thus I have subtracted all save 2, which I must abate from 6 in the second summe, and there will remain 4 thus.

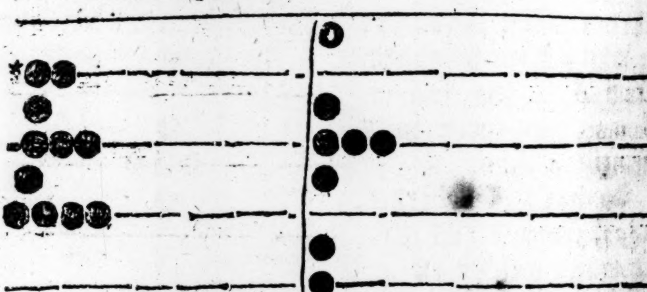
So that if I subtract 2892, from 8746 the remainer will be 5854.

And that this is truely wrought you may prove by Addition : for if you adde to this remainer the same summe that you did subtract, then will the former summe 8746 amount again.

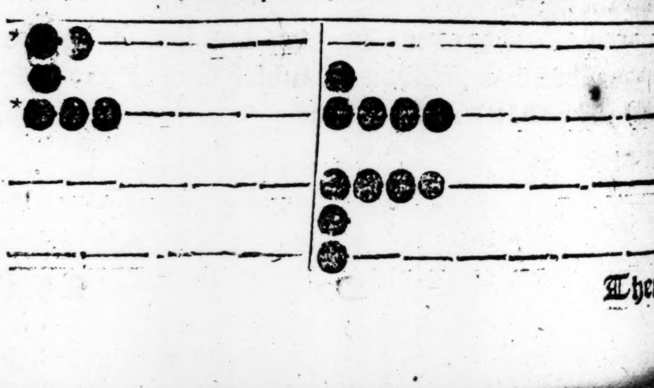
Scholar. That will I prove, and first I set the summe that was subtracted, which was 2892, and then the remainer 5854 thus: A proof of Subtraction.



Then doe I adde the first 2 to 4, which maketh 6 : so take I up 5 of those Counters, and in their stead I set 1 in the space : and one in the lowest line, as here appeareth.

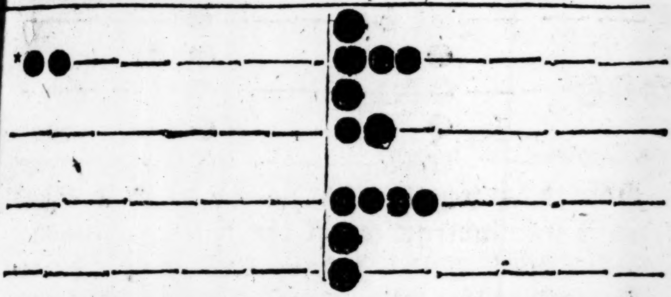


Then do I adde the 90 next above to the 50, and it maketh 140. Therefore I take up those 6 Counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus :

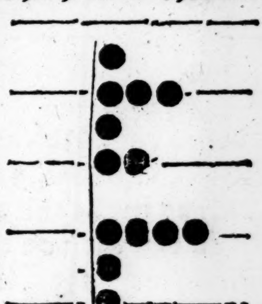


Then

Then doe I come to the hundreds, of which I finde 8 in the firſt ſumme, and 8 in the ſecond, that maketh 1600, therefore I take up thoſe 8 counters, and in their ſtead I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and in the third line, as you may ſée here.



Then is there left in the firſt ſumme but onely 200, and in the ſecond 5000, which is 7000, which I ſhall take up from thence, and ſet in the ſame line in the ſecond ſumme to the one that is there already: and there will the whole ſumme appeare as you may well ſee to be 8746, which was the firſt groſſe ſum: and therefore I do perceive that I had well ſubtracted before.



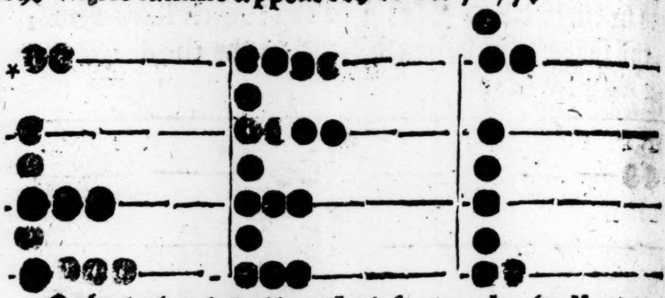
And thus may you ſée how Subtraction may be tried by Addition.

Scholar. I perceive the ſame order here with Counters, that I learned before in figures.

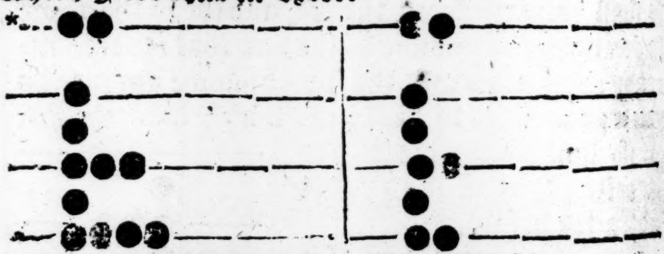
Maſter. Then let me ſee how you can trie Addition by Subtraction.

Scholar. First I will set forth this example of Addition, where I have added 2189 to 4988. And the whole ſumme appeareth to be 7177.

Proof of Addition by Subtraction.



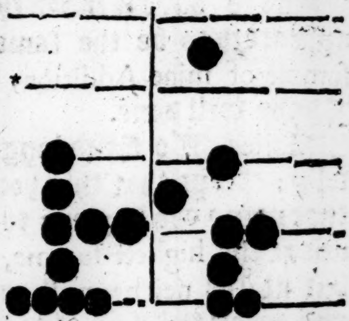
Now to try whether that ſumme be well added or no, I will ſubtract one of the firſt two ſummes from the third. And if I have well done, the remainer will be like that other ſumme: as for example, I will ſubtract the firſt ſumme from the third, which I ſet thus in order.

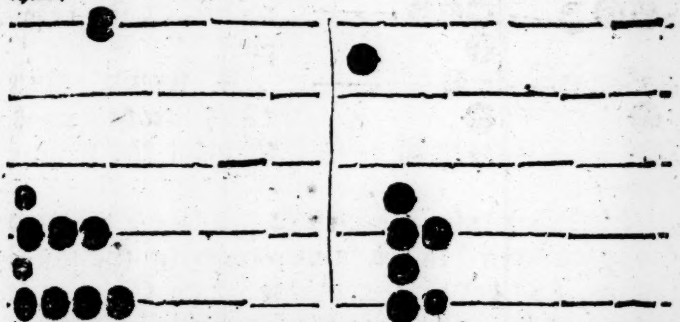


Then do I ſubtract 2000 of the firſt ſumme, from the ſecond ſumme, & then remains there 5000, thus:

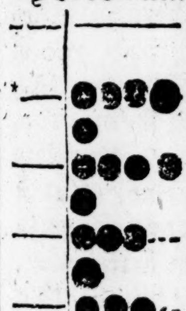
Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is onely 100 alſo: and then in the third line, reſteth nothing, as you may ſee in this example following.

Then in the ſecond line with his ſpace over him I finde 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but only 70, I muſt take it out of ſome higher ſumme, which is here onely 5000: therefore I take up 5000: and ſeeing that is too much by 4920, I ſet downe ſo many in the ſecond roome, which with the 70 being there already do make 4990, and then the ſummes do ſtand thus.





Yet remaineth there in the firſt ſumme 9, to be abated from the ſecond ſume, wherein that place of unites doth appear onely 7: then muſt I abate a higher ſume, that is to ſay 10, but ſeeing that 10 is more then 9 (which I ſhould abate) by 1, therefore ſhall I take up one Counter from the ſecond, and ſet downe the ſame in the firſt line, or lowermoſt line, as you ſee here.

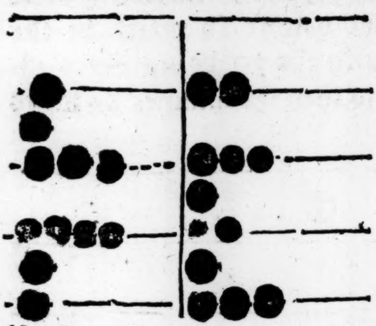


And

And ſo have I ended this work, and the ſumme appeareth to be the ſame which was the ſecond ſumme of mine Addition, and therefore I perceive I have well done.

Another way of Addition.

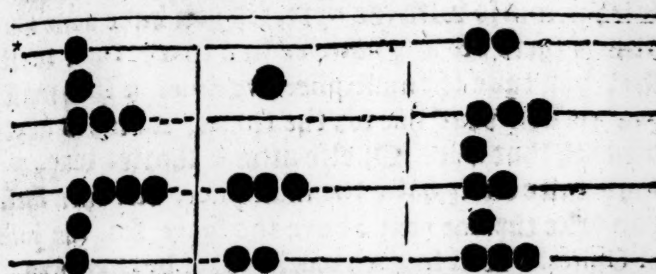
Maſter. To ſtand longer about this, it is but folly: except that this you may alſo underſtand, that many doe beginne to ſubtract with Counters, not at the higheſt ſumme, as I have taught you, but at the neithermoſt, as they doe uſe to adde; and when the ſumme to be abated in any line appeareth greater then the other, then doe they borrow out of the next higher roome, as for example.



I ſhould abate 1846 from 2378, they ſet the ſummes thus:

Firſt they take 6 which is the lower line, and his ſpace from 8 in the ſame roomes in the ſecond ſumme, and yet there remaineth two Counters in the loweſt line. Then in the ſecond line muſt 4 be ſubtracted from 7, and ſo remaineth there 3. Then 800 in the third line, and his ſpace, from 300 of the ſecond ſumme cannot be, therefore doe they abate it from a higher roome, that is, from 1000, and becauſe 1000 is too much by 200, therefore muſt I ſet downe 200 in the third line, after I have taken up 1000 from the fourth line. Then is there yet 1000 in the fourth line of the firſt ſumme, which if I withdraw from the ſecond ſumme, then doe

doe all Figures stand in order thus: 532.



So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both may use which you list.

Multiplication.

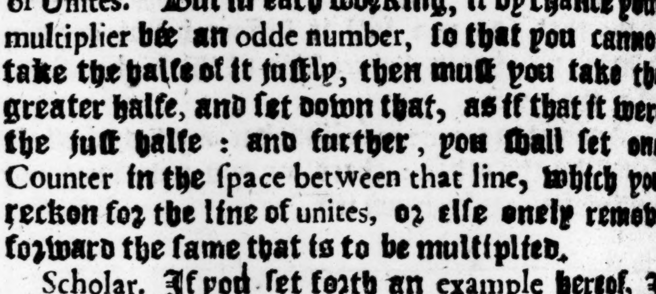
BUt now touching Multiplication : you shall set your numbers *into two* roomes, *(as you did in those other kindes) but so that the multiplier bee set in the first* roome , *then shall you begin with the highest numbers of the second* roome, *and multiply them first after this sort.*

Take the overmost line in your first working as it were the lowest line, setting on it some moveable mark (as you list) and looke how many Counters bee in him, take them up, and for them set downe the whole multiplier so many times as you tooke up Counters: reckoning (I say) that line for the Unites. And when you have done with the highest number, then

then come to the next line beneath, and doe so even with it, and so with the next, till you have done all. And if there bee any number in a space, then for it shall you take the multiplier five times, & then must you reckon that line for the Unites, which is next beneath that space. Or else after a shorter way, ye shall take onely halfe the multiplier, but then shall you take the line next above the space for the line of Unites. But in each working, if by chance your multiplier bee an odde number, so that you cannot take the halfe of it iustly, then must you take the greater halfe, and set down that, as if that it were the iust halfe: and further, you shall set one Counter in the space between that line, which you reckon for the line of unites, or else onely remove forward the same that is to be multiplied.

Scholar. If you set forth an example hereof, I think I shall perceive you.

Master. Take this example: I would multiply 1542 by 365, therefore I set my numbers thus.

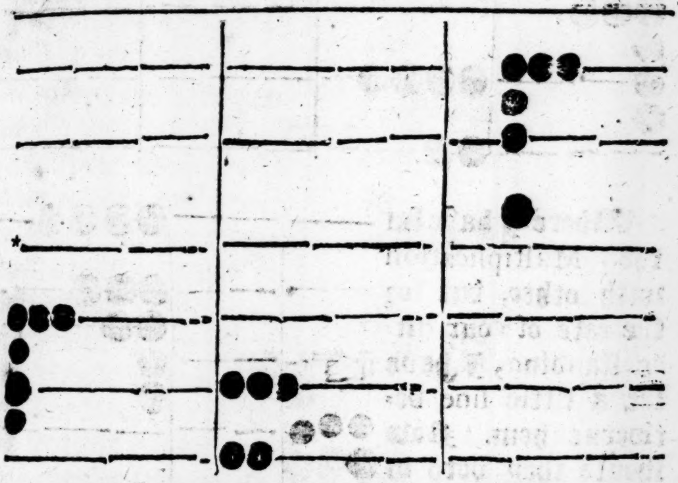


Then first I begin at the 1000 in the highest roome, as if it were the first place, and I take it up setting down for it so often (that is once) the

Multi-

Multiplier, which is 365, thus as you ſée here: where, for the one Counter taken up from the fourth line, I haue ſet downe other ſixe which make the ſumme of the Multiplier, reckoning the fourth line, as if it were the firſt, which thing I haue marked by the Starre ſet at the beginning.

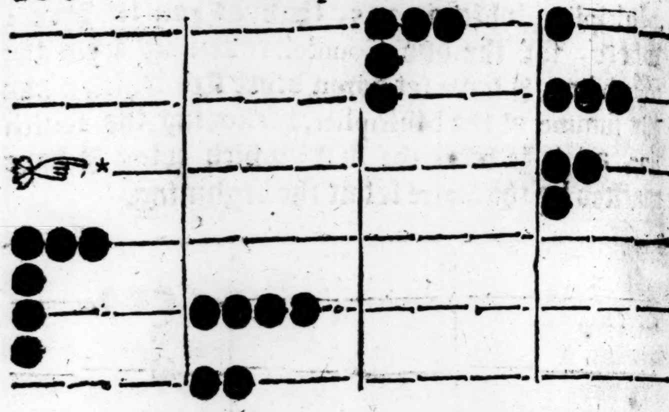
*



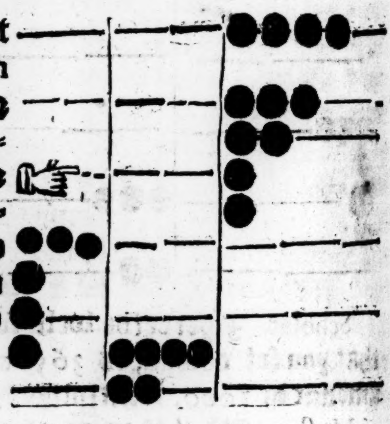
Scholar. I perceiue well, for indeed this ſumme that you ſet downe, is 365000: for ſo much doth amount of 1000, multiplied by 365.

Maſter. Well then goe forth, in the next ſpace I finde one Counter, which I remoue forward, but take it not up, but (as in ſuch a caſe I muſt) ſet downe the greater halfe of my Multiplier (ſéeing it is an odde number) which is 182, and here do I ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſée.

Where



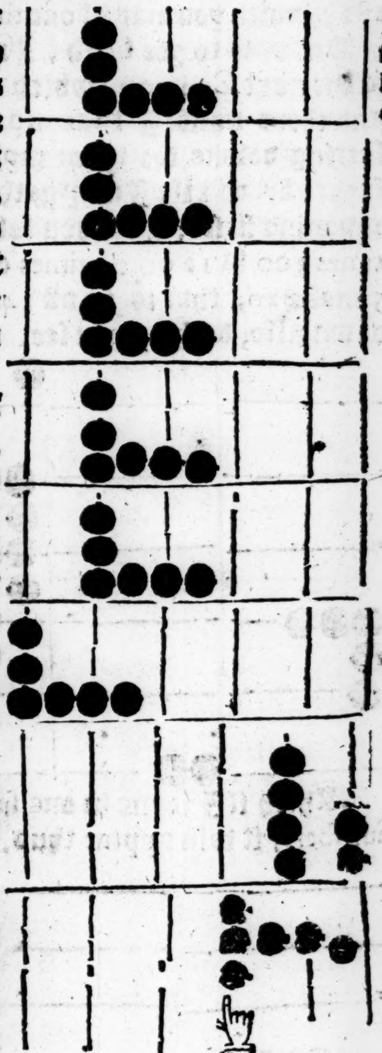
Where I have set the Multiplication with other, but for the ease of your understanding, I have set a little line betweene them. Now should they both in one summe stand thus.



Now

Another forme of Multiplication.

Howbeit, another forme to multiply such Counters in space is this: first to remove the finger to the next line beneath the space, and then to take up the Counter, and to set downe the Multiplier five times, as here you see,

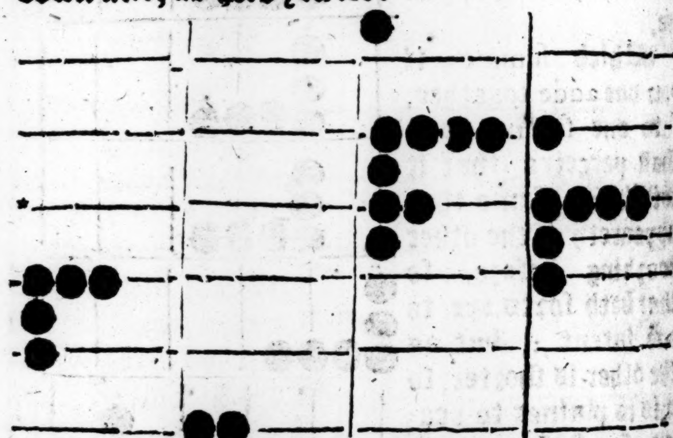


Which summes, if you doe adde together, into one summe, you shall perceive that it will be the same that appeareth of the other working before, so that both sorts are to one intent: but as the other is shorter, so this is plainer to reason, for such as have had small exercise in this Art.

Notwithstanding you may adde them in your minde before you set them down: as in this example you might have said five times 300 is 1500, and five times 60 is 300, also five times five is 25, which all put together doe make 1825,

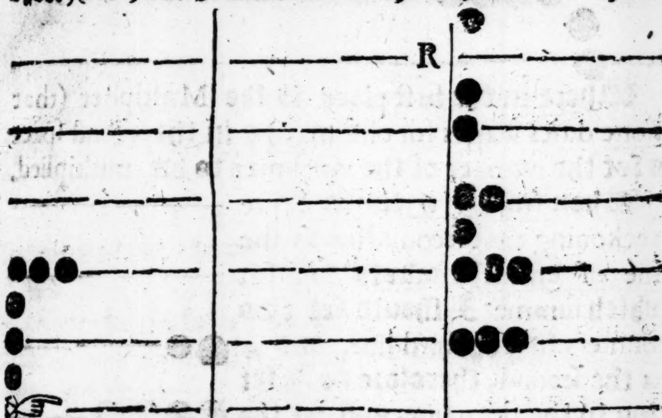
1825, which you may at one time set down if you list.

But now to goe forth, I must remove the hand to the next Counters, which are in the second line, and there must I take up those foure Counters, setting downe for them my multiplier foure times severally, or else I may gather the whole summe in my mind first; and then set it down: as to say, 4 times 300 is 1200: 4 times 60 are 240: & 4 times 5 make 20, that is in all 1460: that shall I set down also, as here you see.



Which if I joyne in one summe with the former numbers, it will appear thus.

Then to end this Multiplication, I remove the finger to the lowest line where are onely 2, them doe I take up, and in their stead doe I set down twice 365, that is 730, for which I set one in the space above the 3 line for 500, & 2 more in the 3 line with that one that is there already & the rest in their order, & so have I well ended the whole sume thus:



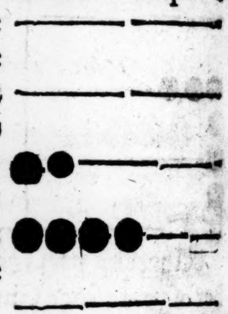
Whereby you see, that 1542 (which is the number of years since Christ his Incarnation) being multiplied by 365 (which is the number of the daies in a year) doth amount to 562830. which declareth the number of daies since Christs incarnation unto the end of 1542 years, besides 385 daies, & 12 hours for leap years.

Scholar. Now will I prove by another example, as this: 40 Labourers (after 6 pence the day for each man) have wrought 28 daies. I would know what their wages doth amount unto. Example of wages.

In this case must I work doubly; first I must multiply the number of the Labourers, by the wages of a man for one day, so will the charge of every day amount.

Then ſecondly, ſhall I multiply the charge of one day by the whole number of dayes, and ſo will the whole ſumme appear: firſt therefore I ſhall ſet the ſummes thus.

Where in the firſt place is the Multiplier (that is one daies wages for one man) & in the ſecond ſpace is ſet the number of the workmen to be multiplied.

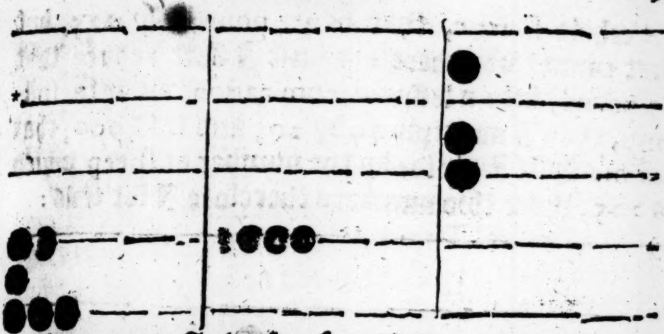


Then ſay: If 6 times foure (reckoning that ſecond line as the line of unites) maketh 24, for which ſumme I ſhould ſet two counters in the third line, and 4 in the ſecond, therefore do I ſet two in the third line, and let the 4 ſtand ſtill in the ſecond line thus.

So appeareth the whole dayes wages to be 240 pence, that is 20 ſhillings.

Then do I multiply again the ſame ſumme by the number of dayes; & firſt I ſet the numbers thus: then becauſe there are counters in divers lines I ſhall begin with the higheſt, and take them up ſetting for them the Multiplier ſo many times as I took up counters, that is twice, then will the ſumme ſtand thus.

Then



Then come I to the second line, and take up those 4 counters, setting for them the Multiplier 4 times, so will the whole summe appear thus:

So is the whole wages of 40 workmen for 28 dayes after 6 pence each day for a man, 6720 pence, that is, 560 shillings, or 28 pound.

Master. Now if you would prove Multiplication, the surest way is by division: therefore will I overpasse it till I have taught you the Art of division, which you shall work thus.

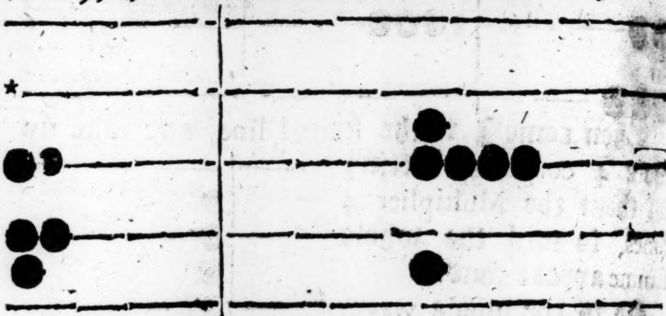
Division.

First, set down the Divisor, for feare of forgetting, and then set that number that shall be divided at the right side, so farre from the Divisor, that the quotient may be set between them: as for example.

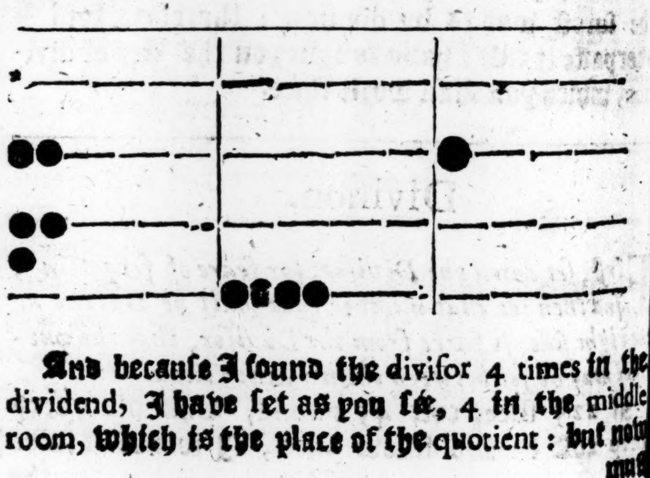
If 225 sheep cost 45 pound, what did every sheep cost? To know this, I would divide the

An example of sheep.

the whole ſumme, that is 45 pound, by 225, but that cannot be: therefore muſt I firſt reduce that 45 pound, into a leſſer denomination, as into ſhillings, then I multiply 44 by 20, and it is 900; that ſumme ſhall I divide by the number of ſheep, which is 225, theſe two numbers therefore I ſet thus:



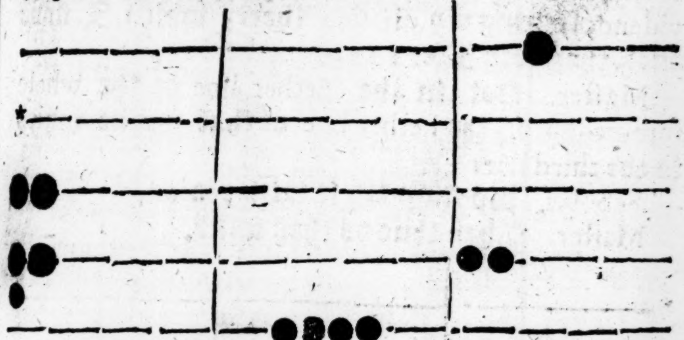
Then begin I at the higheſt line of the dividend, and ſeek how oft I may have the diviſor therein, and that I may do four times: then ſay I, four times 2 are 8, which if I take from 9, there reſteth but 1, thus:



And becauſe I found the diviſor 4 times in the dividend, I have ſet as you ſee, 4 in the middle room, which is the place of the quotient: but now muſt

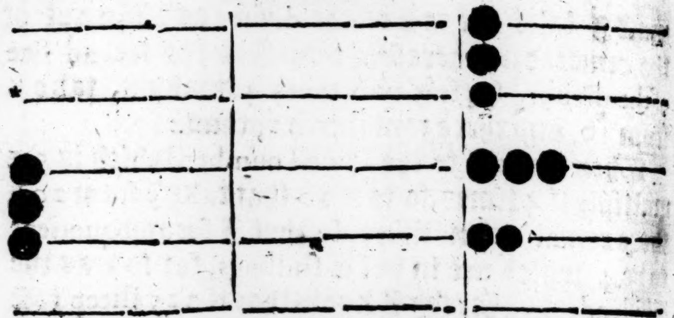
must I take the rest of the divisor as often out of the remainder, therefore come I to the second line of the divisor, saying two times 4 make 8, take 8 from 10, and there remaineth 2, thus:

Then come I to the lowest number, which is 5, & multiply it 4 times, so is it 20, that take I from 20, & there remaineth nothing, so that I see my quotient to be 4, which are in value shillings, for so was the dividend: and thereby I know that if 225 sheep cost 45 pound, every sheep cost 4 shillings.



Scholar. This can I do as you shall perceive by the example. If 160 Souldiers do spend every moneth 68 pound, what spendeth each man? Example of souldiers wages.

First, because I cannot divide the 68 by 160, therefore I will turn the pounds into pence by Multiplication, so shall there be 16320 pence: now must I divide the summe by the number of Souldiers, therefore I set them in order thus:



Then begin I at the higheſt place of the di-vidend, ſeeking my diviſion there, which I finde once, therefore I ſet 1 in the nether line.

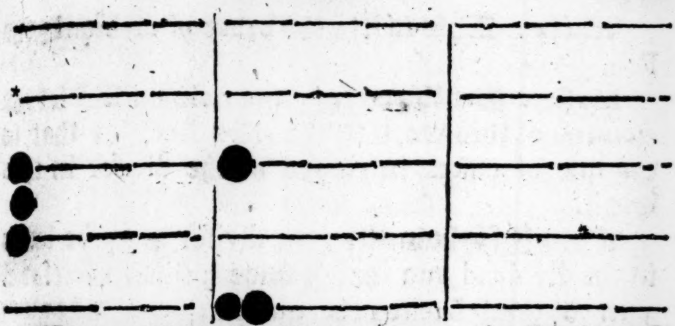
Maſter. Not in the nether line of the whole ſumme, but in the nether line of that worke which is the third line.

Scholar. So ſtandeth it with reaſon.

Maſter. Then thus do they ſtand.

Then ſeeke I againe the reſt, how often I may finde my diviſor: and I ſee that in 300 I might finde 100 three times: but then the 60 will not be ſo often found in 20, therefore I take 2 for my quotient: then take I 100 twiſe from 300, and there reſteth 100, out of which with the 20 that maketh 120, I may take 60 alſo twiſe, and then ſtand the numbers thus:

Where



Where I have set the quotient 2 in the lowest line: so is every Souldiers portion 102 pence that is 8 shillings 6 pence.

Master. But yet because you may justly perceive the reason of division, it shall be good that you set your divisor still against those numbers from which you do take it, as by this example I will declare.

If the purchase of 200 acres of ground did cost 290 pound, what did one acre cost?

First, will I turne the pounds into pence, so will there be 69600 pence. Then in setting down these numbers, I shall do thus:

An example of puchase.

First set the dividend on the right hand as it ought, and then the divisor on the left hand against those Numbers from which I intend to take him first, as here you see, where I have set the divisor two lines higher then his own place.

Scholar. This is like the order of diviſion by the Pen.

Maſter. Truth you ſay, and now muſt I ſet the quotient of this work in the third line, for that is the line of unites in reſpect of the diviſor in this work.

Then I ſee how often the diviſor may be found in the dividend, and that I finde 3 times, then ſet I 3 in the third line for the quotient, and take away that 60000 from the dividend: and further I ſet the diviſor one line lower, as you ſee here.

And then ſeek I how often the diviſor will be taken from the number againſt it, which will be four times and 1 remaining.

Scholar. But what if it chance that when the diviſor is ſo removed it cannot be once taken out of the dividend againſt it?

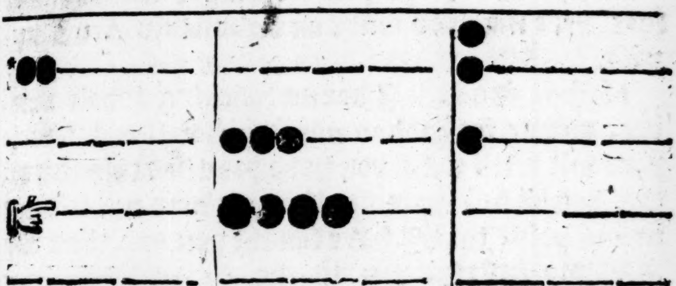
Maſter. Then muſt the diviſor be ſet in another line lower.

Scholar. So was it in diviſion by the pen, and therefore was there a cypher ſet in the quotient: but how ſhall that be noted here?

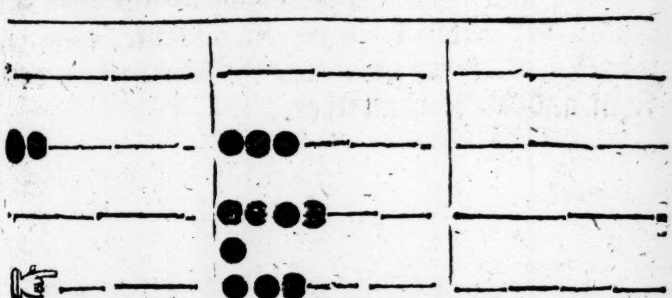
Maſter. Here needeth no token, for the lines do repreſent the places, onely look that you ſet your

quotient

quotient in that place which standeth for unites in respect of the divisor. But now to return to the example: I finde the divisor four times in the dividend, and 1 remaining: for 4 times 2 make 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the middle space for the quotient, I set 4 in the second line, which is in this work the place of unites.



Then remove I the divisor to the next lower line, and seek how often I may have it in the dividend, which I may do here 8 times just, and nothing remain, as in this form.



Where you may see that the whole quotient is 348 pence, that is 29 shillings, whereby I know that so much cost the purchase of one Acre.

Scholar.

Scholar. Now resteth the proofs of Multiplication, and also division.

Master. Their best proofs are each one by the other; for Multiplication is proved by division, and division by Multiplication, as in the work by the pen you learned.

Scholar. If that be all, you shall not need to repeat again that which was sufficiently taught already: and except you will teach me any other feat, here may you make an end of this Art, I suppose.

The reason of all the former rules.

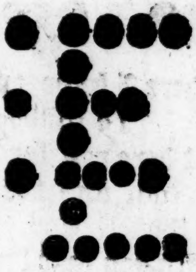
Master. So will I doe as touching whole number, and as for broken number I will not trouble your wit with it, till you have practised this so well, that you be full perfect, so that you need not to doubt in any point that I have taught you, and then may I boldly instruct you in the Art of Fractions or broken numbers: wherein I will also shew you the reasons of all that you have now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings, and pounds, proceeding by no grounded reason, but onely by a received forme, and that diversly, of divers men: for the Merchants use one form, and Auditors another.

Merchants.

Merchants use.

Merchants Accompt.

But first for Merchants form, mark this example here, in which I have expressed this summe, 198 pounds 19 shillings 11 pence. *So that you may see that the lowest line serveth for pence, the next above for shillings, the third for pounds, and the fourth for scores of pounds.*



And further you may see that the space between pence and shillings, may receive but one counter, (as all other spaces likewise do) and that one standeth in that place for 6 pence.

Likewise between the shillings and the pounds one counter stands for 10 shillings.

And between the pounds and 20 pounds, one counter standeth for 10 pounds.

But beside those, you may see at the left side of shillings, that one number standeth alone and betokeneth 5 shillings.

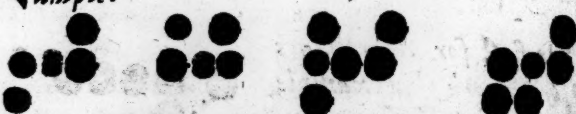
So against the pounds, that one counter standeth for 5 pound. And against the 20 pounds, the one counter standeth for five score pounds, that is 100 pounds: so that every side counter is five times so much as one of them against which he standeth.

Auditors

Auditors Accompt.

Auditors accoumpt.

NOw for the Accompt of Auditors, take this example.



Where I have expressed the same summe 198 pound 19 shillings 11 pence.

But here you see the pence stand towards the right hand, and the other increasing orderly towards the left hand.

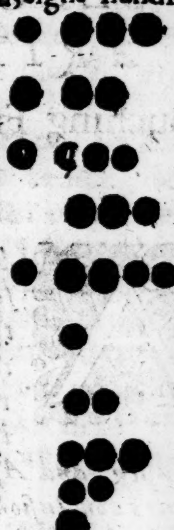
Again you may see, that Auditors will make two lines (yea and more) for pence, shillings, and all other values, if their summes extend thereto. Also you see that they set one Counter at the right end of each row, which so set there standeth for five of that room, and on the left corner of the row, it standeth for 10 of the same row.

But now if you would adde, or subtract after any of both these sorts, if you mark the order of the other feat which I taught you, you may easily do the same here without much teaching: for in Addition you must first set down one summe, and to the same set the other orderly, and in like manner, if you have many; but in Subtraction, you must set down first the greatest summe, and from it must you abate the other, every Denomination from his due place.

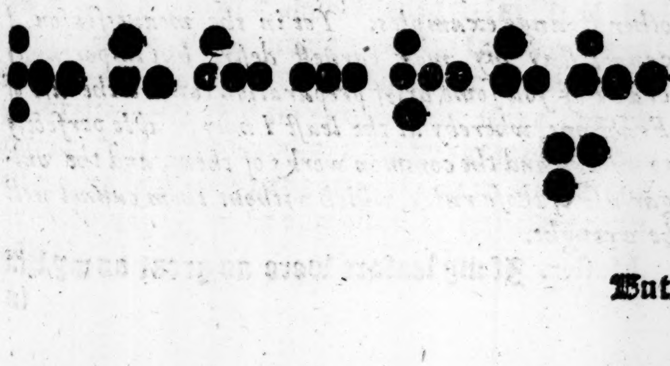
Scholar. I do not doubt but with a little practise I shall attain these both: but how shall I multiply and divide after these forms?

M. You cannot duely do any of both by these sorts: therefore in such case you must resort to your other Arts.

Scholar. They that uſe ſuch Accounts that it exceed 200 in the ſumme, they ſet not 5 at the left hand of the ſcores of pounds, but they ſet all the hundreds in another farther row, & 500 at the left hand thereof, and the thouſands they ſet in a farther row yet, and at the left ſide thereof they ſet the 5000, and in the ſpace over they ſet the 10000, and in a higher row 20000, which all I have expreſſed in this example, which is 97869 pounds 12 ſhillings 9 pence ob, q. Ninetey ſeven thouſand, eight hundred threeſcore and nine pounds, twelve ſhillings and nine pence half peny farthing, for I had not told you before, where neither how you ſhould ſet downe farthings, which (as you ſee here) muſt be ſet in a voide place ſideling beneath the pence, for a farthing one counter, ob. two counters, for ob. farthing 3 counters, and more there cannot bée: for 4 farthings make 1 peny, which muſt be ſet in his due place.



And if you deſire the ſame ſumme after Auditors manner, lo here it is.



But

But in this thing you shall take this for sufficient, and the rest you shall observe as you may by the working of each sort, for the divers wits of men have invented divers and sundry wayes, almost innumerable.

THE SECOND PART OF *ARITHMETICK*, touching Fractions, briefly set fourth.

Scholar.

Arithmeticall fractions.

ALbeit I perceive your manifold businesses doth so occupie, or rather oppresse you, that you cannot as yet compleatly end the Treatise of Fractions Arithmeticall, which you have prepared, wherein not onely sundry works of Geometry, Musick, and Astronomy be largely set forth, but also divers conclusions and naturall works touching mixtures of Metals, and compositions of Medicines, with other strange examples. Yet in the meane season, I cannot stay my most earnest desire, but importunely crave of you some brief preparation toward the use of Fractions, whereby at the least I may be able perfectly to understand the common works of them, and the vulgar use of those rules, which without them cannot well be wrought.

Master. If my leasure were as great as my will is

is good, you should not need to use any importunate craving, for the attaining of that thing, whereby I may be perswaded that I shall any way profit the Common wealth, or help the honest studies of any good Members in the same: wherefore while mine attendance will permit me to walk and talk, I am well willing to help you as I may.

Therefore, first to begin with the explication of this name fraction, what take you it to be?

Scholar. Marry sir, I think a fraction (as I have heard it often named) to be a broken number, that is to say, to be no whole number but part of a number. What a Fraction is

Master. A fraction indeed is a broken number, and so consequently the part of another number, but that must be understood of such another number as cannot be divided into any other parts then fractions: for although I may take the third part of 60, or the 4 part of it, & so of other parts diversly, yet those parts be not properly, nor ought to bee called fractions, because they may be expressed by whole numbers, for the 3 part of it is 20, the 4 part is 15, the 12 part is 5, & so forth of other parts, all which bee whole numbers.

Wherefore properly a fraction expresseth the parts or part onely of a unite, that is to say, that the number which is the whole or entire summe of any fraction may not bee greater then one: and therefore it followeth, that no one fraction alone can be so great, that it shall make 1, as by example I will declare, as soon as I have taught you to know the form how a fraction is expressed or represented in writing. What a Fraction is properly.

Nume-

Numeration.

The expressing of fractions.

But first to begin with expressing of a fraction, which is the Numeration of it: you must understand that a fraction is represented by two numbers set one over the other, and a line drawn between them, as thus, $\frac{1}{3}\,\frac{3}{4}\,\frac{2}{5}\,\frac{10}{17}$, which four fractions you must pronounce, thus, $\frac{1}{3}$ one third part, $\frac{3}{4}$ three quarters, $\frac{2}{5}$ two fifth parts, $\frac{10}{17}$ ten seventeen parts.

Scholar. I understand this form of their expression and pronunciation, but their meaning or valuation seemeth more obscure. Yet I think that by the two first Fractions, I understand the valuation of the two latter Fractions, and consequently of other.

Master. Value them then, that I may perceive your taking of them.

Scholar. $\frac{2}{5}$ betokneth two fift parts, that is to say, if one be divided into 5 parts, that Fraction doth expresse two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be divided into 17 parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$, that is, one third part, doth easily declare, that if one thing bee divided into three parts, I must take out one of them: so $\frac{3}{4}$, that is three quarters, doth declare that one being divided into foure quarters, I must take (for this Fraction) three of those quarters.

If there be no more difficulty in this Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kindes of workes. For I understand that the same kinds of

of workes be in fractions, that bée in whole numbers.

Master. There are the same kindes of works in both, albeit the order of them is divers, as I will anon declare: but yet more in Numeration before we leave it. You must understand, that those two numbers which expresse a fraction, have severall names, the overmost which is above the line, is called the Numerator, & the other beneath the line is called the Denominator.

Numerator.

Denominator.

Scholar. And what is the reason of their divers names? For (in mine opinion) both bée Numerators, seeing both doe expresse the numeration of the fraction.

Master. You are deceived; for one onely (which is the overmost) doth expresse the Numeration, and the Denominator doth declare the number of parts, into which the unite is divided, as in this example; when I say: divide a pound weight of gold betweene foure men, so that the first man shall have $\frac{2}{15}$, the second $\frac{3}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now doe you perceive that by the Denominator (which is one in all foure fractions) it is intended that the pound weight should bée divided into so many parts, I meane 15, and by the 4 severall Numerators, is limited the divers portion that each man should have, that is, that when the whole is parted into 15, the first man shall have two of those 15 parts: the second man thrée of them: the third man foure: and the fourth man six. And so may you sée the severall offices (as it were) of those two numbers, I meane of the Numerator and the Denominator.

And hereby you perceive that a man can have no more parts of any thing then it was divided

into, neither yet aptly so many: so that it were unaptly said, You shall have $\frac{15}{15}$, that is fifteen parts of any thing, seeing it were better said, you shall have the whole thing.

Scholar. So doth it appeare reasonably, for the labour is vain to divide any thing, and then to apply the Division to no use. And much lesse reasonable were it to say $\frac{16}{15}$: for if the whole be divided into 15 parts onely it is not possible to take 16 of them, that is to say, more then all together.

Master. This is true touching the proper and apt use of the name of a Fraction; yet improperly (and after a vulgar acceptation for easinesse in worke) both those formes bee called Fractions, because they be written like Fractions although they bee none indeed: for $\frac{15}{15}$ and generally in such other, where the Numerator and Denominator be equall, are not Fractions, but the whole thing with all his parts: And so $\frac{14}{12}$ is not to be called a Fraction, but a mixt number, of a whole number and a Fraction, for it is as $1\frac{2}{12}$, that is, one whole and $\frac{2}{12}$ parts; as shall be declared in Reduction. Therefore they do abuse the names that call them Fractions, where the Numerator is either equall, or greater then the Denominator.

An improper fraction of a mixt number.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in worke write some numbers, after that sort like Fractions: but they needed not to call them Fractions, but (as they be) whole numbers, or mixt numbers, (that is, whole numbers with Fractions) expressed like Fractions;

or

or as improper Fractions.

Now must you understand, that as no Fraction properly can be greater then one, so in smalnesse under one the nature of Fractions doth extend infinitely, as the nature of whole numbers is to increase above one infinitely, so that not onely one may be divided into infinite Fractions or parts, but also every Fraction may be divided into infinite Fractions or parts, which commonly be called Fractions of Fractions: and they be expressed diversly as for example, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is, thrée quarters of 2 thrée parts of one halfe part. Whereby is signified, that if one be divided into 2 halfes, and the one halfe into thrée parts, and two of those thrée parts be divided joyntly into foure quarters; this fraction of fractions doth represent thrée of those quarters.

Fractions of fractions.

Scholar. I pray you let me prove by an example in common money, whether I doe rightly understand you or no. One Crowne which I take for an whole, doth contain 60 pence; therefore the halfe of it is 30 pence, $\frac{2}{3}$ of that half is 20 pence, whereof $\frac{3}{4}$ is fifteen pence; so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$. of a Crowne: and so is 3 pence $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$. of a shillings.

Master. You perceive this well enough: yet this note I give you by the way, that the form of expressing the fractions is voluntary, and hath no other reason then the will of the Divisor, which form many follow: for some expresse them thus, $\frac{3}{4}\,\frac{2}{3}\,\frac{1}{2}$ without any figure of distinction betwéen them, which form also many follow. Some other doe make lines betwéene every fraction, and adde words of distinction, after this sort, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$,

which form is best.

Some other expresse them thus in slope form, to distinct them from fractions of whole numbers, for if they were in one right line thus, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ then ought it to bee pronounced, three quarters, & two third parts, & a halfe, which maketh almost two whole unites, lacking but one twelfth part. And so is it nothing agreeable with the other fraction of fractions: wherefore it is a great oversight in certain learned men, which doe expresse them so confusedly with such severall fractions, that a man cannot know the one from the other.

$\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$

Therefore some men (as Stifelius *) do expresse without a line, numbers of proportion, being applied to* Addition *or* Subtraction, *because they must be taken as 2, where the line in fractions maketh them to be taken for one: for of the* Numerator *and* Denominator *is made one number.*

Three severall varieties.

Schol. Then I perceive there be thrée severall varieties in fractions: First, when one onely fraction is set for one number, as $\frac{4}{5}$, that is, foure fifth parts. The second is, when there be set two or more severall fractions of one number, as $\frac{4}{9}$ $\frac{2}{5}$, that is, foure ninth parts, and two fifth parts. The third sort is fractions of fractions, as $\frac{4}{9}$, of $\frac{2}{5}$, that is, foure ninth parts of two fifth parts.

Master. You have said well, if you understand well your own words.

Scholar. If it shall please, I will by an example in the parts of an old English Angel, expresse my meaning.

Master. Let me hear you.

Scholar. The old English Angel did contain 7 shillings 6 pence, that is, 90 pence : Now $\frac{4}{5}$ of it is 72 pence : And of the same 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is foure ninth parts, and two fifth parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76 : but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that is, foure ninth parts of two fifth parts, seeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16, for $\frac{1}{9}$ of 36 is but 4, and that taken foure times maketh 16.

Master. This is plainly expressed, and truely, and hereby (I doubt not) but you doe perceive, that as great a difference, as is betweene 16 and 76, so much difference is betweene those two Fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that ye understand these varieties, I will proceed to the rest of the works : First, admonishing you, that there is another order to be followed in Fractions, then there was in whole numbers : for in whole numbers this was the order : Numeration, Addition, Subtraction, Multiplication, Division, & Reduction : but in Fractions, (to follow the same aptnesse in proceeding from the easiest workes to the harder) we must use this order of workes, Numeration, Reduction, Addition, Subtraction, Multiplication, and Division. The order of works in fractions.

Scholar. That Addition and Subtraction should go together, & Division to follow Multiplication, naturall order doth perswade : but why Reduction should be first in order here, next to Numeration, and Addition & Subtraction, in the middle, I desire to understand the reason.

Master. As in the Art of whole numbers, Order would reasonably begin with the easiest, and so

goe forward by degrees to the harvest: even reason teacheth in Fraction the like order. And consider that Addition or Subtraction of Fractions, can very seldome be wrought without Reduction: & contrariwise, Reduction may be wrought without this form of Addition or Subtraction: therefore was it orderly required, that Reduction should goe before Addition and Subtraction, and this reason serveth for the placing of Reduction before the other.

Scholar. Then, if Reduction be the easiest, I pray you declare the forme of it, first by rule, and then by example.

Master. Your request is good.

Reduction of Fractions.

Of Reduction of Fractions, there are five varieties.

Therefore will I now declare the diversities of Reduction of fractions, which commonly hath five varieties, or formes.

1 *First, when there bee sundry Fractions of one intire unite, they must be reduced to one Denomination, and also into one Fraction.*

2 *Secondly, when there be propounded fractions of fractions, they must be reduced likewise into one fraction: for otherwise they cannot bee brought into one Denomination.*

3 *Thirdly, when an improper fraction is propounded, that is to say, a fraction in form, which indeed is greater*

greater then an unite: it must be reduced into apt form, expressing the unite or unites of it, and the proper fraction distinctly. And sometimes also it shall be needfull to convert such a mixt number of unites with fractions, into the form of a fraction, that is, into an improper fraction: which two forms I esteem but as one, because they work one kinde of number.

Fourthly, there happneth sometimes fractions to be written in great numbers, which might be written in lesser numbers: therefore is there a meane to reduce such great numbers into their smallest termes. 4

Fifthly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certain parts, but none of like Denomination *with that* fraction, *then may you reduce the said* fractions *into another, whose* Denomination *shall expresse the common parts of that whole thing.* 5

Scholar. This distinction in Doctrine delighteth me much, but more with hope then present fruit: for as yet I doe not understand scarcely the varieties, and much lesse the practise and use of their works.

Master. Reduction is an orderly alteration of Numbers out of one forme into another, which is never done orderly but for some needfull use, as in every of the said five severall formes, I will distinctly declare.

First therefore, when two, or more severall fractions of any unite be propounded: as for example $\frac{3}{8}$ and $\frac{4}{9}$, because it is hard to tell what proportion of the intire number those two fractions do expresse, therefore was Reduction devised to be a mean whereby these severall fractions might be brought into one Denomi-

The first forme of Reduction

Denomination and fraction.

And in these fractions, this is the Art for bringing them to one Denomination.

How to reduce fractions of divers denominations into one denomination.

Multiply first the Denominators *together, and the totall thereof you shall set twice down under two severall lines for two new* Denominators, *or rather for one common* Denominator. *Then multiply the* Numerator *of the first fraction, by the* Denominator *of the second, and set the totall thereof for the* Numerator *over the first line. Likewise multiply the* Numerator *of the second fraction by the Denominator of the first, and set that totall over the second line for the Numerator of that fraction: and so are these two first Fractions of severall Denominations, brought to one Denomination.*

Scholar. If I understand you, as I thinke I doe my Example shall declare the same. The Fractions, which you propounded were these $\frac{3}{16}$, and $\frac{4}{6}$, whose Denominators (being 16 and 6) I multiply together, and there amounteth 96, which I set under two lines, thus: $\frac{}{96}\ \frac{}{96}$

Then I multiply the Numerator of the first Fraction by the Denominator of the second, saying, 3 into 6 maketh 18, that I set over the first line for a new Numerator, and it will be thus, $\frac{18}{96}$.

Likewise I multiply the Numerator of the second Fraction, by the Denominator of the first, saying, 4 times 16 maketh 64, that I set for the second Numerator, and the Fraction will appear thus, $\frac{64}{96}$.

So that both Fractions brought to one Denomination, must stand thus, $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You have done well.

Scholar.

Scholar. I beseech you let me examine it after my accustomed forme, by common parts of coyne or other measure.

Master. Go to.

Scholar. I have a peece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is, the sixteenth part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Againe $\frac{1}{6}$ of the same peece of gold is 16 pence, so that $\frac{4}{6}$ parts maketh 64 pence, that is $\frac{64}{96}$. And so I finde the summes to agree with the other before.

Master. *So have you now the Art to bring two such fractions into one Denomination : And if there be more then two, then must you multiply all the Denominators together, and set the totall thereof so many times down as there be fractions ; and then to get for each one a new Numerator, multiply the Numerator of the* first, *by the Denominator of the* second, *and the totall thereof multiply by the Denominator of the* third, *and so forth, if there be more. Likewise multiply the Numerator of the* second, *by the Denominator of the* first, *and the totall thereof by the Denominator of the* third. *And in the same sort multiply the Numerator of the* third *into the Denominator of the* first, *and the totall thereof into the Denominator of the* second, *and so forth if there were moe. So these three fractions* $\frac{2}{5}\frac{3}{4}\frac{2}{3}$ *doe make by Reduction these other three fractions of Denomination.* $\frac{24}{60}\frac{45}{60}\frac{40}{60}$. *All which you may bring into one fraction by adding the Numerators together, and putting the totall for the totall Numerator, reserving still that same common Denominator. And those three fractions make one improper fraction, thus :* $\frac{1\,2}{60}$.

Note the Reduction of three fractions, or more, into one.

Scholar.

Scholar. All this I perceive, and also that this last Fraction is more then an unite, and therefore you did call it an improper Fraction.

Master. There be certain other formes of working in this Reduction, which I will briefly touch also, to give you an occasion to exercise your wit therein.

The first variety of Reduction

The first variety is this: When you have made and written down your common Denominator (*as I have taught before*) *then to get a* numerator *for the first, do thus: Divide the* common denominator *by the* denominator *of the first fraction, and the quotient multiplied by the* numerator *of the same yeeldeth a new* numerator *for the first new fraction. So likewise do with the second and the third, and with all the residue, if there be more.*

Scholar. That will I prove in your last example of these three fractions, $\frac{2}{5}\,\frac{3}{4}\,\frac{2}{3}$. When the denominators be multiplied they make 60, for 5 into 4 maketh 20, and 20 by 3 yeeldeth 60. that I set down three times thus: $\overline{60}\,\overline{60}\,\overline{60}$ then to have a numerator, for the first, I must divide 60 by 5 (the denominator of the first) and the quotient is 12, which I must multiply by 2 (the numerator first) and that maketh 24, and so have I for the first Fraction, $\frac{24}{60}$.

Likewise for the second Fraction: I divide 60 by 4, and there cometh 15, which I multiply by 3, and so have I 45, for the second Fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{40}{60}$.

The second variety.

Master. *Another way is this: If it happen so, that the lesser* denominator, *can by any* multiplication *make the greater, then note the multiplier, & by it multiply the numerator over that lesser denominator,* and

and for the lesser Denominator put the greater, as thus in these two fractions $\frac{3}{12}$ and $\frac{2}{3}$, three being the lesser Denominator multiplied by 4, will make 12, which is the greater Denominator: therefore by the same 4 I do multiply 2 which is the Numerator over 3, and that maketh 8: under which I do put 12, being the greater Denominator, which is also made by multiplication of 4 into 3, and so have I these two fractions $\frac{3}{12}$, $\frac{8}{12}$, thus shortly reduced, without altering the one Fraction.

Scholar. This I understand.

Master. Then mark this third way: If the Denominators doe not happen so, that one by Multiplication may make the other, then look whether they both may be parts of any other one number, as in $\frac{5}{12}$, and $\frac{7}{18}$, although the lesser taken but twice, be too much to make 18, yet they both may be parts unto 36, therefore look how many times twelve is in 36, and that quotient being multiplied by the Numerator over 12, the totall shall be put in stead of the Numerator over 12, and for 5 put 15, thus, $\frac{15}{36}$. So likewise look how often is 18 in 36, because it is twice, therefore by 2 multiply 7, which is over 18, and it will be 14: set that for the Numerator, and in stead of 18 put 36; and then your Fractions reduced stand thus $\frac{15}{36}$ $\frac{14}{36}$, in stead of $\frac{5}{12}$, and $\frac{7}{18}$.

The third variety.

And if you will prove whether you have wrought well or no, that may bee proved by Reduction of them again to their former Denominations, which Art shall be taught in the fourth kinde of Reduction, where greater termes of Fractions be reduced into smaller in number, but no smaller in proportion. And if in such Reduction the same termes or numbers come again that were before, then is

Proofe.

is the worke good, else not.

Scholar. Sir I heare your words, but I doe not understand many of them: which if it please you declare.

Master. With a good will, when convenient place serveth, but that must be in the said fourth kind of Reduction, which teacheth how to reduce fractions of fractions into one fraction, and so to one denomination.

The second form of Reduction of fractions of fractions into one fraction and Denomination.

When Fractions of Fractions be propounded, you sha'l multiply the numerators of each into other, and set the totall for the new Numerator, and then multiply all the Denominators likewise, and take their totall for the new Denominator, and so are they speedily reduced.

Scholar. If that be all, then I understand it already, as by this example I will declare. These be the fractions $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ which I would reduce to one denomination, and proper simple fraction.

Therefore begin I with the Numerators, and multiply them together, saying, 3 by 2, maketh 6: and 6 by 6, maketh 36, which multiplyed by 7, yeeldeth 252: that I set over a line for the Numerator thus: $\frac{252}{}$

Then I multiply the denominator, 4 by 3 maketh 12, and that by 7 bringeth 84, which multiplied by 9, yeeldeth 756, the new denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to understand yet. $\frac{252}{756}$

Master. You think so, and no marvell, but anon you shall learn to judge it easily, for this fraction is no more indeed then $\frac{1}{3}$, although it be in greater termes, & therefore more stranger, & more obscure.

And this sufficeth for this Reduction, save that I will shew you by a figure of measure the just rate and reason of this kinde of fractions, and also the due understanding of their Reduction.

The entire measure parted into 9.

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	$\frac{7}{9}$	
1	2	3	4	5	6	$\frac{6}{7}$		
1	2	3	4	$\frac{2}{3}$				
1	2	3	$\frac{1}{4}$					

Here you see the longest measure, (which standeth for the whole & entire quantity) first parted into 9 divisions, whereof 7 are severed by the second measure: and thereof againe are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being divided into foure parts, the lowest measure doth contain $\frac{1}{4}$, so that the same $\frac{1}{4}$ must be named, not $\frac{1}{4}$ of the whole measure, but indeed is $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar. This example is so sensible, that I cannot chose but see it. And furthermore see also, that the same fraction is equall to $\frac{1}{9}$ of the entire measure, as the lines which run up and down do expresly set forth. Also I see here that $\frac{2}{3}$ of $\frac{6}{7}$ is equall to $\frac{4}{9}$, & further yet, that $\frac{2}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$

Master. I am glad that you see it so well, not doubting but you will gather greater light of knowledge hereby.

But now it is time that wee come to the third forme of Reduction, which teacheth of improper Fractions, *that*

The third forme of Reduction of improper fractions.

that is to say, mixt numbers of unites and fractions although they appear like fractions, as this $\frac{26}{5}$, *which doth conclude 5 unites wholly, and* $\frac{1}{5}$ *over. Wherefore first you shall know them, by that the Numerator is greater then the Denominator.*

Scholar Indeed Sir, that appeareth reasonable, that if the Numerator do expresse more parts to be taken of any unite then the Denominator doth signifie that unite to be divided 7 into it must needs follow, that such a Fraction importeth more then the whole, that is to say, the whole with certain parts over: but what Reduction is there in it?

Two severall wayes in this Reduction.

Master. There bee two severall kindes of Reduction, concerning such Fractions. Sometimes it shall bee needfull to convert these fractions into unites, and the proper fraction, that will remaine. And sometimes contrariwise, it shall be meet to reduce mixt numbers, that is, unites written with fractions, into the forme of one simple fraction, & so bee there two wayes.

Scholar. What is the mean of the first way to turn improper Fractions into unites with their proper Fractions?

The fifth way.

Reduction of improper fractions into unites, with their proper fractions.

Master. *That is thus; Your* numerator *being greater then the* denominator, *must be divided by the same* denomination, *and the* quotient *thereof expresseth the* unites: *the remainer shall be put for the* numerator *of the* fraction *that resteth, and the* denominator *must be the same that was before.*

Scholar. For example, I take $\frac{17}{5}$, and dividing 17 by 5, the quotient will be 3, and there will remaine 2.

Master. That you must write thus, 3 $\frac{2}{5}$, where (you see) I have written 3 without any line, as entire

entire numbers ought to bee written, and the 2 that remained, I have set over the former denominator, with a line, as a proper fraction. And this number doth signifie now three unites, & $\frac{2}{5}$ of one.

Scholar. Then if I would by unites here understand Crownes, so it were 3 Crownes, and $\frac{2}{5}$, that is 2 s.

Master. Even so: and therefore $\frac{17}{5}$ did signifie the same: But this happeneth sometimes that when the Reduction is so wrought, there remaineth nothing: And then it is not a mixt number, but a simple intire number, represented like a Fraction.

Scholar. As $\frac{15}{5}$ will make three 3 just, and $\frac{18}{3}$ will make even 6. This I will remember. But now, what is the second form of reduction that you spake of for these sorts of fractions?

The second way.

Master. *Whensoever you have any of these two sorts of* numbers, *that is to say whole* numbers *without* fractions, *or whole* numbers *with* fractions, *and you would turn them into the form of a* fraction, *you must multiply the whole* number *by that* denominator *which you will have to remain still, and to the totall thereof adde the* numerator, *which you have already, and all that you shall set for the new* numerator, *keeping still the former* denominator: *As if you have* 6 $\frac{3}{4}$ *which you would convert into an improper* fraction, *you must multiply 6 by 4, whereof cometh* 24, *and thereto adde the* numerator, *which is* 3, *and so have you* 27 *for the* numerator, *and* 4 *still for the* denominator.

Reduction of whole numbers either alone, or joyned with fractions into improper fractions.

Scholar. Then is $\frac{27}{4}$ equall to 6 $\frac{3}{4}$.

Master. Even just, and so backward (as appeareth by the former Reduction) 6 $\frac{3}{4}$ maketh $\frac{27}{4}$.

Note.

And

And thus one of their reductions may be the proof of the other work.

Scholar. This I perceive: But now if you would turn whole numbers without fractions into any fractions I see not how that may be done, because there is no Denominator to make the multiplication by.

Master. That is well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication must be made: for the proofe whereof I set downe 7, which is a whole number. And if you will have this number converted into any certain fraction, will mée to do it.

Scholar. I pray you reduce 7 into a Fraction.

Master. Then you care not what the Fraction be, so it be some Fraction.

Scholar. No, I passe not for the sort of the Fraction.

Master. Then how can you thinke that you require me to do any thing certain, when you leave me to do as I list? And séeing you stand at that stay, whether thinke you that I must first intend in minde what fraction I will make of it before I can do it indéed?

Scholar. Else you should do ignorantly.

Master. Then will I limit my self (seeing you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the denomination by quarters) and there amounteth 28 to be set for the Numerator and the 4 must be set for the denominator, and the fraction will be thus $\frac{28}{4}$.

Scholar.

Scholar. Indeed I perceive this to be reasonable, for without much triall I understand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turn 8 into 5 parts, it will make $\frac{40}{5}$ which is all one with 8 : for 8 Crownes turned into 5 parts, (that is into shillings) will make 40 shillings, that is, $\frac{40}{5}$ of a Crown.

Master. *Seeing you understand now these three kinds of* Reduction, *I will declare unto you the fourth kind, that is, when* fractions *be written in greater termes then they need, how they may bee brought to lesser termes.* The fourth form of Reduction

Scholar. To write any thing in greater terms then needeth, seemeth to be a fault, and so this Rule seemeth to amend that fault.

Master. It were a fault to do any thing without need, which after must bee redressed : but in this case it is not so, neither did I say absolutely (as you doe) that it needeth not to expresse those Fractions in so great terms, but that the Fractions doe not need, I meane for their value, to be understood : but yet it may be needfull for the case of these workes whereto they be applyed ; as for example, In the first kind of Reduction this was your own example, $\frac{3}{16}$ and $\frac{2}{6}$ which when you would reduce, you were fain to turn them first into one denomination, and so appeared they thus, $\frac{18}{96}$ and $\frac{64}{96}$, where the Fractions (for their own understanding) needed not to be turned out of smaller termes into greater, but yet the easinesse of working needed it.

Scholar. Sir, I understand now, not onely the difference of this need (for the Fractions might better be understood as Fractions severall,

each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you mean by greater terms, and lesser terms, whereof before I was in doubt: for I see you call the Numerator and Denominator, the terms of the fraction.

Termes of Fractions.

Master. *I am glad you understand it so well: now when then you would value any* fractions,*because they may best be done when the terms are smallest, you shall reduce them to the smallest that you can, which thing you may do thus: Divide the greatest of any such two terms by the lesser, and if any thing remain, by that remainer divide the last* divisor *: and if any thing remain now, by that divide the first* divisor *(which was before the remainer of the last* division) *and so continue still, till nothing do remain in the* division *: and then marke your last* divisor, *for it is the* number *that will easily reduce your* fractions, *if you divide both the* numerator *and the* denominator *by the same* number, *and put for the* numerator *the* quotient *of his* division, *and for the* denominator *also his* quotient, *that riseth by his* division.

Scholar. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I divide it by 18, and the quotient is 5, and there resteth 6, what shall I do with this quotient?

Master. Nothing in this worke, but now seeing there remaineth somewhat, by that remainer must you divide the last divisor.

Scholar. If I shall divide 18, (which was the last divisor) by 6, that was the remainer, so is the Quotient 3, and nothing resteth.

Master. As for the Quotient, I omit him yet: but

but because there doth remain nothing, therefore is 6 (which was your last divisor) that number by which you may reduce the fraction propounded.

Scholar. Then as you taught me, I must divide the Numerator 18 by 6, and the quotient is 3, which I must put for the numerator over a line thus: And then by the said 6 must I divide also the denominator 96, and the Quotient will be 16, which I must take for the denominator, and so is the Fraction $\frac{3}{16}$. And so me thinketh this rule doth prove the work of the first Reduction.

$$\frac{3}{16}$$

Master. That is true, if the first Reduction were made of fractions into their least terms, and else not, without some help, as the second number in that place will declare.

Scholar. The second number was $\frac{4}{6}$, which was turned into $\frac{64}{96}$ by that Rule. Now if I shall by this Rule reduce it againe into the least terms, I must divide 96 by 64, and there remaineth 32, wherefore I must take that 32, for the divisor, to reduce the said Fractions. Then do you divide 64 by 32, and the Quotient is 2, which I set for my numerator. Again, I divide 96 by 32, and the Quotient will be 3, and so I have but $\frac{2}{3}$.

Master. Muse not at the matter for you have done well enough: but you think you have not the fraction that you looked for, that is, $\frac{4}{6}$, yet have you one equall to it; as by the parts of a shilling you may prove.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{64}{96}$ and $\frac{4}{6}$ and $\frac{2}{3}$ be all three equall. And now I perceive that because $\frac{4}{6}$

was not written in the least termes that it might be, therefore this Reduction brought forth not it, but that other which is written in the least terms. Now understand I this Rule well. But is there and other way to worke this Reduction?

Another way to work this Reduction

Master. Yes: but first note this, that if you finde no such Divisor, to reduce the fraction till you come to 1, because one doth make no division, therefore that fraction is already in his least termes, as by $\frac{71}{100}$ you may prove, and so $\frac{85}{98}$, and many other like.

Note that to mediate any number is to divide by two.

But now for your better aid to find the due proportion in lesse termes, with more ease for a young learner, you shall mediate or take the halfe of the Numerator, and also of the Denominator as long as you may upon a line, alwayes parting them with a right down dash of your pen as you work, which may easily be done, if the numbers be even, as 2. 4. 6. 8. or 10, but if they be odd (though it be but one of them) then must you abbreviate them by 3. 5. 7, or 9. &c.

And because examples doe most instruct, I have here set downe the manner of two or three, whose last number at the end of the line sheweth the least terme of valuation of that Fraction.

As for example: I woule reduce $\frac{288}{576}$ into his least terme or value, whereupon I set forth $\frac{288}{576}$ with a long line drawn from it, thus,

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And because both the Numerator and the Denominator end in even numbers, I see this may be abbreviated by 2, or 4 or 6, &c. Therefore on the

the other side of the right down dash toward the right hand, I first take the half of the Numerator, saying, the half of 2 is 1, the half of 8 is 4: and again, the half of 8 is 4: which 144 is now a new Numerator, and therefore I part it with a right down dash as before.

Then do I also take the half of 576, in saying, the half of 5 is 2, and the half of 17 is 8, and the half of 16 is 8, and so have I 288 for a new Denominator.

Then beginning again; saying the half of 144 is 72, and the half of 288 is 144: thus continuing the mediation of division by 2, untill you come to the last worke, as appeareth here in the example, where the same is reduced to $\frac{1}{2}$ which is equall to $\frac{288}{576}$.

So the second example $\frac{28}{112}$ first abbreviated by 2, & again by 2, and last by 7, is reduced to $\frac{1}{4}$ which is equall to $\frac{28}{112}$.

$$\frac{28}{112}\ \Big|\ \frac{14}{56}\ \Big|\ \frac{7}{28}\ \Big|\ \frac{1}{4}$$

Again, $\frac{1465}{4395}$ abbreviated first by 5, then by 293.

$$\frac{1465}{4395}\ \Big|\ \frac{293}{879}\ \Big|\ \frac{1}{3}$$

Scholar. Sir, I thanke you much, this is very easie and good for a young learner.

Master. So it is, but yet notwithstanding, if you can without that division by memory, espy the greatest number that may divide exactly both termes of your Fractions proposed, then need you not to use that division, as in this Fraction $\frac{60}{96}$,

I see that 12 is the greatest number that can divide them both: and therefore without any work, by memorie onely, I turn that into $\frac{1}{8}$; but this ability in knowledge is got by exercise.

Yet one other way of easie Reduction in this kinde there is; when your fraction hath any cyphers in the first places of both termes, then may you by casting away the Cyphers, make a briefe Reduction as thus, $\frac{200}{400}$ Here take away the cyphers, and it will be $\frac{2}{4}$, which is the same in value with $\frac{200}{400}$.

Scholar. And so if I have $\frac{400}{6500}$, it will be $\frac{4}{650}$.

Master. You are deceived, for you take away more cyphers from the Numerator then you do from the Denominator, which you may not do.

Scholar. I confesse my fault, which came of too much haste, I was gladder of the Rule then wise in using it: but now I understand it I trust.

Master. Then may I goe in hand with the fifth or last kinde of Reduction, which teacheth how to turn any fraction proposed into any other Denomination that you list, or into any part of common coynes, weights, or measures, or such like.

The fifth kinde of Reduction

For declaration whereof, first you shall marke whether your fraction *be a simple* fraction, *either else a* fraction *of sundry parts, I mean of more terms then two. And if your* fraction *be a* fraction *of* fractions, *or otherwise compound, you must reduce it to one simple* fraction*: And then mark well the* denomination *of that other* fraction, *into which you would turn this: for by that* denominator *you must multiply the* numerator *of your first* fraction, *and the totall* Product *thereof shall you divide by the* denominator *of your first* fraction, *& that* quotient *shall be the* numerator *of the* deno-

To reduce fractions to a denomination appointed.

denominator *proposed* : as for example, *I have this* fraction $\frac{3}{5}$, *which I would turn into ten parts : therefore I multiply this* 10 *by* 3, *that is the* numerator *of my* fraction, *and there ariseth* 30, *which I divide by* 5, *and the* quotient *is* 6, *which must be the* numerator *to* 10, *and so* $\frac{3}{5}$ *will be* $\frac{6}{10}$.

Scholar. This is easie enough to do.

Master. Then shall you see another example of the same fraction that is not so easie : as if I would turn $\frac{3}{5}$ into 8 parts, prove you that work.

Scholar. I must multiply 8 by 3, and there amounteth 24, which I divide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull in this work?

Scholar. I see that when 24 was divided by 5, there remained 4, which I did not passe of, because ye spake nothing of any remainer, but onely of the quotient.

Master. By likelihood you remember what I said to you in Division of whole numbers, that you should not passe of the remainer there but onely note it as a summe that could not be divided without knowledge of Fractions. Wherefore now mark this, that in all divisions of whole numbers, when there is any remainer, you shall set it over a line as a Numerator, & set the divisor for the denominator, and that Fraction doth make the Division compleat, & is part of the quotient : As if I would divide 48 by 5, the quotient will be $9\frac{3}{5}$: so in your former work when 24 was divided by 5, the quotient should be $4\frac{4}{5}$, & so the new Fraction should be thus, $\frac{4}{8}$ & $\frac{4}{5}$ of $\frac{1}{8}$, that is, $\frac{4}{8}$ of the entire number, & $\frac{4}{5}$ of $\frac{1}{8}$

part of any thing, which you may prove by example of some Coyne.

Scholar. Then I take a Crowne, whose $\frac{3}{5}$ is 3 s. Now I would prove whether the 3 s, be $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, I shall have a cumbrous work to do.

Master. Indeed for whole pence, your example is a little troublesome : yet turning the crown into halfe pence, it is easie enough.

Scholar. That will I try.

¶ First I see that $\frac{3}{5}$ of a Crown is 3 shillings which is 36 pence, or 72 halfe pence. Now if I can finde that this Fraction $\frac{4}{8}$ & $\frac{4}{5}$ of $\frac{1}{8}$ be equall unto 3 shillings, then am I fully answered.

Because I cannot take $\frac{4}{8}$ of a Crowne, I turn the Crown into halfe pence, as you willed me, which makes 120, which I divide by 8, my Quotient is 15, which taken foure times, make 60 ob. Now resteth me to have $\frac{4}{5}$ of the $\frac{1}{8}$ part of a Crown, whereof $\frac{1}{8}$ part is 15 ob. the 15 being parted into 5 parts, the Quotient is 3, which taken foure times maketh 12 ob. which with my 60 before amounteth to 72, which are then equall to $\frac{3}{5}$, my desire.

Master. I commend you for your diligence, you might have wrought it thus: either $\frac{4}{8}$ being abbreviated as before I taught is $\frac{1}{2}$. Now halfe a Crown is 2 shillings 6 pence. Now $\frac{4}{5}$ of $\frac{1}{8}$ is a Fraction of Fractions, which if you doe reduce into one entire Fraction, as before you have learned in saying, 5 times 8 is 40, for a new Denominator, and once 4 is 4, for a new numerator : it maketh $\frac{4}{40}$, and abbreviated also make $\frac{1}{10}$. Now the tenth part of a Crown is 6 pence, which put to 2 shillings 6 pence, make also 3 shillings your desire.

But

But now one example more for this Rule, and then we shall end it. If I have $\frac{7}{15}$ of a Soveraign (accounting the Soveraigne 20 shillings) how many shillings is that $\frac{7}{15}$?

Scholar. I must multiply 7 by 20 and that maketh 140, which I shall divide by 15, and the quotient will be $9\frac{5}{15}$, or in lesser termes $\frac{1}{3}$.

Master. That is 9 shillings, and one third part of a shilling, that is 4 pence, as by the same Rule you may prove. And this for this time shall suffice for Reduction. And now I will proceed to Addition.

Addition.

Whensoever you have any Fractions *to be added, you must consider whether they be of one* denomination, *or not, and if they be of one* denomination, *then adde the* Numerators *together, and set that that amounteth for the* numerator *over the common* denominator, *and so have you done: The reason is, because that such differ little in* Addition *or* Subtraction *from the worke of vulgar* denominations *where the* denominators *be of the* number, *as* 3 pence *and* 5 pence *make* 8 pence, *where the* denomination *is not altered. But if the* fractions *be not of one* denomination *or any of them be mixt of whole* numbers *and* fractions, *then must you first reduce them to one* denomination, *and after adde them. And if they be many, then adde first two of them, and so the* summe *that doth amount of the* Addition, *and the third, and then the* 4, &c. *if you have so many.*

Addition of fractions of one denomination.

Scholar.

Scholar. This seemeth easie enough, now that I have already learned to reduce, without which I could never have wrought this: And therefore now I see good reason why you did place Reduction before Addition.

Master. It is well considered, but yet refuse not to expresse your understanding of it by an example.

Scholar. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$ and because the denominators are like (and so needeth no Reduction) I adde 7 to 5, which maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbers, being abbreviated $\frac{2}{3}$.

To adde fractions of divers Denominations.

And if you have many numbers to be added, as here $\frac{3}{8}$ $\frac{4}{5}$ $\frac{9}{10}$, first I must reduce them (because they have divers denominators) into one denomination, and then they will bee thus, $\frac{150}{400}$ $\frac{320}{400}$ $\frac{360}{400}$, or in lesser termes, $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$, which by Addition do make $\frac{83}{40}$, that is $2\frac{3}{40}$.

Master. Now may we go to Subtraction.

Subtraction of Fractions.

Subtraction of Fractions.

SUbtraction *hath the same precepts that* Addition *had, for if the* denominators *bee like, then must you subtract the one* numerator *from the other, and the rest is to be set over the common* denominator, *and so your* subtraction *is ended: but and if you have many* fractions *to be subtracted out of many, then must you reduce them to one* denomination, *and into two severall* fractions, *that is, all*

all that must be subtracted into one fraction, *and the residue into another* fraction, *and then work as I said before.*

Scholar. For the first example I take $\frac{11}{12}$ to be subtracted out of $\frac{17}{12}$, and the rest will bee $\frac{6}{12}$ or $\frac{1}{2}$.

For another example I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus $\frac{24}{32}$ and $\frac{28}{32}$.

Then doe I subtract 24 out of 28, and there resteth 4, which I set over the common denominator for a Remainer, thus $\frac{4}{32}$: that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to bee subtracted from $\frac{7}{8}$ and $\frac{9}{10}$: and because their denominators be divers, I doe reduce them into one denomination thus $\frac{18}{24}$ and $\frac{142}{80}$ $\frac{1408}{1920}$ and $\frac{1641}{1920}$.

Then do I adde the two first, and they make $\frac{1440}{1920}$. Also I adde the two last, and they yeeld $\frac{3408}{1920}$. Then do I subtract 3040 out of 3408, and there resteth 368, so is the remainer, $\frac{368}{1920}$, that is in smaller termes $\frac{23}{120}$ And thus have I done with Subtraction, except you have any more to teach me.

Master. Prove one example or more out of Fractions of divers Denominations.

Scholar. I take the two Fractions $\frac{7}{8}$ to bee subtracted from $\frac{9}{24}$ which being reduced, will stand thus $\frac{168}{192}$ and $\frac{72}{192}$: Now would I subtract 168 out of 72, but I cannot.

168		72
$\frac{7}{8}$		$\frac{9}{24}$
	192	

Master. Then may you perceive that you mistook the Fractions: for you can never subtract the greater out of the lesser, although you may adde, multiply or divide the greater with the lesser.

And

The greatest of two fractions.

And albeit that $\frac{7}{8}$ hath both his terms lesser then $\frac{9}{24}$ yet is $\frac{9}{24}$ the lesser Fraction: for generally if you multiply the Numerator and the Denominators of two Fractions crossewayes, that fraction is the greatest of whose Numerator commeth the greatest summe, as in this example, 7 multiplied by 24 maketh 168, and 9 being multiplied by 8 yeeldeth but 72, therefore is the first fraction 7 the greatest of these two, so can you not subtract it out of a lesser fraction.

But if you should subtract a Fraction out of a whole number, what should you doe?

Scholar. Marry I would reduce the whole number into a Fraction of the same Denomination that my Fraction is, and then work by Subtraction.

Master. So may you doe, but it is much easier, if your Fraction be a proper Fraction, that is to say, lesse then an unite, to take an unite from the whole number, and then turn it into an improper Fraction, and so worke your Subtraction. As if I would subtract $3\frac{2}{5}$ from 4, I may take 1 from 4, and turn it into $\frac{5}{5}$, from which I abate $3\frac{2}{5}$, there will remain $\frac{3}{5}$. And if the first be an improper Fraction, then may I take so many unites from the whole number, that they may make an improper Fraction, greater then that first, and then worke by Subtraction. As if there bee proposed $\frac{10}{3}$, to bee subtracted from 6, becuse $\frac{10}{3}$ is more then 3, & not so much as 4, I must take 4 from 6, and turn them into thirds thus, $\frac{12}{3}$ then abate $\frac{10}{3}$ from $\frac{12}{3}$, there resteth $\frac{2}{3}$: so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$, which is $\frac{10}{3}$: from 6 whole: then set 1 under 6, as thus $\frac{6}{1}$: And then

then to reduce those two Fractions into one Denomination, as here appeareth $\frac{10}{3}$ from $\frac{6}{1}$: Then $\frac{18}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$, which maketh $2\frac{2}{3}$ your desire. And thus will I make an end of the work of subtraction of Fractions, and proceed to Multiplication.

$$\begin{array}{ccc} & 8 & \\ \frac{10}{3} & \times & \frac{18}{3} \\ & 3 & \end{array}$$

Multiplication of Fractions.

THerefore *when any two* fractions *be proposed to be multiplied together, the* numerator *of the one must be multiplied by the* numerator *of the other : and the summe that amounteth thereof must be set for a new* numerator *: likewise the* Denominator *of the one must be multiplied by the* Denominator *of the other, and that that amounteth shall be set for the* Denominator, *and this new third* fraction *expresseth the* Product *of the* Multiplication *of the two first* Fractions *proposed, whereof take this Example,* $\frac{3}{5}$ *multiplied by* $\frac{5}{12}$ *doth make* $\frac{15}{60}$. Multiplication of Fractions.

$$\begin{array}{ccc} & 15 & \\ \frac{3}{5} & & \frac{5}{12} \\ & 60 & \end{array}$$

Scholar. I perceive then that 3 being the Numerator of the first Fraction is multiplied by 5 being the Numerator, of the second Fraction, whereof amounteth 15, the Numerator of the third Fraction. And so likewise 5 being the Denominator of the first Fraction, is multiplied by 12 the Denomination of the second Fraction, whereof amounteth 60 the new denominator, so that

that I perceive how the work is done, I do not perceive how $\frac{15}{60}$ is greater then $\frac{3}{5}$, for if I shall use my former manner of examination by the parts of some coine, I see that $\frac{3}{5}$ of a Crowne is 36 pence, and $\frac{5}{12}$ of a Crowne is 25 pence, whereof the one multiplied by the other, doth make 900 pence, which is 15 Crownes, but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 pence, and that is much lesse then any other of both the first Fractions.

Master. That difference is between multiplication in whole numbers, and Multiplication in broken numbers, that in whole numbers, the summe that amounteth is greater then both the other whereof it came: but in fractions it is contrariwise: for the summe that amounteth is lesser then any of the other two fractions whereof it is produced.

S. I desire much to understand the reason thereof.

Master. Although I purposed to reserve the reasons of works Arithmeticall for the perfect Book of Arithemetick, yet I will shew you this, because of the strangenesse of the work.

You see in whole numbers, that of two numbers, being multiplyed together, is made the third number, which third number doth beare the same proportion to the number multiplied, that the multiplier doth beare to an unite. And so in Fractions the third number which amounteth of Multiplication, beareth the same proportion to each of the two first Fractions, that the other of these two fractions doth bear to an unite.

Scholar. Sir, I understand your words thus: when 40 is multiplyed by 12, there doth amount

480,

480, which 480 doth contain 40 so many times in it, as 12 doth containe Unites, that is to say, twelve times. And so it appeareth that 480 doth contain twelve so many times also as 40 doth contain unites, that is 40 times. But now I see not how the third number in this example of Fractions can containe any of the two former (as it happened in whole numbers) seeing it is lesser then either of them.

Master. No marvell if you cannot see that thing which is not possible to be seen of any man, how the third number in Multiplication of Fractions should be greater then any of the two former Fractions: but yet this may you see (which I said) that the third number in fractions so multiplyed doth beare the same proportion to any of the two former Fractions that the other of those two Fractions doth bear to an unite, as in your example, $\frac{3}{5}$ being multiplyed by $\frac{5}{12}$ doth make $\frac{15}{60}$ Now I say that $\frac{15}{60}$ doth beare the same proporti-on to $\frac{3}{5}$ that $\frac{5}{12}$ doth bear to a unite, as you may in your own forme of examination by Coine, try it: for in an old angell (which in times past was, currant for 7 shilling 6 pence) are 180 halfe pence which I set for the intire unite, whose parts (according to the Fractions aforesaid) are these, for $\frac{15}{60}$ set 45 halfe pence, for $\frac{3}{5}$ take 108 halfe pence, and for $\frac{5}{12}$ put 75 halfe pence. Now doth 45 bear the same proportion to 108, that 75 doth bear to 180, for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserved till another time, when the knowledge of proportions in due order shall be taught: yet in the mean season

I

I will shew you how it commeth to passe, that in Fractions the third summe must needs be lesser then any of the other two.

Consider this, that when a Fraction is proposed, as in the former example $\frac{3}{5}$ if it be multiplied by more then 1, it will make more then one entire number. As if I multiply $\frac{3}{5}$ by 5, that is to say, if I take it 5 times, it will make three entire unites. Example: in a Crown $\frac{3}{5}$ of it maketh 3 shillings, which if I take five times, it will amount to 15 shillings that is, three entire Crownes; so if I take the same $\frac{3}{5}$ but twice, it will yeeld 6 shillings, that is, one entire Crowne, and $\frac{1}{5}$. Now if I take it but once, it cannot be more then it was before, that is 5 shillings. And if I take it lesse then once, it cannot bee so much as it was before. Then seeing that a Fraction is lesse then one, if I multiply a Fraction by another Fraction, it followeth that I doe take the first Fraction lesse then once, and therefore the summe that amounteth, must néeds be lesse then the first Fraction.

Scholar. Sir, I thank you much for this reason. And I trust I doe perceive the thing, as by example of this same Fraction $\frac{3}{5}$ I will expresse. If I take $\frac{3}{5}$ of a Crowne once, that is to say, if I multiply $\frac{3}{5}$ by 1, it will be as it was before, but 3 shillings: so if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then will it be but halfe so much: likewise if I multiply it by $\frac{1}{3}$ that is, if I take but the third part of one, it will yeeld but 12 pence, that is, the third part of the first Fraction.

And

And so to make an end: if I take but the twelfth part of one, that is, if I doe multiply it by $\frac{1}{12}$ it will yeeld but the twelfth part of the first Fraction, which is but 3 pence. And it followeth, that if $\frac{1}{12}$ make 3 pence, then $\frac{5}{12}$ must needs make five times so much, that is, 15 pence, which was the summe that hath given the occasion of all this doubt.

Master. Then I perceive you have sufficient understanding in this sort of Multiplication for this time, wherefore I will proceed to the rest.

In Multiplication *it happeneth sometime, that there bee whole numbers to be multiplyed with Fractions, and may be in two sorts: for either the whole number is severall from the fraction, & is the* multiplier, *or else the whole number is joyned with one, or both of the fractions, and so maketh a mixt number thereof. If it bee in the first sort, then needeth there no* Reduction, *but onely multiply the* numerator *of the fraction by that whole number, and the totall thereof set for the new* numerator. To multiply a whole number into a fraction.

Scholar. I understand you thus. If I have $\frac{6}{23}$ to be multiplyed by 16 then must I multiply that 16 with 6, which is the Numerator, whereof commeth 96, and that must I set for the new Numerator: keeping still 23 for the Denominator, and so the Fraction will be $\frac{96}{23}$ that is $4\frac{4}{23}$.

Mast. And in this sort of work you may abridge the labour thus. If it happen the denominator to be such a number as may evenly be divided by the said whole number proposed, then divide it thereby, and set the Quotient of that division for the former denominator, but reserve still the numerator, and so is the multiplication ended.

Scholar. Then fain this example $\frac{7}{20}$ to be multiplied by 5, and becauſe 5 will juſtly divide 20, therefore I take the Quotient of that diviſion, which is 4, and ſet it in ſtead of 20, and ſo the Fraction will be $\frac{7}{4}$ that is $1\frac{3}{4}$.

Maſter. Which is all one with $\frac{35}{20}$ that would have followed of the other ſort of work.

Scholar. I perceive it very well.

How to multiply mixt numbers.

Maſter. *Now then for the other ſort, where the number is mixt, take this way: firſt to reduce the ſaid whole number and Fraction into one improper Fraction (as I ſhewed you in* Reduction) *and then multiply them together, as if they were proper Fractions.*

Scholar. $13\frac{3}{5}$ being ſet to be multiplyed by $\frac{5}{8}$ firſt I muſt reduce the mixt number, as in this example appeareth, by multiplying 13 by 5, and that maketh 65, whereto I muſt adde the numerator 3, and ſo the Fraction will be $\frac{68}{5}$ which two Fractions now I ſhall multiply after the accuſtomed forme, and it will be $\frac{340}{40}$, or $\frac{34}{4}$.

$13\frac{3}{5}$	340
68 by	5
5 40	8

Maſter. You have done well: and ſo may you ſee, that although moſt part of the formes of multiplication may be wrought without Reduction, yet ſome cannot, as namely, mixed numbers.

Duplation.

And yet one note more I will tell you of Multiplication *before we leave it: That is, whenſoever you would multiply any Fraction by 2, which commonly is called* duplation, *you may doe it not only by doubling the* numerator, *but alſo by parting the* denominator *into half, if it be eaven.*

Scholar. Then if I would double $\frac{5}{12}$ I may chuſe

chuse whether I will make it $\frac{10}{12}$ or else $\frac{5}{6}$. And indeed I see that is all one, but the dividing of the Denominator seemeth the better way to make smaller termes of the Fraction, and so they shall need the lesse Reduction.

Master. It is so: and now I shall not need to tell you that Multiplication is proved by Division, and Division likewise by multiplication: but the like work that I shewed you in multiplication, will I shew you in division.

Division of Fractions.

Whensoever two fractions *bee proposed, that one should be divided by the other, I must set down first the* fraction *that shall be divided (which is called the* dividend) *and then after it the other which is the* divisor : *Then shall I multiply the* numerator *of the* dividend *by the* denominator *of the* divisor, *and that which amounteth I must put for a new* numerator. *Againe I shall multiply the* denominator *of the* dividend *by the* numerator *of the* divisor, *and the* number *that amounteth thereof I must put for the new* denominator. *And this third* fraction *is the* Quotient *of the said* division. Division of Fractions.

Scholar. This seemeth easie in forme, as by example thus: If I would divide $\frac{5}{8}$ by $\frac{2}{6}$, first I multiply 5, (being the numerator of the dividend) by 6, which is the denominator of the divisor,

and thereof riseth 30: then I multiply 8 (being the denominator of the dividend) by 2 being the numerator in the divisor: and so riseth 16, the which I must make a third Fraction thus $\frac{30}{16}$.

	30		
5		2	
8		6	
	16		

Master. Me seemeth you are quicker in understanding now, then you were when I taught you the Art of whole numbers, but that is no marvell: for the more knowledge that a man getteth the readier shall he finde his wit, and be quicker in understanding: but yet of two things I will admonish you, which you might have observed here for the ease of worke and lightnesse of understanding the nature of the Quotient.

Whensoever you divide one Fraction by another, either they be both equall together, or else the one is greater then the other: if they be equall, their Quotient shall be such, that the Numerator and the Denominator of it shall be equall also, And if the two first Fractions be unequall their Quotient shall declare the same by the inequality of the Numerator and Denominator, as in these examples following shall appear.

First, if equall Fractions $\frac{4}{9}$ and $\frac{12}{27}$ be equall together, and if the one be divided by the other, the Quotient will be $\frac{108}{108}$, as you may perceive by that Rule aforesaid,

Now in the unequall Fractions, as $\frac{4}{9}$ and $\frac{3}{10}$ the Quotient will be $\frac{40}{27}$, where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Master.

Master. The reason is this; when any Fraction is divided by another, the Quotient declareth what proportion the Dividend beareth to the Divisor. So $\frac{1}{2}$ divided by $\frac{1}{4}$, maketh 2, which must be sounded, not 2, but twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$. Note how to know the proportion between to numbers.

And note this, that the Numerator in the Quotient representeth the Dividend, and the Denominator representeth the Divisor. And this is alwayes true, whether the greater Fraction be divided by the lesser, or the lesser by the greater. But this proportion will not be exactly known, till you have learned the Art of proportions: notwithstanding somewhat of it I have declared in the Rule of Reduction. But now for the easie remembrance of the Quotient in Division, as soone as you have set downe your two Fractions the one against the other, then make a straight line for the Quotient: and as soone as you have multiplied the Numerator of the Dividend, by the Denominator of the Divisor, set the Number that amounteth over the said line, and then multiply the other two Numbers, and set their totall under the same line.

Scholar. I perceive you would not have me trust to memory till I were better expert, lest oftentimes I happen by misse-remembrance to be abused. This Example I take for that declaration.

If I would divide $\frac{2}{3}$ by $\frac{3}{4}$ I must set the numbers one against the other (as here doth appear) and then make another line for the Quotient in some good

$$\frac{2}{3} \text{ by } \frac{3}{4}$$

S 3 distance

distance, where I may set the numbers of the Quotient, as soon as any of them is multiplied. So then as soone as I have multiplied 2 by 4 which maketh 8, I shall set that 8 over that line, thus: And then multiply 3 by 3, which yeeldeth 9: and that 9 must be set under the same line, and then will the whole Quotient appeare thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$ as 8 is to 9, but how may I perceive that?

$$\frac{8}{9}$$

Master. Although you might better perceive it by the Rule of Reduction, yet this example may be declared in common coines, as in a common shilling of 12 pence, of which $\frac{2}{3}$ maketh 8 pence, and $\frac{3}{4}$ doth make 9 pence, an so you may easily see that their proportions doe agree. And if you had taken this example before when you tooke the example of $\frac{5}{8}$ & $\frac{3}{6}$, your Quotient would appeare (as this doth) more easie to understand; whereas that Quotient being $\frac{30}{16}$, is not an easie proportion for you to perceive, being yet little acquainted with proportions.

Scholar. If there be whole Numbers to be divided by a Fraction, how shall I performe it?

To divide a whole number by a fraction.

Master. *When any whole number shall be divided by a fraction, you must multiply the said whole number with the Denominator of the Fraction, and set the totall thereof for the new Numerator, and for the Denominator set the Numerator of the fraction.*

Scholar. Then 20 divided by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth $\frac{80}{3}$.

$$\frac{20}{1} \text{ by } \frac{3}{4} \quad \frac{80}{3}$$

To divide the fraction by the whole number.

Master. *Even so: but if you would divide the Fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall for*

for the Denominator, without changing the Numerator.

Scholar. Then to divide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth $\frac{20}{23}$ by $\frac{4}{1}$ in this Example $\frac{20}{92}$.

$$\frac{20}{23}\ \text{by}\ \frac{4}{1} \quad \frac{20}{92}$$

Master. You say well. And by the same Example you give me cause to remember another briefe way to doe the same: for if you had divided the said Numerator by 4, and set the Quotient for the Numerator, keeping still the old Denominator, it would have been not only as well done, but also in a Fraction of lesser terms. Another brief way.

Scholar. I guesse it to be even so, by a like work that you taught me in multiplication: And for proof thereof $\frac{20}{23}$ being the Dividend, and 4 the Divisor, I divide the Numerator 20 by 4, and the Quotient is 5, which I set for 20 over 23, thus $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$, as by dividing or abbreviating both these termes by 4, and so reducing them to their least Denomination, I may easily prove: as appeareth by this example $\frac{20}{2}$ $\frac{5}{1}$.

Master. You conceive it well. And if there be mixt numbers, (either one or both) you must first reduce that mixt number into an improper Fraction, and then work as you have learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Division: if the denominators be like, then divide the numerators, as it were in whole numbers, and the Quotient, whether it be Fraction, whole number or mixt, is a

good Quotient for that Division. And generally, if one of the numerators may justly divide the other by that Quotient, multiply the Denominator of the lesser numerator, and set it that doth amount in the rome of the same denominator, and then for a numerator to it, set the denominator of the other Fraction.

Scholar. Then if I would divide $\frac{3}{4}$ by $\frac{12}{17}$ I see that 3 will divide 12, and the Quotient will be 4, by which I must multiply the other 4, that is the denominator under 3, and then it is 16, which is set for the denominator 4, and over it in stead of 3 I must set 17 the other denominator, and so it is thus $\frac{17}{16}$.

Master. And so is $\frac{7}{16}$ in stead of $\frac{51}{48}$, which would have risen by the common worke, as here appeareth.

	51	
3		12
4	by	17
	48	

And now for Mediation (which is to divide by 2) marke this, if the Numerator *be an even number, set the half of it in his place without the* divisor, *and so have you done: and if the* numerator *be not even, then double the* denominator.

Scholar. That is, if I would mediate $\frac{16}{11}$, I may make the Quotient $\frac{8}{11}$, and if I would mediate $\frac{7}{11}$, I must make it $\frac{7}{22}$.

Master. And thus will I make an end of the workes of common fractions for this time, not doubting but you can apply them both to the Rule of progression, and also to the Golden Rule, without any other teaching then you have learned before, which might seem tedious to repeat, in regard you have sufficient knowledge in Reduction,

Addition,

Addition, Subtraction, Multiplication, and Division: And therefore will I goe in hand with the Rule of Proportion, or Golden Rule, which now will appear easie enough.

The Golden Rule direct in Fractions.

Master.

THerefore as touching the Golden Rule for the placing of the three numbers proposed in the question whereby to finde the fourth and for the form of their worke, with other like notes, I referre you to that which you have already learned. The rule of proportion in Fractions.

But this easie form of working by fractions shall you note, that if your three numbers be fractions, for an apt worke and certaine, multiply the numerator of the first number in the question, by the denominator of the second: And all that againe multiply by the denominator of the third number, and the totall thereof shall you keep for to bee the divisor. Then multiply the Denominator of the first number by the numerator of the second, and the whole thereof by the numerator of the third, & the totall thereof shall be your dividend. Note this for a generall Rule.

Now divide this dividend by the divisor which you found out before, and that number shall be the fourth number of the question which you seek for, as in this example.

If $\frac{1}{4}$ of a yard of Velvet cost $\frac{2}{3}$ of a soveragin, esteemed at 20 shillings, what shall $\frac{5}{6}$ cost? A question of velvet.

Scholar. If it please you to let me make the answer, I would first place these three numbers as I learned in whole numbers, thus: $\frac{1}{4}\ \frac{5}{6}$ Z $\frac{2}{3}$

And

And then according to your new rule, I must multiply 3, being numerator in the first number, by 3, the denominator of the second: & thereto commeth 9, which I multiply again by 6, the denominator of the third number, and so have I 54, which I keep for the divisor. Then multiply I 4 the denominator of the first, by 2 the numerator of the second, and there ariseth 8, which againe I multiply by 5 the numerator of the third, and it maketh 40. Then must I divide 40 by 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$ in lesser termes, and then the figure will stand thus:

$\frac{3}{4}\ \frac{2}{3}\ \frac{5}{6}$ Z $\frac{20}{27}$

But what that is in money I cannot tell, except I shall worke it by Reduction, as you taught me.

Master. It forceth not now, you may reduce it when you list, but it were disorderly done here to mingle divers workes together, where we do not seeke the value of the thing in common money, but in apt number, which yee have well done: and therefore will I yet shew you another like way of easinesse in worke, how you may change your three Fractions into three whole numbers, by which you shall worke, as if the question were proposed in whole numbers. The first number you shall finde as I taught you: now to finde the divisor of the second number, take the numerator for the second fraction: and for the third number, take that that ariseth of the multiplication of the denominator of the first, by the numerator of the third, and then worke your question.

A question of Silver.

Scholar. *For example hereof, I put this question, If $\frac{11}{12}$ of 1 pound weight of silver, be worth $\frac{12}{4}$ of a Soveraigne, what is, $\frac{1}{2}$ of 1 pound weight worth?*

For

For the answer, first I place the Fractions in order thus : $\frac{11}{12}$ Z $\frac{12}{4}$ $\frac{1}{2}$

Then to turne these Fractions into whole numbers, I multiply 11, which is the numerator of the first by 4 (the denominator of the second) and there commeth 44, which I multiply by 2 the denominator of the third, and so amounteth 88, which I set for the Divisor in the first place. Then in the second place I set 12, which is the numerator of the second fraction, and in the third place I set the summe that amounteth of 12, being the denominator of the first number, multiplied by one, being numerator in the third number, and so the figure will stand as here you see. 88 Z 12 12

Then to work it forth, I multiply 12 by 12, and there amounteth 144, which I divide by 88, and the quotient will be $1\frac{56}{88}$, or in lesser termes, $1\frac{7}{11}$, and then the figures will stand thus : $\frac{11}{12}$ Z $\frac{12}{4}$ $\frac{1}{3}$ $1\frac{7}{11}$

Master. These two formes now you understand well enough, and as for any other at this time I will not repeat, onely this shall you mark for the proof of this Rule, whether your work be well wrought or no. Multiply the first number by the fourth, and note what amounteth; then multiply the second by the third, and mark what amounteth also. Now if those two numbers so amounting be equall, then is your work well done, else you have erred. And this shall suffice for the former Rule.

The proof of the golden rule.

The

The Backer Rule, or Reverse Rule in Fractions.

The backer Rule in Fractions.

BUt in the Backer Rule, this shall you note for your ease of worke, that you multiply the Numerator of the first by the Numerator of the second, and the whole thereof by the Denominator of the third, and that amounteth thereof, shall be the dividend. Then multiply the Denominator of the first, by the Denominator of the second, and that whole by the Numerator of the third, and that that ariseth thereof, shall be the Divisor. Example of this.

Note this also for a generall Rule.

A question of Loan.

I did lend my friend $\frac{3}{4}$ of a Porteguise, *seven Moneths upon promise that he should do as much for me againe; and when I should borrow of him he could lend me but $\frac{5}{12}$ of a* Porteguise: *now I demand how long time I must keep his money in just recompence of my loane, accounting 13 Months in the year?*

Scholar. The first number must be the first money borrowed, that is $\frac{3}{4}$ of the Porteguise: the second number the 7 moneths, that is $\frac{7}{13}$ of a yeare: and the third number the money that was lent in recompence, that is $\frac{5}{12}$ of a Porteguise: then I set the numbers thus:

$$\frac{3}{4} \; Z \; \frac{7}{13}$$
$$\frac{5}{12}$$

Then (as you taught me) I multiply 3 (being Numerator in the first number) by 7, the Numerator of the second number, and it maketh 21, which I multiply by 12 the Denominator of the third, and so have I 252 for the dividend: then I multiply 4, the Denominator of the first, by 13 the Denominator

tor of the 2^d, & it yeeldeth 52, which I multiply again by 5 the Numerator of the third, & it will make 260, that is the divisor. Then must I divide 252 by 260 so it will be in the small Fraction $\frac{63}{65}$ of a year.

Master. And thus do you see some ease in working, better then to multiply and divide tediously so many Fractions.

Another question yet I will propose, to the intent you may see thereby the reason of the Statute of Assise of Bread and Ale, which in all statute Bookes, in Latine, French and English, is much corrupted for want of knowledge in this Art; for the right understanding whereof I propose this question. Statute of Assise of bread and ale.

When the price of a quarter of wheat is 2 shillings, *the farthing white loafe shall weigh* 68 shillings; *then I demand what shall such a loafe weigh, when a quarter of Wheat is sold for* 3 shillings. Question of bread.

Scholar. This question must be wrought as it is proposed in whole numbers: and not in Fractions.

Master. You seem to say reasonably, howbeit in the Statute of Assise, the rate is made by the proportion of parts in a pound weight Troy else could it not be a Statute of any long continuance, seeing the shillings doe change often as all other moneys doe: but this Statute being well understood, is a continuall Rule for ever, as I will anon declare by a new Table of Assise, converting the shillings into ounces, and parts of ounces.

Wherefore here by a shilling you must understand $\frac{1}{20}$ of a pound weight, and so by a penny $\frac{1}{20}$ of an ounce: wherefore although you might worke this question proposed by whole nmmbers well enough, for that time when the Statute was made, yet

yet to apply it to your time, and make it serve for all times generally, it is best to worke it by fractions, setting for 2 shillings $\frac{2}{20}$, and for 68 shillings $\frac{68}{20}$, and so for three shillings $\frac{3}{20}$, and then will the figure of the question stand thus. $\frac{2}{20}$ $\frac{68}{20}$ $\frac{3}{20}$ In which question, because all the Denominators be like, you shall work onely with the Numerators.

Scholar. Then shall I multiply 68 by 2, whereof commeth 136, which if I divide by 3, the quotient will be $45\frac{1}{3}$: but how shall I make a fraction of that, to stand with the other?

Master. Have you so soon forgotten what was taught you so lately? this is his forme. $\frac{45\frac{1}{3}}{20}$

Scholar. I remember it now, and then it signifieth 45 twenty parts, and the third deale of one twenty part.

Note what a shilling is.

Master. So is it that maketh in shillings 45 shillings 4 pence, whereby you may note one great errour in the Statute Books; which have constantly 48 shillings in that Assise. And by this Rule, if you examine the Statute, you shall finde many summes false. Wherefore for the true understanding of that Statute, and such like, as I have made mention of it, and somewhat recognized it, so doe I wish that all Gentlemen and other students of the Lawes would not neglect this Art of Arithmetick, as unneedfull to their studies. Wherefore to encourage them thereto, and to gratifie both them & all other in generall I will exhibite a Table of that part of the Statutes in two Columnes and in a third Columne, I will adde the correction of those errours which have crept into it.

Here followeth the Table.

The price of a quarter of Wheat.		The weight of a farthing white loafe by the Statute Bookes.			The Correction by just Assise.		
S.	D.	l.	S.	D.	l.	S.	D.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	2	8	0	3	8	0
2	6	2	14	$4\frac{1}{5}$	2	14	$4\frac{4}{5}$
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	$10\frac{2}{7}$
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	$2\frac{2}{3}$
5	0	1	8	$2\frac{1}{2}$	1	7	$2\frac{2}{5}$
5	6	1	4	$8\frac{1}{4}$	1	4	$8\frac{4}{11}$
6	0	1	2	8	1	2	8
6	6	0	16	11	1	0	$11\frac{1}{13}$
7	0	0	19	1	0	19	$5\frac{1}{7}$
7	6	0	18	$1\frac{1}{2}$	0	18	$1\frac{1}{3}$
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	$0\frac{1}{4}$	0	15	$1\frac{1}{7}$
9	6	0	14	$0\frac{1}{4}$	0	14	$3\frac{12}{19}$
10	0	0	13	$7\frac{1}{2}$	0	13	$7\frac{2}{5}$
10	6	0	12	$11\frac{1}{2}$	0	12	$11\frac{3}{7}$
11	0	0	12	$4\frac{1}{4}$	0	12	$4\frac{4}{11}$
11	6	0	11	10	0	11	$9\frac{21}{23}$
12	0	0	11	4	0	11	4

In the common Bookes there is no further rate of Assise made, then unto 12 s. the quarter of wheat, but in an ancient copy of 200 years old (which I have) there is added the rate of Assise unto 20 s. the quarter, but yet was that Assise also either wrong cast at the first penning, or else corrupt since that time, for lack of just knowledge in the Rule of Proportion, *which I will adde here also to gratifie such as be desirous to understand truth exactly.*

The price of a quarter of wheat.		The weight of a farthing white loafe by the Statute Bookes.			The Correction by just Assise.		
s.	d.	l.	s.	d.	l.	s.	d.
12	6	0	11	0	0	10	$10\frac{14}{53}$
13	0	0	11	$0\frac{2}{7}$	0	10	$5\frac{2}{13}$
13	6	0	10	$1\frac{1}{2}$	0	10	$0\frac{8}{9}$
14	0	0	9	7	0	9	$8\frac{4}{7}$
14	6	0	9	$2\frac{1}{3}$	0	9	$4\frac{16}{19}$
15	0	0	9	$1\frac{1}{2}$	0	9	$0\frac{4}{5}$
15	6	0	9	$1\frac{1}{4}$	0	8	$9\frac{8}{31}$
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	$2\frac{10}{11}$
17	0	0	8	3	0	8	0
17	6	0	7	10	0	7	$9\frac{8}{35}$
18	0	0	7	6	0	7	$6\frac{2}{3}$
18	6	0	7	3	0	7	$4\frac{8}{37}$
19	0	0	7	2	0	7	$1\frac{12}{19}$
19	6	0	5	10	0	6	$11\frac{2}{13}$
20	0	0	5	6	0	6	$9\frac{1}{5}$

These

These two Tables I have set severall, because no man should think that I would either add or take away from any Law those parts which might of right seem either superfluous, either diminute: but yet I may not be so curious as to neglect manifest errours, which is not onely my part, but every good Subjects duty with sobriety to correct. And for avoiding of offence, I have rather done it in this private Booke, then in any Booke of the Statutes it selfe, trusting that all men will take it in good part.

Scholar. I would wish so, but I dare not so hope, sith never good man that would reform errour, could reforme the venemous tongues of envious detractors, which because they either cannot or list not to doe any good themselves, doe delight to bark at the doings of others, but I beseech you to stay nothing for their perverse behaviour.

Master. I consider many things that some may object, whereunto I am not unprovided of just answers, but I will not seem so hasty to make the answers before I heare their Objections, but as I trust that men are of a better nature, and more gratefull now then some have been in times past. As I have done in the Statute of Assise of bread in rate of shillings, so will I set forth the like Table in pounds and ounces, and the parts thereof, that it may be easily applyed to all times: But I meane not by this to alter any word of the Statute, (being so good an Ordinance, and of so great continuance) but onely to make it as a kinde of exposition and declaration of the said Statute, trusting that thereby the Statute may be better understood and con- Concerning the following Tables.

T sequently

A pound weight.

sequently better put in execution. And here you shall note, that I have accounted the shillings after the rate of 60 shillings to the pound weight, because I esteem it the most apt for our time. Wherefore in the first Columne you finde the price of Wheat directly against it; in the second Columne, you may finde the weight of a farthing white loafe, in this our time; and if you double the number (as I have done in the third Columne) then have you the weight of the halfe penny white loafe; and so in the fourth Columne) is set the weight of a penny white loafe; It needeth not to tell, that the sight doth testifie how that every Columne is parted into three smaller pillers, whereof the first Columne hath these three titles; pounds, ounces and penny weights. And as in the first Columne 12 pence make a shilling, and 20 shillings make a pound, so in the other three Columnes 20 pence weight maketh an ounce, and 12 ounces make a pound.

GEntle *Reader*, touching the understanding of the *Table* following, wherein according to our time, *Master Record* alloweth 60 *pence* to the *ounce*, and 3 *pound* or 60 *shillings* to the *pound*, and thereupon after the rate of 60 *shillings* to the *pound Troy*, doth he frame or produce this his *Table*, beginning at 3 *shillings the quarter* till it come to 40 *shillings 6 pence the quarter*. and this is his proportion (for that he hath not set downe any one *Example* to continue the worke) hath been hard for many to conceive or comprehend, and therefore the onely chief cause why I have written this *digression* for the better understanding of him therein.

The first thing therefore that is sought for in this *Table*, as in the other aforesaid, is a *Maxime* grounded upon the *Statute*, which is this. When the *quarter of wheat* is sold for *two shillings*, then the *farthing white loafe* shall weigh 68 *shillings*, whereby a *shilling* is $\frac{1}{60}$ meant of a *pound*, and by a *penny* $\frac{1}{60}$ of an *ounce*. Now therefore

fore for a generall Rule, to finde what weight the *farthing white loafe* shall weigh at 3 *shillings the quarter*, till you come to 40 *shillings 6 pence the quarter*, is thus to be wrought. Comming to the *first ground*, and working by the *Backer Rule*, say; If *two shillings the quarter* give, or allow the *farthing white loafe* to weigh 68 *shillings*, what weight ought the *farthing white loafe* to weigh at 3 *shillings the quarter*? Worke, and you shall find 45 *shillings 4 pence*, as before in the correction of the *first table* is noted. Then for the *second work*, say by the *Rule of Three Direct*, if 20 *pence* give one *ounce*, what giveth 45 *shillings 4 pence*? multiply and divide, and you shall finde 544 *ounces*, which 544 *ounces* being multiplied by 3, for 3 *pounds*, or 60 *shillings*, yeeldeth 1632 *ounces*, which divided by 20, produceth 81 *ounces*, & $\frac{12}{20}$ or rather $\frac{3}{5}$ of an *ounce*, equall unto 12 *penny weight*, which is *halfe an ounce*, & 2 *penny weight*, & so maketh in all 6 *pounds*, 9 $\frac{1}{2}$ *ounces* & 2 *penny weight*. Now the next way to continue this *table*, to know the weight of the halfe *penny white loafe*, is thus, multiply 1632 *ounces* by 2, & it bringeth forth 3264 *ounces*, & divided by 20, it yeeldeth 163 *ounces* and $\frac{1}{5}$, which is equall to 13 *pounds* 7 *ounces*, and 4 *penny weight*, as M. *Record his Table noteth*.

Thirdly, for the weight of the penny white loafe, multiply 1632 *ounces* by 4, and divide by 20, and after by 12 as before, and you shall finde 27 *pounds* 2 *ounces*, and 8 *penny weight*, &c. This *Method*, or else by doubling the *farthing white loafe*, for the *weight of the halfe penny white loafe*, and so doubling the *halfe penny white loafe*, for the *weight* of the *penny white loafe*, is the order to continue the *table* to the end thereof.

The price of a quarter of Wheat. li.	s.	d.	The weight of a farthing white loafe. Po.	Ounces.	Peny weight.
0	3	0	6	9	12
0	4	6	4	6	8
0	6	0	3	4	16
0	7	6	2	8	12 $\frac{4}{5}$
0	9	0	2	3	4
0	10	6	1	11	6 $\frac{2}{7}$
0	12	0	1	8	8
0	13	6	1	6	2 $\frac{2}{3}$
0	15	0	1	4	6 $\frac{2}{5}$
0	16	6	1	2	16 $\frac{8}{11}$
0	18	0	1	1	12
0	19	6	1	0	11 $\frac{1}{25}$
1	1	0	—	11	13 $\frac{3}{7}$
1	2	6	—	10	17 $\frac{2}{5}$
1	4	0	—	10	4
1	5	6	—	9	12
1	7	0	—	9	1 $\frac{1}{3}$
1	8	6	—	8	11 $\frac{11}{19}$
1	10	0	—	8	3 $\frac{1}{5}$
1	11	6	—	7	15 $\frac{1}{7}$
1	13	0	—	7	8 $\frac{6}{11}$
1	14	6	—	7	1 $\frac{21}{23}$
1	16	0	—	6	16
1	17	6	—	6	10 $\frac{14}{25}$
1	19	0	—	6	5 $\frac{7}{13}$
,	0	6	—	6	0 $\frac{8}{0}$

The price of a quarter of Wheat.			The weight of a half penny white loafe.			The weight of a half penny white loafe.		
li	s.	l.	Po.	Ounces.	Penny weight	Po.	ounces.	Penny weight.
0	3	0	13	7	4	27	2	8
0	4	6	9	0	16	18	1	12
0	6	0	6	9	12	13	7	4
0	7	6	5	5	5 2/5	10	10	11 1/5
0	9	0	4	6	8	9	0	16
0	10	6	3	10	12 4/7	7	9	5 1/7
0	12	0	3	4	16	6	9	12
0	13	6	3	0	5 1/3	6	0	10 2/3
0	15	0	2	8	12 4/5	4	5	5 2/5
0	16	6	2	5	13 4/11	4	11	6 10/11
0	18	0	2	3	4	4	6	8
0	19	6	2	1	2 2/13	4	2	4 4/13
1	1	0	1	1	16 2/7	3	10	12 4/7
1	2	6	1	9	15 1/5	3	7	10 2/5
1	4	0	1	8	8	3	4	16
1	5	6	1	7	4	3	2	8
1	7	0	1	6	2 2/3	3	0	5 1/3
1	8	6	1	5	3 11/19	2	10	7 2/19
1	10	0	1	4	6 2/5	2	8	12 4/5
1	11	6	1	3	10 6/7	2	7	1 5/7
1	13	0	1	2	16 8/11	2	5	13 5/11
1	14	6	1	2	3 12/23	2	4	7 12/23
1	16	0	1	1	12	2	3	4
1	17	6	1	1	1 8/25	2	2	2 6/25
1	19	0	1	0	11 1/13	2	1	2 2/13
2	0	9	0	0	1 6/7	2	0	3 5/9

Having spoken before for the understanding of the *Table* placed by M. *Recorde*, a man indued with rare knowledge in *Arithmeticall* and *Geometricall proportions*, touching the *Statute of Coynage, and the Standard thereof*, as appeareth in his Epistle of this *Book dedicated* to *King Edward the sixth*, insinuating unto his Highnesse that the *Standard of Coyne* is much altered from the 14. yeare of *King Edward the third* (when this *Statute and Assise* was confirmed) to the *Standard of this our time*. For it appeareth that in *King Edward the thirds time*, when the *Assise of Bread and Drink* was established, that a *Sterling penny, round without clipping*, did then weigh 32 *cornes of wheat dried*, and taken out of the middle of the *eare*, and 20 of these *pence* made an *ounce*, and 12 *ounces* made a *pound Troy*. And so from the *weight of a penny* to 20 *shillings sterling*, which then weighed 12 *ounces*, tooke *Bread his weight* and proportion: And now finding 60 *pence* is an *ounce*. That onely cause (I perceive, for the zeale of a Common-wealth) moved him to set downe the same *Table in this private Booke*; meaning not thereby to alter any word of the *Statute* being so good an ordinance & of so long continuance, but as a kinde of exposition by the way, that thereby the *Statute* may be better understood, and so consequently better put in execution. Which *Assise of his*, is three times greater then the *Statute* now alloweth. Therefore also, to gratifie such as are desirous of knowledge, according to these *prices of a quarter of wheat* I have added to this *Author these three other new Tables following* and reduced their *prices* into their just proportions of *Sterling money*, and also reduced the *money* into known *weight Troy*, according to the Statute. And thereafter according to proportion in my other three *Tables*, have I noted the *just weight* that a *Farthing, Half penny, and Peny white-loafe*, ought to weigh by the *Statute*.

The

The price of a Quarter of Wheat.			The weight of a Farthing white loaf in Sterling money by Assise.			The weight of a Farthing white loaf in Troy weight by Assise.		
l.	s.	d.	l.	s.	d.	Ounces.	Po.	penny weight.
0	3	0	2	5	4	2	3	4
0	4	6	1	10	2 $\frac{2}{3}$	1	6	2 $\frac{2}{3}$
0	6	0	1	2	8	1	1	12
0	7	6	0	18	1 $\frac{2}{5}$	0	10	17 $\frac{2}{5}$
0	9	0	0	15	1 $\frac{1}{3}$	—	9	1 $\frac{1}{3}$
0	10	6	0	12	15 $\frac{2}{7}$	—	7	15 $\frac{2}{7}$
0	12	0	0	11	4	—	6	16
0	13	6	0	10	0 $\frac{8}{9}$	—	6	0 $\frac{8}{9}$
0	15	0	0	9	0 $\frac{4}{5}$	—	5	8 $\frac{4}{5}$
0	16	6	0	8	2 $\frac{11}{10}$	—	4	18 $\frac{10}{11}$
0	18	0	0	7	6 $\frac{2}{3}$	—	4	10 $\frac{2}{3}$
0	19	6	0	6	11 $\frac{2}{13}$	—	4	3 $\frac{7}{13}$
1	1	0	0	6	9 $\frac{3}{5}$	—	4	1 $\frac{3}{2}$
1	2	6	0	6	0 $\frac{8}{13}$	—	3	12 $\frac{8}{15}$
1	4	0	0	5	8	—	3	8
1	5	6	0	5	4	—	3	4
1	7	0	0	5	0 $\frac{4}{9}$	—	3	0 $\frac{4}{9}$
1	8	6	0	4	9 $\frac{5}{19}$	—	3	17 $\frac{5}{19}$
1	10	0	0	4	6 $\frac{2}{5}$	—	2	14 $\frac{2}{5}$
1	11	6	0	4	3 $\frac{17}{21}$	—	2	11 $\frac{17}{21}$
1	13	0	0	4	2 $\frac{1}{11}$	—	2	9 $\frac{1}{2}$
1	14	6	0	3	11 $\frac{7}{23}$	—	2	7 $\frac{7}{23}$
1	16	0	0	3	9 $\frac{2}{3}$	—	2	5 $\frac{2}{3}$
1	17	6	0	3	7 $\frac{12}{25}$	—	2	3 $\frac{12}{25}$
1	19	0	0	3	5 $\frac{11}{13}$	—	2	1 $\frac{11}{13}$
2	0	6	0	3	4 $\frac{8}{17}$	—	2	0 $\frac{8}{27}$

The price of a Quarter of Wheat.			The weight of a half penny white loaf in Troy weight by Assise.			The weight of a half penny white loaf in Troy weight by Assise.		
li.	s.	d.	Po.	Ounces.	Penny weight.	Po.	Ounces.	Penny weight.
0	3	0	4	6	8	9	0	16
0	4	6	3	0	5 1/3	6	0	10 2/3
0	6	0	2	3	4	4	6	8
0	7	6	1	9	15 1/5	3	7	10 2/5
0	9	0	1	6	2 2/3	3	0	5 1/3
0	10	6	1	3	10 6/7	2	7	1 2/5
0	12	0	1	1	12	2	3	4
0	13	6	1	0	1 2/9	2	0	3 4/9
0	15	0	0	10	17 1/5	1	9	15 1/5
0	16	6	0	9	17 1/11	1	7	15 7/9
0	18	0	0	9	1 1/3	1	6	3 2/3
0	19	6	0	8	7 1/13	1	4	14 2/11
1	1	0	0	8	3 1/3	1	4	0 4/5
1	2	6	0	7	5 1/15	1	2	10 2/15
1	4	0	0	6	16	1	1	12
1	5	6	0	6	8	1	0	16
1	7	0	0	6	0 2/9	1	0	1 7/9
1	8	6	0	5	14 10/19	0	11	9 1/19
1	10	0	0	5	8 4/5	0	10	17 1/5
1	11	6	0	5	03 14/21	0	10	7 5/12
1	13	0	0	4	19	0	9	18
1	14	6	0	4	14 14/23	0	9	9 5/23
1	16	0	0	4	11 1/3	0	9	2 1/3
1	17	6	0	4	7 1/25	0	8	14 2/25
1	19	0	0	4	3 2/23	0	8	7 5/28
1	0	6	0	4	0 [illegible]	0	8	1 [illegible]

The weight of a half penny white loaf in Troy weight by Assise.

Scholar. Sir I do thanke you most heartily for this, not onely in mine owne name, and in the name of all students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restituton of the weight in Bread, which long time hath béen abused. And if you know any thing more, wherein you would vouchsafe to declare the errours, and set forth the truth, you cannot but obtain great thankes of all good hearted men that love the Commonwealth.

Mast. I have sundry things to declare, but I have reserved them for a private Booke by it self, yet notwithstanding because the statute of the rate of measuring of grounds is so common, that it toucheth all men, and yet no more common then néedfull, but so much corrupt, that is, too farre out of all good rate; not onely in the English Bookes of statutes, commonly Printed, but also in the Latin Bookes, and in the French also, (for I have read of each sort, and conferred them diligently) I will give you a Table for the restitution of those errours, as may suffice for this present time. And first I will propose one question to you touching the use of that statute, whereby you may perceive the order how to examine the whole statute, and every parcell thereof, and the question is this.

When the Acre of ground doth containe foure Perches in breadth, then must it contain 40 Perches in length. Then do I demand of you, how much shall the length of an Acre be, when there is in the breadth of it 13 Perches. But before you shall answer to this question, I will declare unto you another Statute, which is the ground of the former Statute. And that Statute is this. A question of measuring of ground.

It

Statute measure.

It is ordained, that thrée Barly-cornes dry and round, shall make up the meaſure of an inch: 12 inches ſhall make a foot, & 3 foot a yard, (the common English books have an Elne) five yards and a halfe ſhall make a Perch, and 40 Perches in length, and 4 in breadth ſhall make an Acre. This is that Statute, whereby you may perceive, that the intent of the ſtatute is, that one Acre ſhould containe 160 ſquare Perches. Now let me hear you anſwer to the queſtion.

An Acre.

Scholar. As I perceive by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an Acre, ſo could I make thoſe numbers all in fractions, and ſo worke the queſtion: but ſéeing I may doe it alſo in whole numbers, I take that forme for the moſt eaſe; therefore thus I ſet the queſtion in forme. Then doe I multiply 40 by 4, and it maketh 160, which I divide by 13, and the Quotient is 12 $\frac{4}{13}$.

4 Z 40
13 Z 12

Maſter. Now turne that $\frac{4}{13}$ into the common parts of a Perch, as they be named in the former ſtatute: Howbeit it ſhall be beſt to take one of the leaſt parts in Denomination for avoyding of much labour, as Feete, whereof the Perch containeth 16 $\frac{1}{2}$.

Scholar. Then to returne $\frac{4}{13}$ into Feet, I multiply 16 $\frac{1}{2}$ by 4, and it maketh 66. which I muſt divide by 13, and the Quotient is 5 $\frac{1}{13}$.

Maſter. So I find, that if the Acre hold in breadth 13 Perches, it ſhall containe in length 12 Perches 5 Foot, & $\frac{1}{13}$ of a foote, which is not fully an Inch, for the Inches is $\frac{1}{12}$ of a foot. But here all the Statute Bookes in Latine and Engliſh (that I have ſéene) doe note it to be 13 Perches, 5 foote, and

Note this errour

one

one Inch, which make above 13 Perches to many in the Acre: so that I would have thought the errour to have crept into the Printed Bookes, by the great negligence that Printers in our time doe use, save that in written copies of great antiquity, I doe finde the same; yet have I one French copy which hath 12 Perches $\frac{3}{4}$ and one foote, and that misseth very little of the truth.

Scholar. Then I see it is true that I have often heard say, that the truest Copies of the Statutes, be the French copies.

Master. That is often true, but not generally, as I have by conference tried diversity: but in this Statute the French Booke is most corrupt: in all other places lightly.

But now to performe my promise, I will set forth the Table for measuring of an Acre of ground, onely by such parts as the Statute doth mention, because at this time I doe of purpose write it for the better understanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this Table *following, I have not done as in the other* Statute *before compared by restitution with the faults crept into the* Statute, *but only have written that true measure, which the equity of the* Satute *doth pretend. For it were vile to judge of so noble Princes and worthy Counsellors, as have authorised and set forth this* Statute, *that they would make one* acre *in any forme greater then another, but every one to be just and equall with each other, which is the ground also of my worke: & hereby may all men perceive how needfull* Arithmetick *is to the* Students of Law. *But*

now

now I think best to make an end of these matters for this present time, sith the Table *hath in it no obscurity that I should need to declare.*

The breadth of the Acre.	The length of the Acre.			
Perches	Perches	Feet.	Inches.	Parts of an Inch
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{2}{17}$
18	8	14	8	0
19	8	6	11	$\frac{2}{19}$
20	8	0	0	0
21	7	10	2	$\frac{6}{7}$
22	7	4	6	0
23	6	15	9	$\frac{2}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	6	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The

The breadth of the Acre.	The length of the Acre.			
Perches	Perches	Feet.	Inches.	Parts of an Inch.
28	5	11	9	2/7
29	5	8	6	12/29
30	5	5	6	0
31	5	2	7	28/31
32	5	0	0	0
33	4	14	0	0
34	4	11	7	12/17
35	4	9	5	1/5
36	4	7	4	0
37	4	5	4	8/37
38	4	3	5	1/19
39	4	1	8	4/13
40	4	0	0	0
41	3	14	10	24/41
42	3	13	4	2/7
43	3	11	10	32/43
44	3	10	6	0
45	3	9	2	0

Scholar. Indeed Sir, I understand the Table (as I thinke) by those other which you set forth before. For in the first columne is set the Perches of the breadth of an Acre, and then in the two Columnes following appeareth how many Perches, and how many foote that same Acre must have for his length.

Master.

Master, You take it well : howbeit to speake exactly of breadth and length, and the first Columne doth sometime betoken the breadth, and sometime the length : for properly the longest side of any square doth limite his length, and the shorter side doth betoken the breadth, yet it is no great abuse in such Tables, where a man cannot well change the Title, to let the name remaine, although the proportions of the numbers doe change : for still by the first Columne is expressed the measure of the one side, and by the two other pillars in one Columne, is set forth the measure of the other side. And this shall be sufficient now for the use of the Golden Rule.

The Rule of Fellowship.

NOw will I touch certaine other Rules, which for their severall names may seeme divers Rules, and distinct from this, but indeed they are but branches of it : yet because they have severall workings in appearance, but also pleasant in use, I will give you a taste of each of them. As for the Rule of Fellowship, both single and double, with time and without time, I shall need to say little more then I have already said in teaching the workes of whole numbers : yet an example or two will we have to refresh the remembrance of the same, and to declare certain proper uses and applications of it, as this for one.

The Rule of Fellowship without time.

A question of inequall society.

Foure men got a booty or prize in time of warre, the prize is in valne of money 8190 *pound, and because the*

the men be not of like degree, therefore their shares may not be equall; but the chiefest person will have of the booty the third part, and the tenth part over; the second will have a quarter, and the tenth part over: the third will have the sixth part: and so there is left for the fourth man a very small portion, but such as his lot (whether he be pleased or wroth) he must be content with one 20 part of the prey: Now I demand of you what shall every man have to his share?

Scholar. You must be fain to answer to your owne question, else it is not like to be answered at this time.

Master. The forme to understand the solution of this question, and all such like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be parts of some other of them, for all such parts you may overpasse and take for them all those numbers, whose parts they be: as in this example the shares be these $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{6}$ $\frac{1}{20}$, if I multiply all the Denominators together, beginning with 3, and so goe on unto 20, it will make 144000: but considering that 3 is a part of 6, I will omit that 3, and likewise 10, which is a part of 20, I may overpasse also, & then is there but 3 Denominators to multiply, that is, 4, 6, and 20 which make 480, which summe I take for my worke, because all the Denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ and $\frac{1}{10}$, the $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48, which I put in one summe for the first mans share, and it maketh 208. Then for the second mans share, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole

whole 168. Now for the third man which must have $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole summe: so that if the whole prey had been but 480 pound, then were the question answered: but because the summe was of greater value, by this meanes now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number of that I found by multiplying the Denominators, and in the second place the summe of the booty, And look what proportion is betweene the first number and the second, the same proportion shall be between the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the third place, one of the parts or shares, and then worke by the former Rule of proportion or Golden Rule. And because I have 4 severall parts of the first number, by which I would find out four like parts of the second number, therefore must I make foure severall figures.

The reason of this rule.

Scholar. Now I trust I can answer to your question, as by your favor I will prove:

And to trie it, I set the foure figures thus, marked with A, B, C, D, to shew their order.

A

480 Z 8190
208

B

480 Z 8190
168

C

480 Z 81190
80

D

480 Z 8190
24

And then in each of them I multiply the ſecond number by the third, and divide their totall by the firſt, and ſo amounteth the fourth ſumme which I ſeek for: for if I doe multiply 8190 by 208, it maketh 1703520, which being divided by 480, maketh in the Quotient 3549 for the firſt mans portion.

And ſo working with the other three figures, I finde for the ſecond man $2866\frac{1}{2}$. and for the third man 1365, and then for the fourth man $409\frac{1}{2}$, and ſo every mans ſhare is ſet forth in the figure here annexed.

A

480 Z 8190
208 Z 3549

B

480 Z 8190
168 Z $2866\frac{1}{2}$

C

480 Z 8190
80 Z 1365

D

420 Z 8190
24 Z $490\frac{1}{2}$

And thus I think I have done well.

Maſter, If you miſdoubt your working, and liſt to prove it, adde all the ſhares together; and if they make the totall, then ſeemeth it well done.

The proof of Addition.

3549
$2866\frac{1}{2}$
1365
$409\frac{1}{2}$
8190

Scholar. I may ſet them thus: and then by Addition the laſt ſumme doth amount, that is, 8190, and therefore (as you ſay) it ſeemeth to be well wrought.

U But

But I beséech you, is there any doubt in this triall, that you use that word seemeth?

Master. You may easily coniecture, that if you did assigne the first mans share to the last, and so change all the rest, and one had anothers share, yet would the Addition appeare all one, and therefore is not the proofe exact.

The just Proof.

But if you will make a just proofe for the first mans part, take $\frac{1}{3}$ and $\frac{1}{10}$ of the whole summe, and if it agrée with the number in the figure, then it is well done. And so do for the second, third, and fourth summes, and this proofe faileth not. Now will I propound certaine other questions, which have béen set forth by certain learned men, albeit not without some oversight, which questions I protest heartily, I doe not repeate to deprave those good men, whose labours and studies I much praise and greatly delight in. But onely according to my profession, to séeke out truth in all things, and to remove all occasions of errour as much as in me lieth: and for that cause I will onely name the questions whithout hurting the Authors name.

A question of building

The first question is this.

Foure men did build an house, which cost them 3000 crownes, their shares were such that one man should pay $\frac{1}{2}$ of the summe & six crownes over: the second should pay $\frac{1}{3}$ and 12 crownes over: the third man must lay out $\frac{2}{3}$ abating 8 crownes: and the fourth man should pay $\frac{1}{4}$ and 20 crownes more. Can you answer to this question?

Scholar. No, I cannot sir, and that you know best of any man, for I know no more then you have taught me.

Master.

Master. Then I dare say you cannot doe it, neither yet the best learned man that ever did propose it: for the question is impossible. For declaration whereof, I will be bold to use first the representation of the Numbers in their aptest forme, (although I have not yet taught that manner of worke) because it may appeare plainly that the question is not possible. For here I have set the parts, and added them, and they make the whole sum, and $\frac{1}{4}$ & 30 more. Now how is it possible to divide truely either gaines, either charges, so that the particulars shall be more then the totall? An impossible question.

$\frac{1}{2}$	+	6
$\frac{1}{3}$	+	12
$\frac{2}{3}$	—	8
$\frac{1}{4}$	+	20
$1\frac{1}{4}$	+	30

Scholar. It is against the forme of proofe by addition of parts.

Master. You say truth. And (because you shall perceive it the better) I will trie it after the vulgar forme, as in this figure you see where the $\frac{1}{2}$ with 6 over, is 1506, for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{2}{3}$ would be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one summe doe make 5280 where the totall summe should be but 3000, which summe of 3000 if you divide by $\frac{1}{4}$, you shall have $\frac{1}{4}$ of it, that is 1250, and thereto adde 30 more, then will those three sums make 5280: whereby you may see how this forme (as well as the other) doth declare that the particulars in that question would make more then the whole summe by $\frac{1}{4}$ and 30 more, and therefore can that question not be accepted as a

1506
1012
1992
770
5280

30
2000
2250
5280

possible thing, but yet doe certaine learned men propound such questions, and answer to them: Therefore somewhat to say to their excuse (rather of their good meaning, then for their doing) I will anon declare what may be said for their defence: but in the meane season, I will propound the question as it may be wrought by good possibility.

The former question of building now possible.

As if foure men build a house together, and it cost them 3000 Crownes, and then for the partition they agree thus: that as often as the first man doth pay 6 Crownes, so often the second man shall pay 4, the third man 8, and the fourth man 3. Or else thus, that the first man shall pay double so much as the fourth, and the second man shall pay $\frac{2}{3}$ of the first mans charge: the third man shall pay double so much as the second: (and these two ways are to one end) but further for their agreement it is appointed also, that the first shall give 6 Crownes overplus, and the second 12, and the fourth shall give 20, but the third man shall give no overplus, but shall have 8 Crownes abated of his charge.

Now is the question possible to be assoiled, and this is the way to doe it. Marke the proportion of the severall charges, and set out small numbers in that Rate, by which you may reduce the worke to the Golden Rule, as here in the first forme the numbers are already named, 6, 4, 8, 3: and in the second forme, although they be but plainly named, yet they may be the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Now adde these together, and they make 21, which 21 must be set for the first number in the Golden Rule: for if it with the overplus of each mans charge would make the totall summe of the charges

charges, then were those severall summes the charges of each man, besides his overplus: but now it is not so.

But yet this is true: (so excellent are conclusions Arithmeticall) that look what proportion each of their severall summes doth bear to 21, the same proportion doth the just charges of every man (besides his overplus) beare to the totall of the charges, the overplus being deducted: wherefore this may you note, that before you doe apply the totall of his charges to the golden rule, you must deduct the overplus, which is 6, 12, and 20, that is in the whole 38: but then 8 must be restored for the abatement of the third man, and then remaineth to be deducted 30: take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden Rule, for the second summe: and for the third summe, I must put each of the small numbers before mentioned, which although they be not severall charges, yet they represent them in proportion. And so making for every mans charge a severall question, the figures will be 4, which I marke with four letters, A, B, C, D, thus.

A		B	
21	2970	21	2970
6	$848\frac{4}{7}$	4	$565\frac{5}{7}$

C		D	
21	2970	21	2970
8	$1131\frac{3}{7}$	3	$424\frac{2}{7}$

Where I have set for breifnesse the summe of every mans charge in the fourth place, presupposing that you can tell how to try out the fourth summe

by so many Examples as ye have had.

Scholar. As I trust that I understand this forme, so I desire much to know what may bee said for them that mistooke this Question.

Master. You seem so desirous to know this errour, that you have forgotten to examine, whether this work be without fault.

Scholar. Me seemeth this worke to be well done, because the Addition of the foure severall numbers doth make the totall summe of 2970, which was to be divided into such foure parts.

Master. But then have you forgotten that the first man must pay sixe crownes more besides his share, and the second man 12 Crownes more, the third man 8 crownes lesse, and the fourth man 20 crownes more; for without these, your first totall of 3000 crownes will not be made.

Scholar. Then must I adde to the first mans summe 6 more, and it will bee $854\frac{4}{7}$ and to the seconds summe, I must adde 12, and it will be $577\frac{5}{7}$: from the thirds summe I must abate 8, and then will the summe bee $1131\frac{3}{7}$: then adding unto the fourths summe 20, it will be $444\frac{2}{7}$, and these foure summes will make 3000, which is the whole charge, as in this example it may appear, where first I gather the $\frac{14}{7}$, that maketh 2, and so proceed I in the Addition to the end.

$854\frac{4}{7}$
$577\frac{5}{7}$
$1123\frac{3}{7}$
$444\frac{2}{7}$
3000

Master. Now have you well done, and this worke in the same summes is brought of other Learned men for the true solution of the question, as it was proposed, which (as I said) was impossible

possible: and now examine by these severall summes, and see whether it doth agree with the summes in the Question proposed.

The first man must pay $\frac{1}{2}$ and 6 over of the totall summe: how think you, is $854\frac{4}{7}$ the halfe and 6 more of 3000?

Scholar. No that it is not, for it should bee 1506: and for the second man 1012: and for the third man 1992, and for the fourth man 770: whereof not one summe agreeth to this worke. But I marvell, that so wise men could bee so much over-seen.

Master. It is commonly seen, that when men will receive things from elder writers & will not examine the thing, they seeme rather willing to erre with their Ancients for company, then to bee bold to examine their workes or writings. Which scrupulosity hath ingendred infinite errors in all kindes of knowledge, & in all civill administration, & so in every kinde of Art. But these Learned men did not mean any other thing by this question, then to finde such numbers as should beare the same proportion together, as those numbers in the question proposed did bear one to another: which thing you shall perceive more plainly by another question of theirs, that is this.

A man lying upon his Death-bed, bequeathed his goods (which were worth 3600 Crownes) in this sort: because his Wife was great with childe, and he yet uncertaine whether the Childe were Male or Female, hee made his bequest conditionally, that if the Wife bare a Daughter, then should the Wife have halfe his goods, and the Daughter $\frac{1}{3}$, but if she were delivered of a Sonne, then that Sonne should A question of a Testament.

 have

have $\frac{1}{2}$ of the goods, and his Wife but $\frac{1}{3}$: Now it chanced her to bring forth both a Sonne and a Daughter; the question is, How shall they part the goods agreeable to the Testatour his Will?

S. If some cunning Lawyers had this matter in scanning, they would determine this Testament to be quite voide, & so the Man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it, and also it might have chanced that shee should have brought forth neither Sonne nor Daughter, as often hath béene séene: so is the Will insufficient in that point also.

Master. Such Scanners should séem too cunning and yet not so cunning as cruell: for the minde of the Testator is to be taken favorably for the aid of the Legataries, when there ariseth such doubt. But let us try this work, not by force of Law, but by proportion Geometricall, séeing the Testator did minde to provide for each sort of them.

Scholar. If the Sonne shall have $\frac{1}{2}$ by force of the Testament so must the Mother have $\frac{1}{2}$. Again, because she hath a Daughter also, therefore ought she to have $\frac{1}{3}$, and the Daughter $\frac{1}{3}$, that is both wayes $\frac{1}{3}\frac{1}{3}$, & $\frac{1}{3}\frac{1}{2}$, which commeth to the whole goods, & $\frac{2}{3}$ more. Wherefore it séemeth also impossible.

Master. In this matter the minde of the testator is to be understood that such proportion should be betwéen the portion of the VVife, and the Sonne, as is betwéen $\frac{1}{3}$, and $\frac{1}{2}$, that is, the Sonne must have $\frac{3}{6}$, for $\frac{2}{6}$ to his Mother, so shall he have 3 to 2, that is as much as his Mother, and halfe as much more; and the Mother must have the like rate in comparison to her Daughter.

Then

Then must I finde out 3 numbers in such proportion, that the first may have as much as the second, and halfe as much more (that is) in proportion sesquialtera, and the second to the third in that same proportion: such numbers, be 9, 6, 4.

Scholar. I pray you Sir, how shall I finde out these numbers?

Master. That will I gladly tell you.

VVhatsoever the proportion be of any three numbers, multiply the Termes of that proportion together, and the number that amounteth shall be the middle number of the three: then multiply that middle number by the lesser term, and divide that totall by the greater, and the least number of the three will amount. So if you multiply that middle number by the greater extreame, and divide the totall by the lesser extream, then will the greatest number of that Progression amount. To finde three numbers in any proportion.

Scholar. Then in this example to finde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must divide (as you taught me in Division) $\frac{1}{2}$ by $\frac{1}{3}$, and the Quotient will be $\frac{3}{2}$, that is, $1\frac{1}{2}$. whereby I perceive that the proportion in this question is as three to 2. Therefore as you taught me even now, I multiply 3, by 2, and the summe is 6, which must be the middle number: then I multiply the middle number 6 by 2, which is the least terme, and the summe is 12, that I doe divide by 3, being the greater terme, and the Quotient is 4: so is 4 the least number of the three. Then I multiply 6 by 3, whereof cometh 18, and that I divide by 2, and so have I 9 which is the greatest number of the three. To finde the proportion between 2 numbers.

Master. Another way yet may you finde the

third

third number in any Progression, if you have two of them: for if the middle number be one of them which you have, then multiply it by it self (as in this example, 6 by 6 maketh 36) and that totall divide by the other number which you have, and the third number will be the Quotient.

Scholar. Then I divide 36 (which commeth of 6 multiplied by it self) by 4 the Quotient will be 9: and if I divide 36 by 9, the Quotient will be 4. But what if I know the first number, and the third, and would have the middle number?

Master. Multiply the 2 numbers together, and in their totall you must seek the root of that number, and it shall be the middle number: but because as yet you have not learned to extract Roots, therefore use the first forme which I have taught you, till I teach you to extract Roots. And now go forwards with the answer to the same question.

Note.

Scholar. I perceive then, that the Sonne must not have $\frac{1}{2}$ of the goods, neither the Mother $\frac{1}{3}$, nor yet the Daughter $\frac{1}{3}$, but yet must the goods be divided into such proportion, that the Sonne shall have 9 Crownes for 6 to his Mother, and the Mother shall have 6 crownes for every 4 to her Daughter. Then I apply it to the Golden Rule in three examples, as followeth.

Where the first number is the Addition of those three numbers 9, 6, 4: and the third is one of them severally: the second is the totall of the goods in that Testament: and then by the worke of the Golden Rule, I finde out the fourth

19 3600
9

19 3600
9

19 3600
4

number

number in every worke: that is for the Sonne 1705 $\frac{5}{19}$, for the Mother 1136 $\frac{16}{19}$, and for the Daughter 757 $\frac{17}{19}$: the which summes added together, doe make the summe of the whole goods as may be seen by this Example.

1705 $\frac{5}{19}$
1136 $\frac{16}{19}$
757 $\frac{17}{19}$
3600

And this (me thinketh) I doe perceive, because in this Case there is a necessary remedy devised against an urgent inconvenience: therefore those learned men thought they might use the like liberty in that other question.

Master. Your guesse is good, but they had so good reason for them in the one, as they have in the other: As in another example of theirs, it may better appeare, as in this.

A man left unto his three Sonnes 7851 *crowns to be parted in such sort, that the first Sonne should have* $\frac{1}{2}$, *the second Sonne* $\frac{1}{3}$, *and the third Sonne* $\frac{1}{4}$, *which is not possible: for* $\frac{1}{2}$ *and* $\frac{1}{3}$ *and* $\frac{1}{4}$ *doe make* $\frac{26}{24}$, *or* $\frac{13}{12}$, *that is,* 1 $\frac{1}{12}$, *so it is more then the whole, but reduce these* Fractions *into one denomination, the least that they will come to, and they will be* $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$ *and so may you part the goods into such proportion as these* 3 *Numerators beare together, that is, the first to have* 6 *for every* 4 *to the second, and the second to have* 4 *as often as the third hath* 3: *and so their portions will be for the first,* 3623 $\frac{7}{13}$, *for the second* 2415 $\frac{8}{13}$, *& for the third* 1811 $\frac{10}{13}$: *& these* 3 *shares added together, will make the totall summe of the whole goods, as you may easily see in this example.* Another question of a Testament.

3623 $\frac{7}{13}$
2415 $\frac{8}{13}$
1811 $\frac{10}{13}$
7851

Another

Another question is there proposed thus.

Another like question.

There are 450 crownes to bee divided betweene three men, so that the first man must have $\frac{1}{2}$ and $\frac{1}{4}$, the second man $\frac{1}{3}$ and $\frac{1}{4}$; the third man shall have $\frac{1}{4}$ and $\frac{1}{5}$.

Scholar. I marvell that any man should be so overseen, to propose that question as a thing possible, sith $\frac{1\,1}{2\,3}$, $\frac{1\,1}{3\,4}$, $\frac{1\,1}{4\,5}$, doe make 1 $\frac{12}{13}$, that is almost double the whole summe.

But I perceive it might be thus proposed: that as often as the first man did receive 50 crownes, so often the second man should receive 35, and the third man 27: for $\frac{1}{2}$ and $\frac{1}{3}$ is equall to $\frac{5}{6}$: and so is $\frac{1}{3}$ and $\frac{1}{4}$ equall to $\frac{35}{60}$, and $\frac{1}{4}$ and $\frac{1}{5}$ is $\frac{27}{60}$ and so working the question the three figures will appear in this forme:

112	450
50	200 $\frac{50}{56}$
112	450
35	140 $\frac{35}{56}$
112	450
27	108 $\frac{27}{59}$

whereby the first mans portion is found to be 200 $\frac{50}{56}$: the second mans part is 140 $\frac{35}{56}$ the third mans share 108 $\frac{27}{56}$: which in the whole doth make 450 crownes to be divided betweene them.

Master. And thus you are (I thinke) sufficiently instructed in the Rule of Fellowship.

The

The Rule of Alligation.

Now will I goe in hand with the Rule of Alligation; *which hath his name, for that by it there are divers parcells of sundry prices, and sundry quantities alligate, bound or mixed together: whereby also it might bee well called the* Rule of Mixture; *and it hath great use in composition of Medicines, and also in mixtures of Metalls, and some use it hath in mixtures of Wines: but I wish it were lesse used therein then it is now a dayes. The order of this rule is this.* The rule of Mixture

When any summes are proposed to bee mixed, set them in order one over another, and the common number (whereunto you will reduce them) set on the left hand; then marke what summes bee lesser then that common number, and which bee greater, and with a draught of your penne evermore linke two numbers together, so that one bee lesser then the common number, and the other greater (for two greater or two smaller cannot well be linked together) and the reason is this, that one greater and one smaller, may be so mixed, that they will make the meane or common number very well: but two lesse can never make so many as the common number, being taken orderly: no more can two summes greater then the mean, ever make the mean in due order, as it shall appeare better to you hereafter. And as it is of necessity to linke every smaller (once at the least) with one greater, & every greater with one smaller, so it is at liberty The reason of this Rule.

to linke them oftner then once, and so may there be to one question many solutions. When you have so linked them, then marke how much each of the lesser numbers, is smaller then the mean or common number, and that difference set against the greater numbers, which be linked with those smaller, each with his match still on the right hand, and likewise the excesse of the greater numbers above the mean, you shall set before the lesser numbers which be combined with them.

Then shall you (by Addition) bring all these differences into one summe, which shall be the first number in the Golden Rule, and the second number shall be the whole masse that you will have of all those particulars: the third summe shall be each difference by it selfe, and then by them shall be found the fourth number, declaring the just portion of every particular in that mixture: As now by these Examples I will make it plaine.

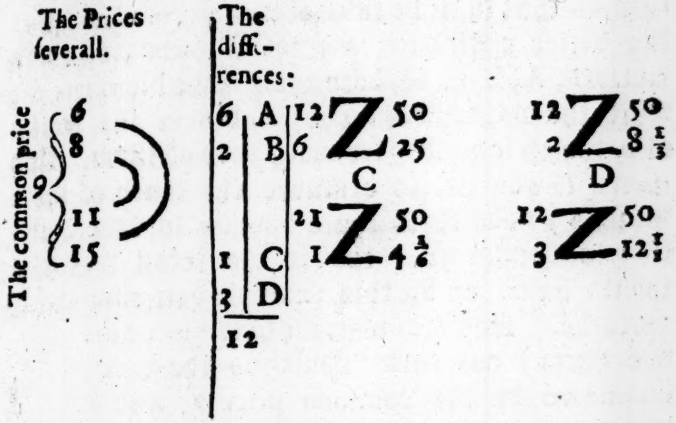
A question of mixing of Wines.

There are foure sorts of Wine, of severall prices one of 6 pence *a gallon, another of* 8 pence, *the third of* 11 pence, *and the fourth of* 15 pence *the gallon, Of all these Wines would I have a mixture made to the summe of fifty gallons, and so the price of each gallon may be* 9 pence. *Now demand I, how much must be taken of every sort of Wine?*

Scholar. If it shall please you to worke the first example, that I may marke the applying of it to the Rule: then I trust I shall bee able not onely to doe the like, but also to see the reason in the order of the worke.

Master.

Master. Marke then this forme, and the placing of every kinde of number in it.



Here (you see) I have set downe the severall prices, which be 6, 8, 11, 15, and have linked together 6 with 15, and 8, with 11. The common price 9, I have set on the left side, and the difference betwéen it, and every particular price I have set on the right hand, not against the summe (whose difference it is) but against the summe that it is linked withall, so the difference of 15 above 9, is 6, which I have set, not against 15 but 6, that is linked with 15, & the difference betwéen 6 & 9 (that is 3) I have set against 15. So likewise the difference betwéen 8 and 9 is but 1, that I have set against 11; and the difference of 11 above 9 (which is 2) I have set against 8. Then adde I all those foure differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe

of Gallons that I ſhould have, and the third ſumme is every particular difference. Now if you worke by the Golden Rule, you ſhall finde the number of Gallons that ſhall be takn of each ſort of wine: For the better diſtinction wereof I have ſet theſe Letters, A, B, C, D, both againſt the Numbers for which the workes doe ſerve, and over the worke alſo, which ſeverally ſerve for each of them. And now (if you liſt to examine the truth of theſe workes) adde theſe foure ſummes together, and they will make 50. that is the totall which I would have, as by this example you may eaſily perceive. And (for to prove how the prices doe agrée) doe this: Multiply the totall ſumme 50, by the common price 9, and it will make 450: then kéep that ſumme by it ſelfe, and afterward multiply every ſeverall ſumme of Gallons, by the price belonging to the ſame Gallons, and if that ſumme doe agrée with this, which you have kept firſt, then is your worke well done. As here 25 is the number of Gallons of 6 pence price, multiply then 25 by ſix, and it maketh 150, which you ſhall ſet downe, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of Gallons, and it will make $66\frac{4}{6}$: ſo againe $4\frac{1}{6}$ multiplied by 11, doth make $45\frac{5}{6}$. And laſt of all, $12\frac{1}{2}$ multiplied by 15, maketh $187\frac{3}{6}$: and theſe added together doe make 450, as in the example annexed you may ſée, wherefore ſeing it doth agrée with the former ſumme of 50, multiplyed by 9, I may juſtly affirme this worke to be good, and well done.

The proof of this rule

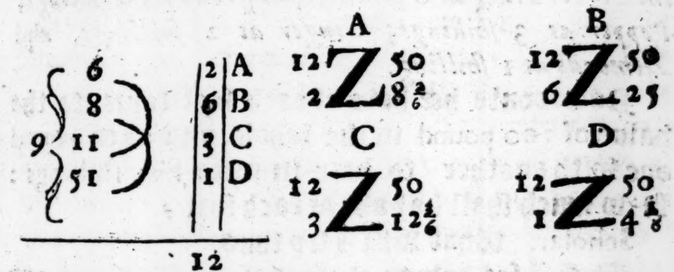
25
$8\frac{1}{5}$
$4\frac{1}{6}$
$12\frac{1}{2}$
50

150
$66\frac{4}{6}$
$45\frac{5}{6}$
$187\frac{3}{6}$
450

And

And now to prove how you can doe the like: I propound the same Question, onely willing you to use some other forme of combining or linking the summes. The variation of this question.

Scholar. That shall I prove with your favour, and therefore I combine 8 with 15, and 6 with 11; and then the form will be as followeth.



Whereby amounteth the same summe in totall of the differences as did before: and yet now the differences be altered as the combination is changed, whereof I understand the reason by your former worke. And therefore here appeareth no strange thing, but that now I have $8\frac{2}{6}$ gallons of 6 pence, and 25 gallons of 8 pence, and 12 gallons and $\frac{3}{6}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{6}$ of 15 pence: so that multiplying $8\frac{2}{6}$ by 6, it maketh 50, and then 25 multiplyed by 8, maketh 200: likewise $12\frac{3}{6}$ multiplyed by 11 yeeld $137\frac{3}{6}$, and $4\frac{1}{6}$ multiplyed by 15, maketh $62\frac{3}{6}$, which 4 summes added into one, will yield in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of Gallons) by 9 the common or mean price.

50
200
$137\frac{1}{2}$
$62\frac{1}{2}$
450

Master. Seeing you conceive this worke so

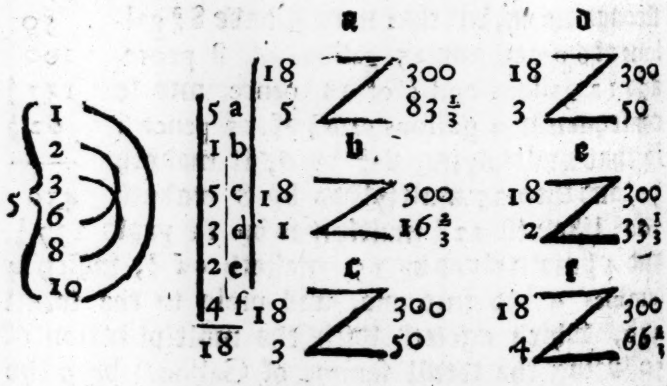
well, I will propound another example unto you of more variety in the Alligations or combinings, as thus.

Aqueſtion of Spices. *A Merchant being minded to make a bargain for ſpices, in a mixt maſſe (that is to ſay) of Cloves, Nutmegs, Saffron, Pepper, Ginger, and Almonds: the Cloves being at 6 ſhillings, Saffron at 10 ſhillings, Pepper at 3 ſhillings, Ginger at 2 ſhillings, and Almonds at 1 ſhilling.*

Now would hee have of each ſort ſome, to the value of 300 pound in the whole, and each pound one with another to bear in price five ſhillings: How much ſhall he have of each ſort?

Scholar. That will I try thus.

Firſt I ſet downe thoſe 6 ſeverall prices, and at the left hand I ſet the common price five ſhillings. Then I linke them thus, one with 10, two with 6, and 3 with 8: as in the example following.



Maſter. I had minded to have combined them in more variety: but I am content to ſee your own

own work first, and then more varieties in combination may follow anon.

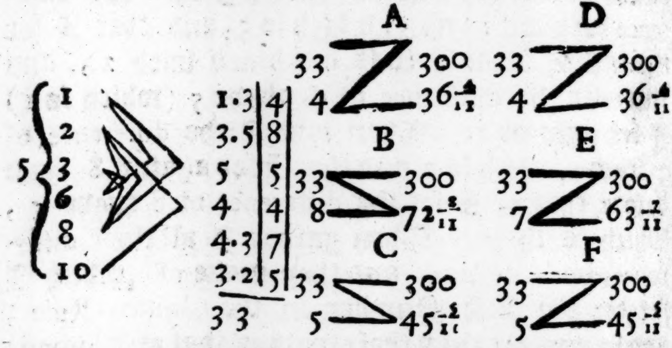
Scholar. Then to continue as I began I seek the difference betwéen 1 and 5, (which is 4) and that I set against 10, then against 1 I set 5, which is the excesse of 10 above 5, so I gather the difference betwéen 2 and 5, which is 3, and that I set against 6, because it is combined with 2, and likewise the difference of 6 above 5 (which is 1) I set against 2. Then take I the difference of 3 from 5, which is 2, and that I set against 8: and before that 3, I set the difference of 8 above 5, which is thrée. Then gather I all these differences by Addition, and they make 18, which I set for my first Number in the Golden Rule, and so appeareth by those workes, that of Almonds I must take $83\frac{1}{3}$ pound, of Ginger $16\frac{2}{3}$ pound, Pepper 50 pounds, of Cloves 50 pound, of Nutmegs $33\frac{1}{3}$ pound, and of Saffron $66\frac{2}{3}$ pounds.

$83\frac{1}{3}$
$33\frac{1}{3}$
150
300
$266\frac{2}{3}$
$666\frac{2}{3}$
1500

Then for triall hereof, I multiply every parcell by his severall price, as $83\frac{1}{3}$ which is the summe of Almonds, I multiply by one which is their price.

Also $16\frac{2}{3}$ the summe of Ginger, I multiply by two, which is the price of it: and so each other in his kinde, as this Table annexed doth represent, and then adding them all together I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300, by the common price 5, wherefore it appeareth well wrought.

Master. Now I will make the alligation to prove your cunning somewhat better: but because you shall not thinke your selfe pressed too much, I will also note the differences, as by this Example you may see, where I have alligated 1 with 6



and 8, and therefore have I set against 1 both their differences, that is 1 and 3: Likewise, because 2 is combined with 8 and 10, I set before him their differences, 3 and 5. Against 3 I have set onely 5, which is the difference of 10 with whom 3 is combined onely. Likewise 6 is onely alligate to 1, and therefore is the difference of 1 from 5, which is 4, onely set against it: 8 is linked with 1 and 2, and therefore hath set against him, both their differences, 4 and 3: and 10 is joyned with 2 and 3, therefore hath he their differences, 3 and 2. And because of ease for you, in another columne I have set the differences reduced into one number, for every severall sort, and have also added them together whereby appeareth that they make 33, & so consequently you see the works of the Golden Rule set forth. For the six Drugges

I

I have added the letters A. B. C. &c. as before.

But I would not wish you to cleave still to these Elementary aides, but accustome Memory to trust her self: so shall occasion of negligence best be avoyded. And as for the proof try it at more leisure, because the time now is short, and you sufficiently instructed in that proof. And there resteth divers things behinde yet, of which I would gladly give you some taste, before your departure. Note.

Scholar. But if it may please you to let mee see all the variations of this question, before you goe from it, for methinketh I could vary it two or three wayes more yet.

Master. I am content to see you make two or three variations: but I would be loth to stay to see all the variations: for it may be varied above 300 ways, although many of them would not well serve to this purpose.

Scholar. I thought it impossible to make so many variations.

Master. Marvell not thereat, for some questions of this Rule, may be varied above 1000 wayes; but I would have you forget such fantasies till a time of more leisure. And now goe forward with some variation of this question. Note.

Scholar. For the first variation, I linke the first number 1 with 1 and 10, and 2 I combine with 9 and 10: then joyne I 3 with 6, 8, and 10, as in this forme.

3.5	8
1.5	6
1.35	9
3.2	5
4.2	6
4,32	9
	43

A: 43 / 8 Z 300 / $55\frac{35}{43}$

B: 43 / 6 Z 300 / $41\frac{37}{35}$

C: 43 / 9 Z 300 / $62\frac{34}{45}$

D: 43 / 5 Z 300 / $34\frac{38}{43}$

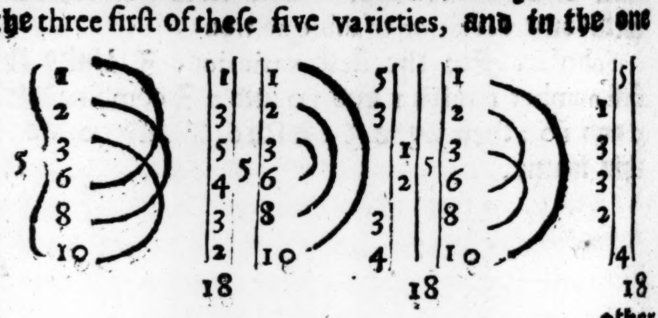
E: 43 / 6 Z 300 / $41\frac{37}{43}$

F: 43 / 9 Z 300 / $62\frac{34}{43}$

And so doth there appeare the proportion of weight for every kinde of Drugge in this mixture. Now for the tryall.

Master. Nay stay there: you shall not need to make tryall in one example so often, or if you list to doe it by your selfe, I am content. But now set forth (for declaration that you conceive the Rule) two or three examples of severall Combinations, and then will we passe to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these be the varieties: in which as the combinations are severall, so doth it plainly appeare, that the differences by which the proportion of each severall kinde is taken are also severall. And yet I see in the three first of these five varieties, and in the one



other

5	1	1.3.5	9	5	1	1.3	4
	2	5.3	8		2	4.5	9
	3	5	5		3	3	3
	6	4	4		6	4.3	7
	8	4.3	7		8	4.5	9
	10	4.3	9		10	3	3
			42				35

other before, the totall summe of the differences to be one, that is to say, 18, whereby I perceive that the variety of their mixture doth depend on the variety of their differences severall, and not of the variety of their totall summe.

Master. So is it. And seeing you conceive it so well, I will make an end of this Rule, onely exhibiting unto you one Question or two of the mixture of Metals, that by it you may devise others like, and exercise your selfe therein also, because the use of it serveth often in businesse of charge, not so much for Goldsmiths, as of coynage in Mints. First, I demand of you this question: *If a Mint-master have Gold of 22 Karects, and some of 23 Karects, some of 24: Againe, some 15, some 16, and some of 18 Karects, and would mix them, so that he might have 100 Ounces of 20 Karects: How much must he take of each sort?*

Scholar. To know that, I answer in order thus:

20	15	2	20	100	20	100
	16	3	2	10	5	25
	18	4	20	100	20	100
	22	5	3	15	4	20
	23	4	20	100	20	100
	24	2	4	20	2	10
		20				

Master. You have wrought the question well, but how chanced you made no doubt of that new name Karect.

Scholar. Because I thought it out of time to demand such questions now seeing you make so much hast to end: and againe in this case the proportion of the number is sufficient for my purpose in this worke, trusting that another time you will instruct me as well of this, as of sundry other things, which as I have heard you talke of, so I have a great desire to them.

Mast. Your answer is reasonable, and your 10
request and trust (with Gods helpe) I intend to 15
satisfie, & now to go forward with this matter, 20
let me see your examination of this last work. 25
Scholar. First for the one part I adde toge- 20
ther all the particular sums, as they appeare in 10
the worke, and they make 100, as here by their ———
Addition doth appeare. 100

And so it seemeth that the sums are well gathered: but for the further triall of them I multiply
150 first 20 which is the common or meane sum
240 of the Karects by 100, which is the sum of the
360 whole Masse, which I would have, and it
550 maketh 2000. Then I multiply every parti-
460 cular sum by the Karects, that it doth contain,
240 as 10 by 15, and that maketh 150.
——— Likewise I multiply 15 by 16, and it
2000 yieldeth 240: so 20 by 18, maketh 360. And
25 by 22, yeeldeth 550: likewise 20 by 23, bringeth forth 460: and last of all, 10 multiplyed by 24, yieldeth 240: which summes all joyned together make 2000, that doth agree with the like summe

before

before, wherefore I may well say, that the worke is good. And now (if it please you) I would set forth some varieties of this question) to prove my wit.

Master. Goe to, let me see.

Scholar. Here be foure varieties,

20	15	3.4	7
	16	3	3
	18	2	2
	22	2	2
	23	5.4	9
	24	5	5
			28

20	15	2.3	5
	16	3.4	7
	18	4	4
	22	5	5
	23	5.4	9
	24	4.2	6
			36

20	15	2.3.4	9
	16	4	3
	18	3	3
	22	5	5
	23	5.2	7
	24	5 4	9
			36

20	15	4	4
	16	4	4
	18	2.3.4	9
	22	2	2
	23	2	2
	24	5.4.2	11
			32

And more yet could I make, but not like to the number that you speak of in the variation of the other question.

Master. That will I teach you at more leisure, seeing it is a thing rather of pleasure then of any necessity.

But now for your exercise in this Rule, one other question I will propose. A Mint-master hath six Ingots of silver, of sundry finenesse, some of four Ounces fine, and some of five Ounces, some of six, and other of eight, some of 11, *and other of* 12, *and his desire is to mixe* 500 *pounds weight, so that in the whole mass every*

A question of mixing of silver.

every pound weight should beare 9 Ounces of fine silver: How much shall he take say you of every sort of silver?

Scholar. To finde out that, I set the numbers thus in order.

9	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	5.4	9
	12	3.1	4
			23

And gathering the differences it will appeare, that, of the first sort there must be 43 $\frac{11}{23}$ of the second like much: of the third sort 65 $\frac{5}{23}$: and of the fourth sort as much: of the fifth sort 195 $\frac{15}{23}$ and of the sixth sort 86 $\frac{22}{23}$, which in the whole will make 500 pound weight, and in ounces after 9 ounces fine 4500, that is of the first sort 173 $\frac{21}{23}$, and of the second sort 217 $\frac{9}{23}$, of the third sort 391 $\frac{7}{23}$, of the fourth sort 521 $\frac{17}{23}$, of the fifth sort, 2152 $\frac{4}{23}$, and of the sixth sort 1043 $\frac{11}{23}$, which all together doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

Master. This is well done of you, therefore now make three or foure varieties, and so an end of this Rule.

Scholar. These four varieties I set for examples.

9	4	3	3	9	4	2.3	5
	5	3	3		5	2	2
	6	3	3		6	2	2
	8	2	2		8	2	2
	11	1	1		11	5.4.3.1	13
	12	5.4.3.	12		12	5	5
			24				29

Master.

9<	4	23	5
	5	3	3
	6	2	2
	8	2	2
	11	5.3.1	9
	12	5.4	9
			30

9<	4	3	3
	5	3	3
	6	2.3	5
	8	2.3	5
	11	3.1	4
	12	5.4.3.1	13
			33

Master. And by these it appeareth, that you can finde out more, with which I will not now meddle, save onely (for to shew you an easie help drawing the lines of Combination) I will set forth 2 varieties here.

9<	4	2	2
	5	2.3	5
	6	3.2	5
	8	3	3
	11	5.4.3	12
	12	4.3 1	8
			35

9<	4	3	3
	5	2.3	5
	6	2.3	5
	8	2.2	5
	11	4.3	8
	12	5.4.2.1	12
			38

And this shall suffice now for the Rule of Alligation or mixture : for by these examples may you easily conjecture such other as do appertain to it, as well for the due working, as for variety of drawing the lines of Combination.

Scholar. Sir, albeit it pleased you erewhile to put me from my musing at the many varieties that may fall in these Combinations, and termed them phantasies, yet my phantasie giveth me, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable : Therefore ye may well think, that at another time convenient

I

I will request you to aid me herein.

Master. Truth it is, that this consideration may fall in practice as well Politick as Philosophicall, and sundry waies in them be applied: Therefore when time shall fall fit, for the discussing of this consideration, you shall not want my helping hand.

The Rule of Falshood.

The occasion of the name.

NOw will I briefly also teach you the Rule of Falsehood, *which beareth his name, not for that it teacheth any fraud or falshood, but for that by false numbers taken at all adventures, it teacheth how to finde those* true numbers *you seek for.*

Scholar. So might any other Rule be called the Rule of Falshood, for they worke by wrong numbers, and by them finde out the right numbers: so doth the Rule of Alligation, the Rule of Fellowship, and the Golden Rule, partly.

Mast. In the Golden Rule, the Rule of Fellowship, & the Rule of Alligation, although the numbers that you worke by, be not the true numbers that you seek for, yet are they numbers in just proportion, and are found by orderly worke, whereas in this Rule the numbers are not taken in any proportion, nor found by orderly worke, but taken at all adventures.

And therefore I sometimes being merry with my friends, and talking of such questions, do call unto them such Children or idiots, as hapned to be in the place, and so take their answer, declaring that

that I would make them solve those questions, that seemed so doubtfull.

And indeed I did answer to the question, and worke the Triall thereof also by those answers which they happened at all adventures to make: which numbers seeing they bee taken as manifest false, therefore is this Rule called the Rule of false Positions, and for brifenesse, The Rule of Falshood: which Rule for readinesse of remembrance, I have comprised in the few verses following, in form of an obscure Riddle.

Ghesse at this worke as hap doth lead,
By chance to truth you may proceed,
And first worke by the question,
Although no truth therein be done.
Such falshood is so good a ground,
That truth by it will soon be found.
From many bate too many moe,
From too few take too few also:
With too much joyn too few again:
To too few adde too many plain:
In crosse wise multiply contrary kinde,
And all truth by falshood for to finde.

The sense of these Verses, and the summe of this Rule is this.

When any *question is proposed* appertaining to this *Rule*, first imagine any number that you list, which you shall name the *first position*, and put it in stead of the true number, and then worke with it as the *question* importeth: and if you have missed, then is the last number of that worke either too great or too little: that shall you note as here after shall be taught

The exposition of the Rule.

taught you, and you shall call it the *first errour.*

Then begin againe, and take another number, which shall be called the *second position*, and worke by the *question*: if you have missed againe, note the excesse or default as it is, and call that the *second errour.* Then multiply crosse-wise the *first position* by the *second errour*, and againe, the *second position* by the *first errour*, and note their totalls severally by the names of totalls: Then marke whether the two errours were both alike, that is to say, both too much, or both too little: or whether they be unlike, that is, the one too much, and the other too little: for if they be like, then shall you *subtract* the one totall from the other (I mean the lesser from the greater) and the remainer shall be your *dividend*: so must you abate the lesser errour out of the greater, and the residue shall be the *divisor*. Now divide the *dividend* by that *divisor*, and the *quotient* will shew you the true number that you seek for. But and if the errours be unlike, then must you adde both those totalls (which you noted) together, and take that whole number for the *dividend*, so shall you adde both errours together, and that whole number shall be the *divisor*, and the *quotient* of that division shall give you the true number that the *question* seeketh for, and this is the whole Rule.

Scholar. This Rule seemeth so unlike any other, that without some example I shall not easily understand it.

Master. With a good will: propose halfe a score sundry questions and examples of variety, for the better understanding of the worke hereof: and for the first take this example. *A mason*

was

was bound to build a wall in 40 dayes, and it was covenanted so with him, that every day that he wrought, he should have for his wages 2 shillings 1 peny, *and every day that he wrought not, he should be amerced* 2 shillings 6 pence, *so that when the wall was made and the reckoning taken of the dayes that he wrought, and of the other that he wrought not, the Mason had clearly but* five shillings five pence for the worke. *Now do I demand how many dayes he did work of those 40, and how many he did not worke.*

A question of Masonry the first example.

Scholar. I pray you expresse the order of the worke, that I may partly by imitation, and partly by comparing it with the Rule, be able again to doe the like.

Master. This order shall you keep in the worke of this Rule: first take some number (as you list) at adventure; as for example, I say he played 12 dayes, and wrought 28 dayes. Now cast you the wages of every day and see whether it will agree with the summe of 5 shillings 5 pence.

Scholar. The 28 dayes that he wrought after 25 pence the day, yieldeth 700 pence: Then 12 dayes that he wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Master. He had but 65 pence, and by this supposition he should have had 340: therefore is this summe too much by 275, which summe I must set down after this sort as you see here, where first I have made a crosse (commonly called S. Andrews crosse) and

12
X
275†

at

at the over corner on the left hand I have set the first position 12: and at the other corner under it I have set 275 which is the first errour, with this figure †, which betokeneth too much, as this line ——— plaine without a crosse line betokeneth too little.

On the right hand of the crosse I have left two like roomes for the second position and his errour. Therefore to prosecute the worke, I suppose he played 16 dayes, and wrought 24.

Scholar. I was a while in doubt why you named the dayes of his working, seeing they be not set in the figure: and I doubted how you knew them, or else whether you did suppose them at all adventures, as you did the dayes that he played: but now I gather that seeing 40 dayes is the whole time limited, then the dayes that he played being supposed, the rest of 40 must needs be the dayes that he wrought, and therefore 28 followed 12 of necessity, and 24 followeth 16 also of necessity, but yet I scarce perceive why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working dayes, or both of idle, but now examine you this second position.

Scholar. If he played 16 dayes, then abating 16 times 30 pence, the sum will be 480 pence, and for 24 dayes that he wrought, every day yielding 25 pence, the totall is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therefore it is too much by 55: that

that must be set on the right hand of the figure, at the neather part, and over it on the same side 16, which is the second position, thus. 12 16 275† 55†

And as I gather by your words, it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

Master. So were it. But this shall you marke, that, of what nature soever the two positions be, of the same nature is the quotient. Therefore when the positions in this question are 12 and 16, which both being numbers of the playing dayes, the quotient shall declare the true number of playing dayes: whereas if the positions had been 28 and 24, which are supposed to be the working dayes, then would the quotient declare the true number of the working dayes, and not of playing dayes, as it will doe now. And therefore to continue the work of this question, and to finde the true number of playing dayes, I must multiply crosse-wise the first position by 55 that is the second errour, and the totall will be 660. Then I multiply 275 and 16, and it yeeldeth 4400. Now because the errours are alike, that is to say, both too much, I must subtract 660 out of 4400, and so remaineth 3740, which is the dividend. Againe I must subtract the lesser errour 55 out of 275, that is the greater errour, and there will remaine 220 which will be the divisor: then dividing 3740 by 220 the quotient will be 17. Wherefore I say now constantly, that 17 is the true number of dayes that the Mason played: and then it followeth that he wrought 23 dayes: and so is the question answered.

Now for the order of triall of this work, there needeth none other triall but only this, to worke with this The proof of this rule

 number

number according to the question, and if it agree, then appeareth the number to be that you would have.

And here now seeing he wrought 23 dayes, and must have for every day 25 pence, the whole summe commeth to 575. Then again, seeing he played 17 dayes and must abate 30 pence for every day, the whole sum of the abatement will be 510. Therefore I subtract 510 out of 575, and there will remaine 65, which maketh 5 shilling 5 pence, the cleere wages of the Mason for his worke according to the question.

Scholar. Now I trust I understand the worke and the Rule so well (and the better by this proof) that I can be able to doe the like: And for a proof, I take the same question, all save the last number, where I will suppose that he had 10 shillings for his wages cleere. And now to guesse of the number of the dayes he wrought, I suppose first that he wrought 20 dayes: then say I if he wrought 20 dayes, his wages must be 500 d. then did he play other 20 dayes, for which must be abated 600 d. and then he looseth 100 d. And so am I at a stay, for it is not like to your former worke.

Master. You should have required of me some question, and not have taken a question of your owne phantasying, untill you were more expert in this Art, for so might you as well happen to an impossible question, as on a possible: but now to go forward, consider that this number is too little by 220, seeing he should gain by your supposition 120 pence, and in this position he looseth 100, those both make 220, which you shall set downe for the first errour, with this signe—, betokening too little, as

as here in this forme following doth appeare.

20

—220

And now for the rest goe forward your selfe once againe.

Scholar. As my errour hath uttered my folly, so it hath procured me better understanding.

Now therefore considering this position not to solve the question, I take another, supposing that he wrought 30 dayes. Then for his wages he must be allowed 750 pence, and for the 10 dayes which he wrought not, he must abate 300 pence, and so remaineth cleer 450 pence, but it should be onely 120 pence, therefore it is too much by 330, which I set downe in the figure with the former position and his errour, and the figure appeareth thus:

Now first, I multiply in crosse wayes 20 by 330, and it will be 6600.

20 30

—220- 330+

Then again I multiply 30 by 220, and it will be also 6600. Wherefore if I shall subtract the one out of the other, there will remain nothing to be the Dividend.

Master. In this you forget your selfe againe; for in as much as the signes in the errours be unlike, therefore must you worke by Addition, adding together those two totalls to make the Dividend, and also adding the two errours to make the Divisor. And because you shall no more forget this part of that Rule, take this brief remembrance.

Unlike require Addition;
And like desire Subtraction.

Scholar. You mean, that if the errours have like signes, then must the Dividend, and the Divisor

bee made by Subtraction, as is taught before: And if those signes bee unlike (as in this last example they bee) then must I by Addition gather the dividend and the divisor. Therefore must I adde 6600 to 6600, and it will bee 13200, which will be the dividend. Then againe I adde 220 to 330 and it will be 550, which must be the divisor: wherefore dividing 13200 by 550, the quotient will be 24, whereby I know that the Mason wrought 24 dayes, and then it followed, that he played 16 dayes.

Master. Examine your worke, whether it be agreeable to the question or no.

Scholar. For 24 dayes worke the wages must be 600 pence, and for 16 dayes which the Mason wrought not, there must be abated 480 pence, and then remaineth cleare to the Mason 120, as the question importeth: wherefore it is evident that 24 is the true number of dayes that he wrought.

Master. Although you seem now to understand this worke, yet to acquaint your minde the better with the new Trade of this Rule, I think it good to propose to you 5 or 6 examples more before I make and end of it.

Scholar. Sir, I thanke you that you doe so consider my commodity and profit in knowledge, for undoubtedly it is practice and exercise that maketh men prompt & expert in every kinde of knowledge.

Master. You say well, so that they follow some certain precepts to governe and rule their practice by, else may practice procure custome of errour, and a repugnance to exactnesse of knowledge: namely, as long as the errour is not plainly known to the vulgar sort. But return to your work.

There

There is a servant that hath bought of Velvet and Damask for his Master 40 yards, the Velvet at 20 shillings *a yard, and the Damask at* 12 shillings, *and when he commeth home, his Master demandeth of him, how much he hath bought of each sort : I cannot tell (saith he) exactly : but this I know that I paid for Damask* 48 shillings *more then I paid for Velvet: now must you guesse how many yards there is of each sort.* A question of wares, the second example.

Scholar. Although the guesse seemeth difficult, yet I will prove what I can doe : for I remember your saying, that it forceth not how fond or false the guesse bee, so it be somewhat to the question, and not an answer of a contrary matter.

Therefore first I imagine that he bought 20 yards of damask, for which hee should pay after the former price 240 shillings: then must he needs have of Velvet other 20 yards, (to make up the 40 yards) and that would cost 400 shillings. So that the totall of the price of the Damask is lesse then the summe paid for Velvet 160 shillings, and should bee more by 48. Therefore the first errour is 208 too little. Then begin I againe, and suppose he bought of Damask 30 yards, that cost 360 shillings, then had he but 10 yards of Velvet, which cost 200 shillings : and now the price of the Damask is greater then the price of the Velvet by 160 shillings, and should be but 48, therefore is the second errour 112 too much, which I set in forme of figure, as here doth appeare. Then do I multiply in crosse wayes 208 by 30, and the summe will be 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much

20	30
208—	112+

much as the signes of the errours be unlike, I know I must worke by Addition, therefore adde I these two totalls together, and they make 8480, which is the Dividend: then adde I also the two errours together, 208, and 112, and they make 320, which is the Divisor: wherefore dividing 8480 by 320, the quotient will be 26 $\frac{1}{2}$, which is the true summe of yards of Damask that he bought, and in Velvet 13 yards $\frac{1}{2}$, and that appeareth by examination, thus: 26 $\frac{1}{2}$ yards of Damask at 12 shillings the yard maketh 318 shillings: then in Velvet he had but 13 yards and $\frac{1}{2}$ and cost 270 shillings, at 20 shillings the yard. Now subtract 270 out of 318, and there will remaine 48, which is the number of shillings that the Damask did cost more then the Velvet.

Master. Now shall you have a question of another kinde.

A question of debt, the third example.

There are three men that do owe money to me, and I have forgotten what the totall summe is, and what the particulars be.

Scholar. Why, then it is impossible to know the debt.

Master. Peace, you are too hasty, there is more help in it then yet you see, I have three severall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the first & the second to amount to 47 pound, the debt of the first man and the third man did make 71 pound, and the second man his debt with the third, did rise to 88 pound. Now can you tell what every man did owe, and what was the whole summe?

Scholar. Nay, in good faith, but as I perceive that it must be found by conjecture, so will I guesse at

at it ſuppoſing that the firſt man did owe 20 pound, and the ſecond man 30, and the third.

Maſter. Nay ſtay there, you are too farre gone already: you may not ſuppoſe a ſeverall ſumme for every man, for it is enough to ſuppoſe one ſumme for the firſt man, and let the other riſe as the queſtion importeth. Therefore ſeeing you ſet the firſt man his debt to bee 20 pound, the ſecond man cannot owe 30 pound, for the declaration is, that their debts added together did make 47 pound, ſo muſt the ſecond man his debt be but 27 pound. Now the ſecond debt with the third muſt make 88: therefore ſubtract 27 out of 88, and there will remain 61, as the third man his debt. Then ſaith the declaration, that the firſt and third mans debts do make 71: but by this ſuppoſition they make 81, that is 10 too much, which I muſt ſet for the firſt errour. Now worke you the ſecond poſition.

Scholar. I ſuppoſe the firſt mans debt to be 24 pound: then muſt the ſecond mans debt (by your declaration) be but 23 pound, ſeeing both they make but 47 pound. And the ſecond man his debt with the third, doe make 88 pound, and the ſecond man oweth but 23: therefore the third man muſt owe 65 pound. Now the third mans debt with the firſt, ſhould make by the declaration 71 pound, and they doe make 89 pound, that is 18 pound too much, and that is the ſecond errour, which I ſet down with the firſt, and their poſitions in this forme, and then I doe multiply in croſſe wayes 20 by 18, and it is 360. And 10 by 24 maketh 240. Alſo becauſe the ſignes of the errours be like,

20	24
10†	18†

like, I must worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the Dividend: then do I subtract 10 out of 18 by the same reason, & so is the Divisor 8, which is found 15 times in 120: therefore I say that the first man did owe 15 li. and then the second man must owe 32 li. for those two doe make 47 li. and the third mans debt is 56: for so much remaineth if I abate 15 out of 71, or if I take 32 out of 88.

The fourth example.

Master. *For the fourth example, take this easie question for the variety in work. Two men having severall summes, which I know not, doe thus talke together: the first saith to the second, if you give me 2* shillings *of your money, then shall I have three times so much money as you. The second man answereth, It were more reason that our summes were made equall, and so will it be if you give mee 3* shillings *of your money. Now guesse what each of them had.*

Note.

Scholar. I imagine that the first had 9 s.

Master. Consider evermore in your imagination that you take a likely summe, as in this question, take such a summe, that having 2 added unto it, may be divided into three parts even.

Scholar. Why? I remember you said before, it forceth not how fondly soever I guessed.

Master. As for the possibility of the solution, it is truth: but for easinesse in worke, the aptest numbers are most convenient.

Scholar. I thought no lesse, and therefore I took 9 as an apt number to be parted into three: but I perceive I should have considered the aptnesse of that partition after the addition of two unto

unto it, and then 7 had béen more méet.

Master. That is truth, and then should the second man his summe be 5: for although he have now but the third part of 9, that is 3, yet you must remember that he lent the first man 2, & so had he 5.

Scholar. Then to go forward: if the second man had thrée of the first man, then should he have 8 and the first man but 4; so hath he double to the first man, yet he said in the question they should have equall: wherefore it appeareth that he hath 4 too much.

Wherefore I note that errour with his supposition, and guesse again that he hath 10 shillings: whereunto I adde 2 shillings borrowed of the second man, and then he hath 12 shillings, so the second man hath remaining but 4, whereunto if I adde the 2 that he lent to the first man so had he but 6 shillings at the beginning.

Then take 3 shillings from the first man, and give to the second, and then hath the first man but 7, & the second hath 9, which are not equall, but there are 2 too many, wherefore I set down both the positions with their errours, as before you see, and multiply acrosse so commeth there 40 and 14: and because the signes be like, I take 14 out of 40, and so resteth 26 to be divided: then likewise I take 2, out of 4, and there resteth 2, by which I divide 26, and the quotient will be 13, which is the summe that the first man had. And so appeareth that 2 being added thereto; the summe will be 15, so hath the second man but 5, and before he had 7: then take 3 from the first, and put to this 7, and so have

7	10
4†	2†

each

each of them 10, and that is equall as the question would.

The fifth example: a question of Lambs.

Master. *For the fifth example take this question. One man said to another, I think you had this yeare two thousand Lambs: so had I said the other; but what with paying the tythe of them, and then the severall losses, they are much abated: for at one time I lost halfe as many as I have now left, and at another time the third part of so many, and the third time* $\frac{1}{4}$ *so many. Now guesse you how many are left.*

Scholar. Because here is mention made of certaine parts, I must take a number that may have all these parts, that is to say, $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{4}$ which will be 24, howbeit 12 hath the same parts. Therefore I take first 12 to be the number that doth remaine, so hath he lost 6, 4, and 3 that is 13, and the whole 25, but it should be 2000.

Master. You are deceived yet still, you have forgotten the 10 part, which must be defalked, that is 200, so there remaineth but 1800, and now go on again.

Scholar. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 to few, which I set for the errour. Then for the second position I take 24, whose halfe is 12, the third part 8, and the quarter 6, whereby riseth 50, which is too little by 1750 therefore I set downe both the positions, with their errours thus:

12	24
—1775	1750—

And multiply in crosse wayes 1775 by 24, whereof commeth 42600. Also I multiply 1750 by 12, and there ariseth 21000. And because the signes are like

like, I doe subtract the one from the other, and so remaineth the Dividend 216000. Then doe I subtract 1750 out of 1775, and there resteth 25, by which I divide 21600, and the quotient is 864, whereof the halfe 432, and the third part is 288, the quarter is 216; which all being added together, will make 1800: And if you adde thereto the tenth which was abated before, then will the whole summe be 2000.

864
432
288
216
1800

And now doth there come a question to my memory which was demanded of me, but I was not able to answer to it: And now me thinketh I could solve it.

Master. Propose your question.

Scholar. *There is supposed a Law made, that (for furthering of tillage) every man that doth keepe sheep, shall for every ten sheep ear and sow one Acre of ground: and for his allowance in sheep pasture, there is appointed for every foure sheep one Acre of pasture. Now is there a rich Sheep-master which hath 7000 Acres of ground and would gladly keep as many as he might: by that Statute I demand how many sheep he shall keep.*

A question of sheep and tillage: the sixth example.

Master. Answer to the question your self.

Scholar. First, I suppose hee may keep 500 sheep, and for them hee shall have in Pasture after the rate of four sheep to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this errour is too little by 6825. Therefore I guesse againe that he may keep 1000 sheep, that is, in Pasture 250 Acres, and in tillage 100 Acres, which make 350, that is too little by 6650. Both these errours with their positions, I set down as

you

you see, and multiply them crosse 6825 by 1000, and it maketh 6825000, also I multiply 6650 by 500, and there commeth 3325000, which summe I subtract out of the former, and there remaineth 3500000 for the Dividend: likewise I subtract the lesser errour out of the greater, and there resteth 175, by which I divide 3500000 (the Dividend aforesaid) and the quotient will be 20000, so that by this rate he that hath 7000 Acres of ground, may keep 20000 sheep.

500	X	1000
6825—6650—		

Master. You have done well, notwithstanding both this last question, and the next before might bee wrought without the second position by the Rule of proportion as this: When in this question you found in the first errour that for 500 sheep there must be 175 Acres, then might you reduce it to the Golden Rule, thus:

Another way of working.

175	Z	500
7000		20000

If 175 Acres will admit in allowance 500 sheep, then 7000 will have 2000. And so by one position, with the help of the Golden Rule may you answer that question.

Likewise for the question of Lambs, when you had found that 12 came of 25, you might have set the figure as followeth, and have said:

25	Z	12
1800		864

If 25 do leave but 12, what shall 1800 leave? and it would appear to be 864.

Scholar. Sir, I thanke you for this aid, for it doth much shorten the worke of this Rule.

Master. Yet againe I will shew you an other way to answer to this last question without the Rule of

Another way yet.

of falſe poſition, and that by the Rule of Fellowſhip, for it appeareth in the propoſing of the queſtion, that ten ſheep muſt have in paſture 2 Acres and $\frac{1}{2}$, and for them muſt there be eared but one Acre; ſo it followeth, that for 2 Acres eared, there muſt bee 5 ſet to paſture: and if you put them both into one ſumme, they will make 7. Therefore look what proportion 7 being the totall, doth bear to 5 and to 2, ſuch proportion ſhall any totall in this queſtion beare to the paſture ground, and the eared ground.

Scholar. This ſerveth wondrous aptly. Therefore to prove it, I demand this by the former ſuppoſition: If a man have 300 Acres, how much ſhall he leave in paſture, and how much ſhall he turne to tillage? You ſay that as 7 is to 5, ſo ſhall 300 be to the Acres of paſture: and as 7 is to 2, ſo is 300 to the Acres of tillage, whereof for both I have ſet examples here following, whereby appeareth that of paſture, there ſhall bee 214 $\frac{2}{7}$ Acres, and of Tillage 85 $\frac{5}{7}$, which both ſummes added together, doe make 300.

7 Z 5
300 Z $214\frac{2}{7}$

7 Z 2
300 Z $85\frac{5}{7}$

Maſter. *Now take another Example: A man hath three ſilver Cups with one Cover, the Cover weigheth 18 ounces, the ſecond Cup weigheth even halfe the weight of the firſt and the third. Now if the Cover be put to the firſt Cup, they weigh juſt as much as all the three cups do weigh: & if the Cover be joyned with the ſecond Cup, they weigh as much as the ſecond twice, & the third: & if the Cover be put to the third cup they will make twice as much as the firſt & ſecond cup. Now try you what was the juſt weight of every cup.* Another queſtion: the ſeventh example.

Scholar.

Scholar. I do ſet the weight of the firſt Cup to be nine Ounces, then in as much as theſe two (that is to ſay, the cover and the firſt Cup) doe weigh the weight of the three Cups, I ſee that the three Cuppes muſt weigh 27 ounces, for ſo much is 18 and 9. Alſo becauſe the firſt and the third doe weigh double ſo much as the ſecond, therefore it is the third part of that weight, that is 9, and then would it follow, that the third Cup alſo ſhould weigh 9 ounces; but then the queſtion ſaith that the Cover being joyned to the ſecond Cup, they weigh as much as the ſecond twice, and the third once, that ſhould be 27, and ſo it doth; that being joyned with the third Cup, they ſhould weigh twice as much as the firſt and the ſecond, that ſhould be 36, and they weigh but 27, ſo is that errour 9 too little. Then beginne I again, and ſay, that the firſt Cup doth weigh twelve ounces which I joyne with the Cover, and they make thirty ounces: then ſeeing the ſecond is of that weight, it muſt needs weigh ten ounces, and the third muſt weigh 8 ounces, ſeeing the firſt and the third muſt weigh 20 ounces. Now put I the Cover to the ſecond Cup, and they weigh 28 ounces, which ſhould be even ſo: then joyne I the cover with the third Cup, & ſo ſhould it weigh twice the firſt and the ſecond, that is 44 ounces, and they weigh but 26, that is 18 too little: thoſe errours with their poſitions I ſet down, and multiply in croſſe wayes 9 by 12, whereof commeth 108: Alſo 9 by 18, and that yeeldeth 162: and in as much as the ſignes be like, I abate the leſſer out of the greater, and there doth remaine

9	12
9—	18—

remaine

remaine 54. Then doe I also abate the lesser errour from the greater, & so remainth 9, by which I divide 54, and the quotient is 6, which I take for the true weight of the first Cup, which being joyned with the Cover, must weigh as much as the three Cups, so doe they weigh but 24 ounces. Then seeing the second Cup is the third part of that weight, for the other two Cups (you say) must weigh double his weight, the weight of the second Cup is 8 ounces, and so the weight of the third Cup must be 10 ounces. Now put the Cover to the second Cup, and it will make 26 ounces: that must be the weight of the second twice, and the third once, that is, twice 8, and once 10, and so is it. Again put the Cover to the third Cup of 10 ounces, and they must weigh, twice as much as the first and the second, that is 28, and so is all agreeable.

Master. Then answer to this Question.

There is a Cisterne with four Cocks, containing 72 barrels of water: and if the greatest Cocke be opened, the water will avoid clean in six houres; at the second Cocke it will aske eight houres: at the third Cock it will avoid in no lesse then nine houres: and at the smallest it will require twelve houres. Now I demand in what space will it avoid, all the Cocks being set open? Aquestion of water: the eight example.

Scholar. First I imagine it will avoid in two houres.

Master. Then must there avoid by the first Cock $\frac{1}{3}$ of the water, that is 24 Barrels, and by the second Cock $\frac{1}{4}$, that is 18, and by the third Cock $\frac{2}{9}$, that is 16 Barrels, and by the smallest Cock $\frac{1}{6}$, that

that is 12 Barrels, all which ſummes put together, do make 70, as by their Addition it doth appeare, but it ſhould be 72, therefore the errour is too few.

Scholar. Then will I begin again by your favour, becauſe I think I underſtand the work, and put three houres for the due time: ſo ſhall there run out at the greateſt Cock $\frac{1}{2}$, that is, 36 Barrels, and at the ſecond hole $\frac{3}{8}$, that is 27, and at the third Cock $\frac{1}{3}$, that is 24, and at the ſmalleſt hole $\frac{1}{4}$, that is 18 Barrels, which all together doe make 105, and ſhould be but 72, ſo is it too much by 33: therefore do I ſet the errours in order of the figure with their poſitions, and worke by multiplication, in croſſe, ſaying, 2 times 3 is 6, & 2 times 33 mak 66, & becauſe the ſignes are unlike, I muſt adde theſe 2 totalls together, which make 72: alſo I adde the two errours, and they make 35, by which I divide 72, and the Quotient riſeth $2\frac{2}{35}$, whereby I ſee that all the Cocks being ſet open, the water will avoid in two houres, and $\frac{2}{35}$ of an hour.

24
18
16
12
—
70

2 3
×
2 — 33 †

Maſter. This exerciſe maketh you to grow expert in the Rule. Therefore I will enure you ſomewhat more with a queſtiou or two.

A queſtion of partners: The ninth example.

There were two men that had been partners, and had in account between them 300 Duckets; whereof the one ſhould have for his part 180, & the other 120: but in the parting of them, they fell at variance; ſo that each of them catched as many as he could: yet afterward being reconciled they agreed that he which hath gotten moſt part of them, ſhould lay down $\frac{3}{4}$ of them again, and he that had gotten leaſt, ſhould lay down $\frac{1}{3}$ of thoſe which

which he had taken, and then parting them into two equall parts, each man to have halfe thereof, and so had they their just portions as they ought to have: now I demand of you what each of them had gotten by the scrambling.

Scholar. I suppose he that had least, got 108 Duckets, then the other had 192: wherefore in laying down again of the 192, there was put down $\frac{1}{4}$, that is, 144, and so had he left but 48. Also of the 108, there was laid down 36, that is $\frac{1}{3}$, & so he had left 72. Then I put together 144 & 36, and it maketh 180, which I part into two parts even, and so commeth 90 to be given to each of them: which summe put to 72 maketh 162; and joyned to 148, it maketh 238: and now I doubt how I shall go forward.

Master. You need not to take but one of them, which you list, the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take to any of the other summes, remembring that you make comparison to the same in the second worke: as for example of the first part. If you compare 138 with the lesser summe due, that is, 120, so is it 18 too much, and if you compare it with the greater summe, then is it 42 too little. Again, if you compare 162 to the greater summe, the errour will be 18, as it was in the other: but it will have a contrary signe: and if you compare it with the lesser summe, it will be 42 too much: so that the errour both wayes is either 18 or 42: & as for the signes it little forceth, for in them is nothing considered here, but likenesse and unlikenesse, which in this case doth neither Note.

further or hinder : But now go on with the worke.

Scholar. If it be ſo, then am I out of my greateſt doubt. Then I ioyne that 90 (which I found as the half of the latter partition) unto 48, which is left with the one man and ſo hath he 138, which (I may ſay) is 18 too many, for the leaſt ſhould be but 120 that errour do I note, and then make a new poſition, ſuppoſing the one man to have 204, and the other to have 96, wherefore of the 204, there muſt be laid down 153, and ſo remaineth with him 51. Alſo of the 96, there muſt bee laid down $\frac{1}{3}$, that is 32, and ſo reſteth with that man 64: Now of the 153 and 32, I make one ſumme, as 185, which I muſt divide into two equall parts, and ſo each man ſhall have $92\frac{1}{2}$, whereunto if I adde their former portions reſerved, then the one ſhall have $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherefore take the leſſer ſumme now again, as I did before, that is, $143\frac{1}{2}$ and find that he hath to many by $23\frac{1}{2}$, for he ſhould have but 120, and ſo have I for my two poſitions two errors, which I ſet down as here may be ſeen, each error under his poſition, and then by the Rule I doe multiply in croſſe wayes 108 by $23\frac{1}{2}$, and there riſeth 2538, which I note, then again I multiply 96 by 18, and thereof amounteth 1728.

108		96
	X	
18 †		$23\frac{1}{2}$ †

Now becauſe the ſignes are both like, that is, both too many, I muſt worke by Subtraction, and ſo abating 1728, out of 2538, there will reſt for the Dividend 810 ; then for the Diviſor I ſubtract 18 out of $23\frac{1}{2}$, and there remain-

eth

eth $5\frac{1}{2}$, by which I divide 810, and the quotient will be $147\frac{3}{11}$, which is the just portion of him that had the least summe. And if I doe subtract it out of 300, being the totall summe, then will there remain $152\frac{8}{11}$, as the portion that the other did get.

Master. For the proofe of this worke, you may chuse whether you will examine those numbers according to the forme of the questions or else worke by other two positions, for to finde the second number: and if those positions bring the same numbers that did amount by the first two positions, then doth each work confirm other.

Scholar. By your patience, I will prove both wayes, not onely to seeke their agreement but also to accustome my minde to those workes: for I perceive it is exercise that must be the chiefe engraver of these Rules in my memory.

Master. You consider it well: then go to.

Scholar. First, I will by two other positions try to finde the portion of him which had most.

Master. Although you may doe it with any positions, yet to see the agreement of your worke the better, take the same positions that you did before, comparing them now to the greater, as you did before unto the lesser.

Scholar. Then I suppose that he that had most, had 192, so had the other 108. Now if I take $\frac{1}{4}$ out of 192, that will be 144 and there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is 36, there will remain to him 72: then joyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to

each of those two mens portions remaining with them, the one shall have 138, and the other 162, of which two I take the greater (that is 162) and see it to be 18 too few; for it should be 180, that errour I note under this position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{1}{4}$ of 204 and it will be 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is, 32, and there remaineth to him 64; now put I that 32 to 153, and it yeeldeth 185: which being parted in equall values, maketh $92\frac{1}{2}$ to be added to each mans remainder, and so the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greatest summe, and it is $23\frac{1}{2}$ too little, that do I note also, and set both these errours under their positions, as in this Example following doth appeare.

And then multiplying 192 by $23\frac{1}{2}$, there doth arise, 4512.

Againe, I multiply 204 by 18, and it maketh 3672, which I doe subtract out of 4512, because the signes be like, and there resteth 840 for the dividend, then subtracting 18 out of $28\frac{1}{2}$ there will remain $5\frac{1}{2}$, which I must take for the Divisor. And so dividing 840 by $5\frac{1}{2}$ the quotient will be $152\frac{8}{11}$, whereby I have found an agreeable summe to that which I found by the former positions, for him that had most, which I doe subtract out of 300, that is the totall, there will rest $147\frac{3}{11}$, which was the portion of him that had the least part.

192	204
—18	—$23\frac{1}{2}$

Master. So by divers positions, you see, that one doth

doth confirme the work of the other. Now, examine those two numbers by the forme of the question, and so shall you prove your work good also.

Scholar. If that he which gat most had $152\frac{8}{11}$, then must he lay down $\frac{3}{4}$ of this summe, that is $114\frac{6}{11}$, and so shall remain with him but onely $38\frac{2}{11}$. The other which had least, that is $147\frac{3}{11}$, must put down of his summe $\frac{1}{3}$, that is $49\frac{1}{11}$, and so doth there remain with him yet $98\frac{2}{11}$. Then do I adde together $114\frac{6}{11}$, and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$ which I must part into equall parts, and that will be $81\frac{9}{11}$, to be given to each of them: putting $81\frac{9}{11}$ unto $38\frac{2}{11}$, there doth amount 120 just, which is the true Portion of him that should have the lesser summe: and adding $81\frac{9}{11}$, $98\frac{2}{11}$, the totall will be 180, the true portion of the other. And so is the work by this proofe also tryed to be good. And this I marke by the way, that in their scrambling, he got most (as it chanceth often) that ought to have had least by just partition.

Master. Let your study be to learn truth and just Art of proportion, and to distribute and part according thereunto, as often as occasion shall be ministred. And here would I make an end of this Rule, save that I remember one pleasant question which I cannot overpasse, which I will declare somewhat largely, because you shall as well understand some reason in the pleasant invention, as apt proceeding in the witty working thereof.

Hiero *King of the Syracusans in Sicilia had caused to be made a Crown of Gold of a wonderfull weight, to be offered for his good successe in wars: in making whereof the Goldsmith fraudulently took out a certain* The tenth example of Gold and Silver.

 portion

portion of Gold, and put in Silver for it, so that there was nothing abated of the full weight, although there was much of the value diminished.

Which thing at length being uttered (as no evill can alwayes lie hid) the King was sore moved: and being desirous to know the truth without breaking of the Crown, proposed the doubt to Archimedes, unto whose wit nothing seemed unpossible, which although presently he could not answer unto, yet he had good hope to devise some policy for that invention, and so musing thereon, as he chanced to enter into a Bain full of water to wash him, he observed, that as his body entred into the Bain, the water did runne over the Tub, whereby his ready wit of such small effects conjecturing greater works, conceived by and by a reason of solution to the Kings question, and therefore rejoycing exceedingly, more then if he had gotten the Crown it selfe, forgat that he was naked, and so ranne home, crying, as hee ranne, ἔυρηκα, ἔυρηκα, I have found, I have found. And thereupon caused two massie pieces, one of Gold, and another of Silver, to be prepared, of the same weight that the said Crown was of: and considering that Gold is heavier of nature then Silver, and therefore Gold of like weight with Silver, must needs occupie lesse rome, by reason it is more compact and sound in substance, he was assured that putting the masse of Gold into a vessell brimfull of water, there would not so much water runne out, as when he should put in the silver masse of the like weight. Wherefore he tried both, and noted not onely the quantities of the water at each time, but

but also the difference or excesse of the one above the other, whereby hée learned what proportion in quantity is betwéen Gold and Silver of equall weight. And then putting the Crown it self into the vessell of water brim-full (as before) marked how much water did run out then, and comparing it with the water that ranne out when the Gold was put in, noted how much it did excéed that: and likewise comparing it to the water that ranne out of the Silver, marking how much it was lesse then that, and by those proportions found out the just quantity of Gold that was taken out of the Crown, and how much Silver was put in stead of it: but séeing Vitruvius which writeth this History, doth not declare the particular worke of this tryall, it shall be no inconvenience to suppose an example for declaration sake, wherein although the true & just proportion be not expressed, yet the forme of triall shall be truly set forth. And for an example, I suppose the weight of the Crown to be 8 pound, and so of each the other two Masses. And when the Masse of Gold was put into the water, I imagine that there ranne out two pound of water: and when the masse of Silver was put in, I suppose there ran out 3 pound $\frac{1}{2}$. Againe, when the Crowne was put in, there ran out two pound $\frac{1}{4}$: Now to know what quantity of Silver was in the Crown, work by the Rule of false position, and imagine that there was two pound of Silver, then must there be six pound of Gold, then say thus by the Rule of Proportion. If eight pound of Gold do expell two pound of water, what shall six pound expell? and it will be 1 pound $\frac{1}{2}$. Againe, for the Silver; if eight pound of Silver

expell three pound $\frac{1}{2}$ of water, what shall two pound of Silver put out? it will be $\frac{7}{8}$, now adde those two weights of water together, and they will make two pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{3}{4}$, so is it too much by $\frac{1}{8}$.

Scholar. Now do I understand the worke as I thinke, therefore I pray you let me worke the rest of the question. And because this first supposition did erre, I note that position & his errour, & take a new position, esteeming the Silver to be but one pound, so must there be in gold 7 pound. Then say I, if eight pound of Gold do yeeld two pound of water, what shall seven pound yeeld? and it will be 1 ponnd $\frac{1}{4}$. Again, if 8 l. of Silver expell 3 pound $\frac{1}{2}$ of water, what shall 1 pound expell? and it will be $-\frac{7}{16}$. Now must I adde those two summes together, and they make two pound $-\frac{1}{16}$, and they should make 2 pound $\frac{1}{4}$, so is it too litile by $-\frac{1}{16}$. Therefore I set the positions with their errors in order as here followeth: And then I multiply in crosse wayes 2 by $-\frac{1}{16}$ and it maketh $\frac{1}{8}$: Likewise 1 multiplyed by $\frac{1}{8}$ - maketh $\frac{1}{8}$. And because the signes be unlike, I must adde these two summes which make $\frac{1}{4}$: and that is the dividend.

2 1

X

$\frac{1}{8}$ † $-\frac{1}{16}$

Againe, I must adde $\frac{1}{8}$ to $-\frac{1}{16}$, and it will be $-\frac{1}{16}$, that is the Divisor. Now I shall divide $\frac{1}{4}$, by $-\frac{3}{16}$ and the quotient will bee $\frac{16}{12}$, that is, 1 $\frac{1}{3}$: whereby I know that there was but 1 pound and $\frac{1}{3}$ of Silver into the Crown, and so much Gold taken out for it.

Master. Prove it now by examination, according to the question.

Scholar. If there were 1 pound $\frac{1}{3}$ of Silver, then was

was there of Gold 6 pound $\frac{2}{3}$. Now say I by the Rule of proportion: if 8 pound of Gold expell two pound of water, what shall 6 pound $\frac{2}{3}$ expell, it will be 1 pound $\frac{2}{3}$.

$8 \; Z \; 2$
$6\frac{2}{3} \quad 1\frac{2}{3}$

$8 \; Z \; 3\frac{1}{2}$
$1\frac{1}{3} \quad \frac{7}{12}$

Againe, if 8 pound of Silver expell three pound $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expell? It will bee $\frac{7}{12}$. Now must I adde together 1 pound $\frac{2}{3}$ and $\frac{7}{12}$, and they will make 2 pound $\frac{9}{36}$, that is, 2 pound $\frac{1}{4}$, according to the supposition of the question: whereby I perceive the work to be well done. And I cannot but much rejoyce of this excellent invention, so my desire is kindled vehemently to be perfectly instructed in every part thereof, & namely in this point, whether the proportion between water and Gold be such that for 8 pound of Gold put into a vessell full of water, there shall runne out two pound of water, and for as much Silver, whether 3 pound $\frac{1}{2}$ of water would avoid.

Master. I perceive your meaning, and conjecture your imagination to be thus, that if you knew the exact proportion betwéen Gold and Silver, and water, both in their weight and quantities, then could you easily finde out the mixtures of them, which thing I have reserved for another worke that intreateth of such matters especially. And at this time you must consider that you learn Arithmeticke, which intreateth of the manner to solve doubtfull questions touching number, without regard what matter is signified by that number: else were it necessary in Arithmeticke, to teach all Arts, séeing in it may be moved questions of all Arts.

But seeing you are so desirous to know these things,

A question of the proportion of gold, silver, and quick-silver, unto water.

things, I will tell you in such a sort, that you shall practise your Art in finding it, and propose it in forme of a question. Gold beareth a greater proportion to water then Silver doth, and their two proportions be in proportion together, as 48 to 25. But to help you somewhat in this Riddle, you shall note that the proportion of Quick-silver unto water, is the just middle number proportionall in progression Geometricall between the proportion of Gold and Silver unto water.

And this proportion is $\frac{220}{21}$. Now if you will know the just numbers of these 3 proportions, then must you finde out 3 numbers in Progression Geometricall, whereof the middlemost must be $\frac{220}{21}$, & the first must be unto the last, as 25 to 48. And thus I will leave you to find those numbers when you be at leisure.

Scholar. Yet Sir, I thank you heartily for thus much, for now I see the possibility to finde them out. Howbeit, because this question seemeth strange, if it might please you to instruct me somewhat in the order of working for it, I should the more easily finde the true working.

Master. You desire too much if you will study for nothing: Therefore to occasion you to study the better, I will leave this doubt wholy to your own search: But as touching the generality of the Rule, Archimedes needed not to take two Masses of gold and Silver equall in weight with the Crown, for the proportion might as well be found in any other weight, yea, although the Masse of Gold were of one weight, and the Masse of Silver of another. As for example: if the Crown were of 8 pound weight as I did suppose, and I have not

so

so much other fine Gold, but onely one pound, and trying that by water, and finding that it doth expell but $\frac{3}{4}$ of an ounce of water, yet then by it I may inferre, that 8 pound of Gold would expell 6 ounces of water. And likewise of Silver, whereof if I had but two pound, and finde that it doth expell three ounces of water, then might I affirme, that 8 pound would expell 12 ounces, that is, one pound weight: and so is it good as if the three Masses were all of one weight. And thus for this time I will make an end of this other part of Arithmeticke.

Scholar. Although I cannot sufficiently thank you for this, yet your promise made me to looke for the Art of Extraction of Roots, whereof hitherto I have learned nothing.

Master. I will not breake my promise, but intend (God willing) to performe it within this three or foure moneths, if I perceive this my paines to be well taken in the mean season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the mean time you shall take this Addition, not for the second part of Arithmeticke which I promised, but for an augmentation of the first part, unto which I would have annexed the Extraction of Rootes square and cubie, namely for Examples of the Statute of Assise of Wood, but that in the second part I must write of divers other Roots, and thought it best to reserve those Rules also with their Examples unto the same second part.

Scholar.

Scholar. Sir, although I cannot recompence your goodnesse, yet I shall alwayes doe mine endeavour to occasion you not to repent your benefit on me thus imployed.

Master. That recompence is sufficient for your part.

FINIS.

The third Part,

OR,

Addition to this BOOKE,

Entreateth of briefe Rules, called Rules of Practise, of Rare, Pleasant, and Commodious effects abridged into a briefer method then hitherto hath *been published.*

With divers other necessary Rules, Tables, and Questions, not onely profitable for Merchants, but also for Gentlemen, and all other Occupiers whatsoever, as by the Contents *of this Booke* may appear.

Set forth by JOHN MEILIS, *School-master.*

The first Chapter of this Addition entreateth of brief Rules, called Rules of Practise, whith divers necessary questions, profitable not onely for *Merchants*, but also for all other Occupiers whatsoever.

THE working of Multiplication in practise, is no other thing then a certain manner of Multiplying of one kinde by another: whereupon is broght forth, the Product of the proposed number, which is accomplished by the meanes of Division in taking the *halfe*, the *third*, the *forth*, the *fifth*, or such other parts of the summe, which is to be multiplied. Rule. 1.

And for the better understanding of such conversions, you shall understand that in the manner and use of these Rules of Practise, you ought first to know the even or aliquot parts of a shilling, which in this Table following doth appear.

Item	6	*pence is the*	$\frac{1}{2}$	*of a shilling.*
	4		$\frac{1}{3}$	
	3		$\frac{1}{4}$	
	2		$\frac{1}{}$	
	1		$\frac{1}{2}$	

Wherein as you see according to the order of these rules of Practise: at 6 pence the yard of any thing, you must take $\frac{1}{2}$ of your number which is to be multiplied, and the product that commeth thereof shall be

be shillings, if any unite doe remain it is 6 pence.

For 4 d. take the $\frac{1}{3}$ of the number that is to be multiplied, and the product also produceth shillings, if any unites doe remain, each one shall be worth in value 4 pence. The like is to be understood of the other 3, &c.

I *Example.*

At 6 d the yard, what	379 yards?
	189 s ——— 6 d

II

At 4 d the yard, what	104 yards?
	34 s ——— 8 d

III

At 3 d the yard, what	5014 yards?
	1253 s ——— 6 d

IV

At 2 d the yard, what	532 yards?
	88 s ——— 8 d

V

At 1 d the yard, what	409?
	34 s ——— 1 d

Here you may see in the first example, that 379 yards at 6 d. the yard, are worth 189 s. 6 d in taking the $\frac{1}{2}$ of 379. And in the second example the 104 yards at 4 pence the yard, are worth 34 s, 8 d, in taking the $\frac{1}{3}$ of 104. Likewise in the third example, 5014 yards at 3 d the yard, bringing forth 1253 s. 6 d in taking the $\frac{1}{4}$ of 5014. Also in the fourth Example at 2 d the yard, maketh 88 s. 8 d.

And lastly, in the fifth Example: 409 yards at 1 d the yard amounteth to 34 s. 1 d. in taking the $\frac{1}{12}$ of 409. and

409. and ſo is to be done alſo of all other queſtions the like, when the number of the pence is any of the even or aliquot parts of 12 d.

Item, to bring the Products of theſe ſhillings, and all other the like into pounds is very eaſie in dividing of it in your minde by 20. for it is to be underſtood that as often as 20 is found in that Product, ſo many pounds doth it contain : which with facility to performe, alwayes ſtrike off the figure towards your right hand, with a right down daſh of your pen, for the 0 that appertaineth to the 20. And then begin at the left hand, in taking the half off the reſt. And if that at the laſt any unite doe remain, the ſame ſhall be joyned with the figure that is cut off which ſhall repreſent the odde ſhillings contained in that work.

As for example, in your third queſtion at 3 d the yard, which amounteth to 1253 s. 6 d. the Product whereof maketh, 62 li. 13 s. 6 d. as here you may ſee is eaſily performed by this Example.

1
125|3
62--13 —— 6

Alſo for the working of one peny the yard, it is ſomething harſh and hard to take the $\frac{1}{12}$ of ſome Products : therefore to eaſe that hard worke, you ſhall firſt bring your delivered ſumme into groats by taking $\frac{1}{4}$ part of the product, and if any unites remain of that $\frac{1}{4}$ part as ſometimes there may they are pence : and muſt be ſignified with a line from the *groats* with their title of *pence* ; and becauſe that 60 *groats* maketh a *Pound* or twenty ſhillings, *ſtrike* off the firſt figure toward your right hand, for the 0 that appertaineth to 60 (as you did even now for the 0 that belongeth to 20 :) Then in taking the $\frac{1}{3}$ of that product, if there doe remain any unites, the ſame ſhall

you joyne with the figure that you cut off, esteeming them as *groats*, which keep in your minde, and by taking the $\frac{1}{3}$ part of them, you shall turn them into *shillings*, and so have you done: *As for Example*, by a Question or two hereafter proposed, shall more plainely by the worke appear.

At 1 d, the yard, what	5 4 3 6 8 yards?
	1 3 5 9 2 *groats.*
$\frac{1}{6}$ li	2 2 6-10 s.-8 d.

Here in taking the $\frac{1}{6}$ part of 1 3 5 9, in coming to the last worke, the $\frac{1}{6}$ part of 39 being taken, the *remainder* is 3, which joyned with the two that was cut off, maketh 32 *groats*, which converted into *shillings*, by taking the $\frac{1}{3}$ part, maketh as appeareth 10 *shillings* 8 d. Many other wayes there are, but none more apt for a *young learner* to understand then this: wherefore this one way well impressed in memory is better then 20 wayes doubtfully understood.

At 1 peny the yard, what	4 5 3 3 yards?
	1 1 3 | 3 *groats*-1 d
$\frac{1}{6}$ li.	1 8 —— 17-9 d
At 1 peny the yard, what	6 4 7 6 8 yards?
	1 6 1 9 | 2 *groats*
li	269--1 7-4 d

Now followeth also to be understood that if the number of pence be not an aliquot part of 12, *you must reduce them into some aliquot part of* 12: *and after the aforesaid manner, you shall make of them two or three Products, as need shall require, and adde them together into one summe. And here for thy furtherance appeareth a note of the order of their parts,*

parts, as they are to be taken.

for pence	take	and	or
5	3	2	4 & 1
7	4	3	6 & 1
8	4	4	6 & 2
9	6	3	4·4 and 1
10	6	4	4·4 and 2
11	6	4& 1	4·4 and 3

Here in the first note of this *Table* at 5 d. you shall first take for 3 d the $\frac{1}{4}$ of the number that is to be multiplyed: and likewise for 2 d. the $\frac{1}{6}$ of the same number, adding together both the Products: But if you will worke by 4 and 1, you must for 4 d, first take $\frac{1}{3}$ of the number that is to be multiplied: and for 1 d, take the $\frac{1}{12}$ of the whole summe, or rather, which is better, for 1 peny you may take the $\frac{1}{4}$ of the product which did come of the 4 pence: because that 1 d, is the $\frac{1}{4}$ of 4 pence. The totall summes of these two numbers shall be the solution to the *Question.* And in like manner it is to be done of all others, as by these Examples following shall appear.

I

At 5 d the yard, what	758 yards?
3 d	189 —— 6 d
2 d	126 —— 4 d
shillings,	325 —— 10 d

Otherwise,

At 5 d the yard what	758 yards?
4 d	252 —— 8 d
1 d	63 —— 2 d
shillings,	315 —— 10 d

II

At 7 pence the Ell, what	563 Elles ?
4 d	187 — 8 d
3 d	140 — 9 d
shillings,	328 — 5 d

III

At 8 d the pound, what	112 pound ?
4 d	37 — 4 d
4 d	37 — 4 d
shillings,	74 — 8 d

Otherwise,

At 8 d the pound, what	112 pound ?
6 d	56 — 0
2 d	18 — 8
shillings,	74 — 8 d

IIII.

At 9 pence the Elle, what	356 Elles?
6 d	178 — 0
3 d	89 — 0
shillings	267 — 0 d

V.

At 10 pence the peece, what	795 peeces ?
6 d	397 — 6
4 d	265 — 0
shillings	662 — 6

VI.

V I.

At 11 pence the pound, what	7576 pounds?
6 d	3788 —— 0
4 d	2525 —— 4
1 d	631 —— 4
	6944 —— 8 d
Pounds	347 — 4 s — 8 d

1 Here in this first example, where it is demanded (at 5 d the yard) what will 758 cost? First, for 3 d, I take the $\frac{1}{4}$ of 758 : and thereof commeth 189 s. 6 d : Then for 2 d I take the $\frac{1}{6}$ of the same 758, which amounteth to 126 s. 4 d, these two summes added together, doe make 315 *shillings* 10 *pence* : and so much are the 758 *yards* worth at 5 d the *yard*.

Item also for the same again : First for 4 d, I take the $\frac{1}{3}$ of 758 ; and thereof commeth 252 s. 8 d : then for 1 peny I take the $\frac{1}{4}$ of the same 758, that is to say, of 252 s, 8 d, and it yeeldeth me 63 s. 2 d : which both added together maketh 315 s — 10 d, as before.

2 *Item, for* 7 d *there is taken the* $\frac{1}{3}$ *and the* $\frac{1}{4}$ of the whole summe which is to be multiplyed, and adde them together, that is to say, first, for 4 pence there is taken $\frac{1}{3}$ of 563 : which comes to 187 s. - 8 d, as appeareth by the worke, and for 3 d there is taken the $\frac{1}{4}$ of the whole summe, which amounteth to 140 s — 9 d. Both which products added together, doe make 328 s. — 5 d. and so much comes 563 *Elles* to at 7 d the *Elle*.

3 *Item, for the first* 8 d. *there is taken for* 4 d. the $\frac{1}{4}$ of the whole summe, and another $\frac{1}{4}$ for the other 4 d. which added together, as in the example doth

evidently appear, amounteth to 74 s.-8 d.

Again, for the second work of 112 li. there is taken first the $\frac{1}{2}$ of the whole summe for 6 d. which comes to 56 s. then for that 2 d. you have to take $\frac{1}{8}$ of the whole summe, or if you will, the $\frac{1}{3}$ of the product that came of 6 d. either of which maketh 18 s.-8 d. these two summes being added together doe make 74 s. 8 d. as in the third example appeareth.

4 *Item, for* 9 d, *there is taken for 6 pence* the $\frac{1}{2}$ of the whole summe, and the $\frac{1}{4}$ of the whole summe for 3 d. or otherwise for the 3 d. you may take the $\frac{1}{2}$ of the product that came of 6 d. because 3 pence is the $\frac{1}{2}$ of 6 d. which added together as plainly appeareth in the *fourth example*, amounteth to 267 s. 0 d.

Item for 10 d. *first there is taken for* 6 d. the $\frac{1}{2}$ of the whole summe, which amounteth to 397 s.-6 d. Then for 4 d. there is found 265 s. both which added together, make 6 6 2 *shillings*, 6 d, as appeareth in the *fifth example:* It may also be wrought, *as appeareth by the second note in the Table*, by 4 d. twice taken, and the $\frac{1}{2}$ of the product of 4 d. or else by the $\frac{1}{6}$ of the whole summe, *&c.*

Item, for 11 *d. there is first taken the* $\frac{1}{2}$ *for* 6 d. then the $\frac{1}{3}$ of the whole summe for 4 d. Lastly, the $\frac{1}{4}$ of the last product for 1 d. All which 3 summes added together, maketh in shillings 6944 s- 8 d. and in pounds 347-4 s-8 d.

Item, likewise by the same reason, when you will multiply (by shillings) any number that is under 20. *you shall have in the Product pounds, if you know the even or aliquot parts of* 20. *which are here in this little Table set down to sight.*

Item,

$$\text{Item, s.}\left\{\begin{array}{c}10\\5\\4\\2\\1\end{array}\right\}\text{is the}\left\{\begin{array}{c}\frac{1}{2}\\\frac{1}{4}\\\frac{1}{5}\\\frac{1}{10}\\\frac{1}{20}\end{array}\right\}\text{of one pound.}$$

So that for 10 s. *which is the* $\frac{1}{2}$ *of a pound*, you may take the $\frac{1}{2}$ of the number which is to be multiplyed, and you shall have in your product pounds, if an unite doe remain, it shall be worth ten *shillings*.

Likewise for 5 *shillings you must take the* $\frac{1}{4}$ of the number which is to be multiplied, and if there do remain any *unites*, they shall be fourth parts of a *Pound*, every unite being in value five *shillings*.

For 4 *shillings take the* $\frac{1}{5}$ *of the number* which is to be multiplied: and if there doe remain any *unites*, they shall be fift parts of a *pound*, each unite being in value 4 *shillings*.

For 2 *shillings you must take the* $\frac{1}{10}$ *of the* number to be multiplied, wherefore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same number (which is nearest your right hand) from all the other figures with a small right down line or dash with a Pen, and so have you done: for all the other figures which doe remain toward your left hand from the same figure that you do separate, shall be *pounds*: and that figure so separated towards your right hand, shall be so many pieces of 2 s. the which figure you must double to make thereof the true number of shillings, as by the Example shall appear.

Finally, for 1 *shilling needeth small work*, for it is so many shillings as be proposed in the summe, which to bring into pounds, hath been already taught in the *first rule*.

Example.

At 10 s. the piece, What		6543 pieces ?
$\frac{1}{2}$	li.	3271 10 s.
At 5 s. the Elle, What		4373 Elles ?
$\frac{1}{4}$	li.	1093 5 s.
At 4 s. the yard, What		7839. yards?
$\frac{1}{5}$	li.	1567. 16 s.
At 2 s. the pound weight, What		752|7 pound
$\frac{1}{10}$	li.	752 — 14 s.
At 1 s the piece, What		775|3 pieces ?
$\frac{1}{20}$	li.	387 13 s.

Next followeth in order to be understood, that if the number of shillings be not some even or aliquot part of 20, you must then convert the same number of shillings into the aliquot parts of twenty, and thereof make two or three products as need shall require : which done, adde them together, and bring them into Pounds. And here for thy furtherance, I have set down a note of the order of their parts, as they are to be taken.

s					s		
3	of	2 & 1	or	5 & 1	13	of	10 2 & 1
6		4 & 2		5.2.1	14		10. & 4
7		5 & 2		4.4.1	15		10. & 5
8		4 & 4			16		10. 5. 1
9		5 & 4			17		10. 5. 2
11		10 & 1			18		10. 4. 4
12		10 & 2			19		10. 5. 4

*For 3 s. according to the tenure that you see is ex*pressed in the Table, you must first take for 2 *shillings*

the

the $\frac{1}{10}$ of the number that is to be multiplied. Then for one *shilling* you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part : which two summes added together produceth the effect desired.

Item, for 6 shillings according to the note set forth in the Table, first for 4 s. I take the $\frac{1}{5}$ of the number that is to be multiplied : Then for 2 s. the $\frac{1}{2}$ of the product that came of 4 s. and adde them together.

Or else as appeareth also in the Table, for 5 shillings you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ part of the product that came of 5 *shillings*, and adde them together.

Item, for 7 s. first for 5 s. take $\frac{1}{4}$ of the product that is to be multiplied, then for 2 s. take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, *&c.*

Item, for 8 s. according to reason, and the intent of the *Table*, for the first 4 s. take the $\frac{1}{5}$ of the product, and the same number again for the other 4 s. and adde them together.

Item, for 9 shillings : first for 5 shillings, take *the* $\frac{1}{4}$, then for 4 shillings take the $\frac{1}{5}$: and adde them together.

Otherwise, as you see by the intent of the Table, work twice for 9 shillings as was taught even now for 8 : and then take the $\frac{1}{4}$ of the last product for the 1 shilling : but 5 and 4 is the shorter.

Item, for 11 s. first dispatch 10 s. for which you must take the $\frac{1}{2}$ of the product : then lastly, for 1 shilling take the $\frac{1}{10}$ part of the summe produced of the $\frac{1}{2}$ of the product, and adde them together.

Item, for 12 shillings, where I will end with the first part of my Table. For take the $\frac{1}{2}$ for 10 *shillings.* And

And then for 2 *shillings*, take the $\frac{1}{5}$ of the summe that came of 10 *shillings*, take and adde them together, or else if you please for 2 *shillings* you may take the $\frac{1}{10}$ of the whole given number.

To write more of the manner of taking the true parts, I omit. The desirous practitioners will (no doubt) conceive it. Also the *Table* is some aid to help the unperfect, whereupon by and by I will set down three or four of these notes in *Examples*, and the rest I will leave to thine own industry and practice, to labour upon.

This is the order most commonly used in practice, when the number of shillings is not an *aliquot* part of a pound. But (*loving Reader*) after I have touched the even or *aliquot* parts of a pound that falleth out in pence and shillings, I will deliver two new Rules that shall drown this common order quite and clean: wherein shall be comprehended in one line, or working both of even and odde parts of shillings under 20 without regard whether it be an *aliquot*, or not an *aliquot* part; which two Rules (when they come in place) I commit to thy friendly judgement in working.

Now follow the examples upon the notes aforesaid.

At 6 shillings the yard, what	3215 yards?	
4 shillings	643	
2 shillings	321	10
li.	964	10 s.

Other-

Otherwise by Multiplication of 6.

3215
6 shillings 1929|0
li ———— 964 —— 10 *shillings.*

At 7 shillings the Ell, what 4563 Ells ?
5 shillings 1140 —— 15
2 shillings 456 —— 6
li. 1597 —— 1 shilling.

Otherwise by Multiplication of 7.

4563
7 s 3194|1
1597 —— 1

At 8 s. the piece, what 7563 pieces ?
4 s 1512 —— 12
4 s 1512 —— 12
pounds 3025 —— 4 s.

Otherwise by Multiplication of 8.

7563
8 s 6050|4
pounds 3025 ———— 4 shillings.

At 13 s. the piece, what 401 pieces ?
10 s 200 —— 10
2 s 40 —— 2
1 s 20 —— 1
pounds 260 —— 13

Other-

Otherwise by Multiplication.

	401
13 s.	1203
	401
	5213
pounds	260——13 s.

These and such like questions of compound number, which I have here in this fourth rule for orders sake set down, for that it hath been heretofore a common course of work, I account but superfluous. For in the eight and nineth Rules of this my simple Addition shall appear, that the given price or any even or odde number of shillings, either under or above 20 shall be wrought at one or two workings at the most, how difficult soever the question be.

3 Rule. To reduce pence into pounds at one operation.

Item, *there resteth yet a kinde of practise, how to bring pence into pounds at the first working, whereupon you must understand that 240 pence maketh one pound, or 20 s. In consideration whereof I cut off the last figure or 0, and there remaineth but 24 (of which 24) 8 d. is the* $\frac{1}{3}$ *part thereof, 6 d. is the* $\frac{1}{4}$ *part, 4 d. is the* $\frac{1}{6}$ *part, and 2 pence is the* $\frac{1}{12}$ *part thereof.*

Whereupon if it were demanded what 1486 yards; or pounds of any thing commeth to, at 8 pence the yard; in pricking or cutting off the first figure towards your right hand, for the 0 that appertaineth 240. There is remaining of the said summe 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 li. and there resteth 1, which 1 I put to the 6. that I prick or cut off, and it maketh 16 pieces of 8 pence which I double to make into groats, & they make 32, whereof

whereof the $\frac{1}{2}$ part maketh 10 s. and there remaineth $\frac{2}{3}$ s. which is 8 d, whereby it followeth, that the 1486 yards at 8 pence the yard maketh 49 li. 10 s. 8 d. as by the example shall appear.

Item, for 6 d. take the $\frac{1}{4}$ part of the number from the prickt figure; and if any unites remain, they are so many six pences, whereof taking the $\frac{1}{2}$ they are shillings, if there doth remain yet one, it is in value six pence.

Item, for 4 d. take the $\frac{1}{3}$ part of the number from the prickt figure; if any unites doe remain, they are so many groats, which to convert into shillings, take the $\frac{1}{3}$ part. And if any yet remain, they are thirds of shillings, each one in value being worth 4 pence.

Item, for 3 pence take the $\frac{1}{4}$ part from the prickt figure, if any unites remain, they are so many pieces of 3 pence, whereof in taking the $\frac{1}{4}$ part, maketh shillings: if any thing yet remain, they are the fourth parts of shillings, each one being in value 3 pence,

Item, for 2 pence: as appeareth also by the Table, take the $\frac{1}{6}$ part of the number from the prickt figure: if any thing remain, they are so many pieces of 2 pence, which by taking the $\frac{1}{6}$ part, you shall turne into shillings, and if any unites remain, they are so many 6 parts of shillings, or pieces of two pence, whether you will.

If one cost 8 pence, what	1486 ?
maketh pounds	49 — 10 — 8d.
If one cost 6 pence, what	7865 ?
maketh pounds	196 — 12 - 6 d.

At

At 4 pence the yard, what	8736 yards?
maketh pounds.	145--12--0 d.
If one cost 3 pence, what	9874 worth?
maketh pounds	123--8--6d.
At 2 d, the Ell, what	7894 Ells to?
maketh pounds	65--15--8d.

6 Rule. *BUt if your number of pence be not an aliquot or even part of 24, then must you bring them into the aliquot parts of 24. and make thereof diverse products which must be added together, as by the question hereafter following shall appear.*

Item, for 5 d. first take for 3 d. then for 2 d. and adde them together, according to the instruction of the *second Rule*: or else first take for 4 d, then 1 d.

Item, for 7 d. first take for 4 d. then for 3 d. and adde them together.

Item, for 9 d. first take for 6 d. then for 3 d. and adde them together.

Item, for 10 d. first take for 6 d. then for 4 d. and adde them together.

Item, for 11 d. first take for 8 d. then for 3 d. and adde them together: as by these Examples.

Examples.

1 If one yard cost 5 d. what	7591|6
4 pence	126--12
1	31--13
maketh pounds	158--5

Other-

Otherwise.

1——5——759|6

3 pence 94——19

2 pence 63——6

maketh pounds 158——5 s.

2 If one cost 7 d. what 98|7 worth?

4 pence 16——9

3 pence 12——6——9

maketh pounds 28——15——9d.

Otherwise.

1————7——98|7

6 pence 24——13——6

1 peny 4——2——3

maketh pounds 28——15——9d.

3 If one cost 9 d, what 98|7 worth?

6 pence 24——13——6

3 pence 12——6——9

maketh pounds 37——0——3d.

Otherwise,

1——9——98|7?

6 pence 24——13——6

3 pence 12——6——9

maketh pounds 37——00——3

4 If one cost 10 pence, what 98|7?

6 pence 24——13——6

4 pence 16——9——0

maketh pounds 41——2——6

5 If

5 If one cost 11 pence, what 9817?

8 pence	32 — 18 -- 0
3 pence	12 — 6 -- 9
maketh pounds	45 — 6 -- 9

But if you haue any shillings and pence to be multiplied together, then are you to take for the shillings according to the instruction of the third Rule: and for the pence according to the first Rule before mentioned: unlesse you can spie the advantage thereof, and thereby helpe your selfe; as appeareth in this second example, where first I worke for 6 d. which is to be rebated out of the given number, and I have 719 li. 11 s. my desire.

At 19 s. 6d. the yard, what 738 yards?

	738		*Otherwise by Rebating.*
10 s.	369 — 0		
5 s.	184 — 10		738
4 s.	147 — 12	6 d.	18 — 9 s.
6 d.	18 — 9	li — 7 — 19 — 11 s.	
pounds	719 — 11 s.		

The like again is done by Rebating, as by these two examples appeareth.

At 18 s. the Ell, what 418 Ells?

2 s.	41 — 16
pounds	376 — 4 s.

At 16 s. the Ell, what 517 Ells?

4 s.	103 — 8
pounds	413 — 12 s.

And

And now I will touch a little the even part of a pound, that falleth out in pence and ſhillings, whereof for thoſe parts you ſhall take ſuch like parts out of the given number that is to be multiplied, as the price of that given number beareth in proportion to a pound, which alſo for their better aid is here ſet down.

s.	d.			
1 s.	8 d.	*is the*	$\frac{1}{12}$	*part of a pound.*
2	6		$\frac{1}{8}$	
3	4		$\frac{1}{6}$	
6	8		$\frac{1}{3}$	

Item, firſt for 1 ſhilling 8 pence take the $\frac{1}{12}$ part of the given number, and if any thing do remain they are twelve parts of a pound, each one being in value 1 ſhilling 8 pence.

Item, for 2 ſhillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplied; and if any thing doe remaine, they are eight parts of 1 pound, each one being in value 2 ſhillings ſix pence.

Item, for 3 ſhillings 4 pence, as appeareth by the Table, you muſt take the $\frac{1}{6}$ part of the given number, and if any thing do remain, they are 6 parts of a pound, each one being in value 3 ſhillings 4 pence.

Item, for 6 ſhillings 8 pence take the $\frac{1}{3}$ part of the number that is to be multiplied: And if any unites doe remaine, they are thirds of a pound, every one being worth 6 ſhillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreviation into the proportionate parts of a *pound*, as 16 ſhillings 8 pence maketh $\frac{5}{6}$: which 16 ſhillings 8 pence is eaſily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats.

Then set 60 the groats of a pound under 50: cutting off the the two Cyphers as is here performed.

16—8
3
5|0
6|0

And then have you brought 16 *shillings* 8 pence into the knowne parts of a *pound*, which maketh

But yet gentle Reader, for thy further instruction, I have hereunto annexed in a *Table*, how pence and shillings bear proportion to a pound, which I commit to thy friendly benevolence; it will be some aid unto the ungrounded Practitioner: but I count him the best workman that can presently reduce his given price into the known and proportionate parts of a pound.

A Table of the Aliquot parts of a pound or 20 shillings.

s.	d.	l.	s.	d.	l.
0	2	1/120	8	4	5/12
0	3	1/80	8	9	7/16
0	4	1/60	9	0	9/20
0	6	1/40	10	0	1/2
0	8	1/30	11	0	11/20
1	0	1/20	11	3	9/16
1	3	1/16	12		3/5
1	8	1/12	12	6	5/8
2	0	1/10	13	0	13/20
2	6	1/8	13	4	2/3
3	0	3/20	13	9	11/16
3	4	1/6	14	0	7/10
3	9	3/16	15	0	3/4
4	0	1/5	16	0	4/5
5	0	1/4	16	8	5/6
6	0	3/10	17	0	17/20
6	6	13/40	17	6	7/8
6	8	1/3	18	0	9/10
7	0	7/20	18	4	11/12
7	6	3/8	18	9	15/16
8	0	2/5	19	0	19/20

Here follow four examples upon the four Notes delivered.

At 1 s. 8 d. the yard, what 3884 yards?

maketh pounds 323 — 13 — 4d.

At 2 s. 6 d. the yard, what 4563 yards?

maketh pounds 570 — 7 — 6d.

At 6 s. 8 d. the Ell, what 7562 Ells?

maketh pounds 2520 — 13 — 4d.

Now by custome you are able to work by all sorts of summes being delivered in shillings and pence, as one shillings one peny, two shillings two pence three shillings three pence, and so of all other: wishing you to have some consideration of your questions, when they are set down, for there are many subtile abbreuiations, and great advantages to be gotten, and easily to be perceived.

As of 3 s. ———— 8 d. of 2 s. and 1 s. 8 d.
Of 4 s, ———— 2d. of 3 s. ———— 4d. & 10 d. which 10 d. is $\frac{1}{4}$ of 3 s ———— 4d.
Of 5 s. ———— 8 d. of 4 s. 1 s. ———— 4d.
Of 5 s. 10 d. of 5 s. and 10 d. which 10 d. is $\frac{1}{6}$ of 5 s.

And by this mean when you have taken one product, you may oftentimes upon the same take another more briefly then upon the summe which is to be multiplied &c.

8 Rule.

NOw (Gentle Reader) that you have seen the vertue of the even or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise hereafter followeth

followeth a briefe and easier method for any even number of shillings, either under or above 20, then ever yet hath been published; Notwithstanding M. Humphrey Baker, *whose travell is worthy commendation, and whom for knowledge sake I reverence, hath in some part touched this first part, though not in this method. The work of the Rule both pleasant, ready, and brief, as by the variety of the examples delivered thereupon shall appear. And first I will set forth a question, thereby the better to expresse or teach you the order thereof: which is this.*

If one cost 6 s. what 8574?

1	6 s.	8574
maketh pounds		2572 —— 4 s.

To the understanding of this example, after you have set down your given number in form of the Rule of 3, with a line drawn under it, you shall presently set a prick under your first figure 4 toward your right hand drawing from the prick, as heretofore hath been practised, a little short line, thereto set down the shillings anon, which done, multiply the first figure 4 by 6, the value of your price; (which here you see standeth in sight above the line) it maketh 24, which is one pound foure shillings. The one pound keep to carry to the next place, and the foure shillings set down at the end of the prescribed line towards your right hand. Thus have you done now with 6 above the line, and also with 4 in the first place (for the prick under 4 doth signifie that 4 hath done his office.) Then secondarily for a generall Rule take but the $\frac{1}{2}$ of the given price, which here is the number that shall now continue the rest of the multiplication, and end the work, whereupon I multiply

M. *Iohn Mellis* his first Rule.

Note a generall rule

3 into 7, ſtanding in the ſecond place it maketh 21, and with the 1 pound I kept in 22; ſet down 2 and keep 2 in mind, working according to the Rule of multiplication, delivering the tenths in minde in their due place, which done, the product from the prick to your left hand repreſenteth the pounds, and the other at the end of the ſhillings, as appeareth by the examples.

If one yard coſt 2 s. what		7536?
1	2 s.	7536
maketh pounds		753 —— 12 s.
If one yard coſt 4 s. what		8792?
1	4 s.	8792
maketh pounds		1758 —— 8 s.
If one piece coſt 6 s. what		9537?
1	6 s.	9537
maketh pounds		2861 —— 2 s.
If one piece coſt 8 s. what		7509?
1	8 s.	7509
maketh pounds		3003 —— 12 s.
If one coſt 12 s. what		5794?
1	12 s.	5794
maketh pounds		3476 —— 8 s.
If one coſt 14 s. what		3705?
1	14 s.	3705
maketh pounds		2593 —— 10 s.
If one coſt 18 s. what		5703?
1	18 s.	5703
maketh pounds		5132 —— 14 s.

If

If one cost 22 s. what		953 ?
1	22 s.	953
maketh pounds		1048 —— 6 s.

Let these suffice (gentle Reader) for an entrance into even numbers. And now I will shew the like rule for any odde or uneven part of a pound.

To help you to the understanding of these other questions that hereafter follow: where in my first Example the given number is 6487 at 3 s. the yard: I multiply 3 above the line into 7, it maketh 21. The one shilling is set down, and the 1 pound I keep. Now am I to take the ½ of three, which because it is an odde number I cannot.

Therefore I shall keep and continue my multiplication by three still, and work by the ½ of the rest of the given figures or number, to wit, 648. And first the ½ of 8 which is 4 multiplied into 3, maketh 12, there to joyn the 1 li. in minde, it maketh 13. set down 3, keep one. Then again multiply by two the ½ of four it maketh six, and with one in minde it maketh 7. Then lastly, take the ½ of six, which is 3. saying, 3 times 3 is 9, which 9 set down, and so is the question answered as appeareth by the practice, and examples following. Mr. John Mellis his second Rule.

At 3 s. the yard, what		6487 ?
1	3 s.	6487
maketh pounds		973 ——— 1 s.

If one yard cost	5 s. what	4269 ?
1	5 s.	4269
maketh pounds		1067 ——— 5 s.

At 7 s. the Ell, what	6489 ?	
1 7 s.	6489	
maketh pounds	2271	3s.
If one Ell cost 9 s. what	2807 ?	
1 9 s.	2807	
maketh pounds.	1263	3s.
At 11 s. the Pistolet, what	8263 ?	
1 11 s.	8263	
maketh pounds	4544 ——	13 s.
If one piece cost 13 s. what	4629 ?	
1 13 s.	4629	
maketh pounds	3008 ——	17 s.

But now note (gentle Reader) when the given price falleth upon an odde number. as 3, 5, 7, 11, 13, &c, then it is to be presupposed that the given summe to be multiplyed, must be a sum made of even numbers, 2, 4 6, 8, 10, &c. else cannot the question be wrought at one line or working.

Providing alwayes that it may bear an odd figure in the first place towards your right hand, as appeareth in these six examples, which last were wrought, and such like, &c. which may beare an odde number for the price, and be done at one line or working very well.

But if the given price be an odde number, and the sum to be multiplyed odde numbers also: then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall Rule. First, worke for the even number, contained in that question, or given price accor-

A generall Rule.

ding

ding as you have learned, and then afterwards for the one odde shilling, take the $\frac{1}{2}$ of the summe given to be multiplied, omitting the first prickt place, as was taught for the working of one shilling in my first Rule of Practice, and adde those two together, and you shall have your desire.

Examples.

At 3 s. the yeard, what	7539 yards?	
2s.	753	18
1s.	376	19
maketh pounds.	1130	17 s.

At 7 s. the Ell, what	7539?	
5 s.	7539	
	2261	14
2 s.	376	19
maketh pounds	6238	13

At 13 s. the yard, what	7534?	
10 s.	3767.	0
2 s.	753.	8
1 s.	376.	14
maketh pounds	4897	2

And thus have I abridged into these two rules how to bring any nūber of shillings whatsoever they be, into pounds, with a briefer Method, then ever yet hath been published which I commend unto thy friendly censure and judgement in the use and practice thereof. Note this well..

If

If one cost 6s. 5d. what	1231 ?
6s.	369 — 6
4d.	20 — 10 — 4
1d.	5 — 2 — 7
maketh pounds	394 — 18 — 11
At 14s. 2 d. what	2825 ?
14s.	1977 — 10
2d.	23 — 10 — 10
maketh pounds	2001 — 0 — 10
At 16s. 4d. what	2531 ?
16s.	2024 — 16
4d.	24 — 3 — 8d.
maketh pounds	2066 — 19 — 8
At 3s. the Pistolet, what	8325 ?
maketh pounds	1248 — 15s
At 7s. the Crown, what	6529 ?
	2284 — 3s.
At 9s. the piece, what	6567 ?
maketh pounds.	2955 — 3s.

These three last questions may seeme something harder, yet they are easie enough, if you mark them well, If I should explain them, then are they too easie. Therefore I leave them to whet the minds of the desirous.

10 Rule. ITem *when any one of the summes, which is to be multiplied, is composed of many Denominations, and the given number but of one figure alone; then shall you multiply all the Denominations of the other summe*

summe by the same one figure, beginning first with that summe which is least in value toward your right hand, and bring the product of those pence into shillings, and the product of the shillings into pounds, as by this example appeareth.

At 3 li. 7s. 4d. a yard, what are 9 worth?
maketh pounds. 30--6s-0d.

B*Ut if any of the summes that are to be multiplied, there be a broken number; First worke for the whole according to the instructions that you have learned, and then take such part of the given price, as that broken number beareth in proportion to the price, as in the examples following. After you have wrought for 3s. & for 6d. then are you to take the ½ of 3s. 6d. for the ½ yard, and adde that to the summe: So adding all the 3 products together, which make 43 li. 2s. 9d. the just price of 245 ½ Ells, and thus must you doe of all other.* 11 Rule.

At 3s. 6d. the Ell, what	245 ½?
3 s.	36——18
6d.	6——3
½	1—9
maketh	43—9--5¾

At 16s. 4d. the piece, what	14 ¾?
16s.	11——4
4d.	0—4—8
¾	12——3
maketh pounds	12——0—11

If

If one peice cost 4 li. 3 s. $6\frac{1}{2}$ d. what 12 pieces?

4 li.	48		
3 s.	1	16	
6 d.		6	6
$\frac{1}{2}$			
maketh pounds	50	2	6

The proofe.

If 12 pieces cost 50 li. 2 s. 6 d. what one piece?
maketh pounds 4 —— 3 —— $6\frac{1}{2}$

14 Rule. Item, *touching the manner how to understand the order of this question, and others the like, first seek how many times* 12 *is contained in* 50, *which is* 4 *times, and so resteth* 2 *Pound, which* 2 *Pound converted into shillings, and joyned with the other* 2 *shillings, maketh* 42 *shillings: wherein is found* 12 *three times resteth* 6 s. *which turned into pence, putting thereto the* 6 *pence in the first place, it maketh* 78, *wherein* 12 *is found sixe times, resteth* 6 d. *which containeth* 12, *but* $\frac{1}{2}$ *a time, put that* $\frac{1}{2}$ *to the* 6d. *and then the solution is* 4li. 3s. $6\frac{1}{2}$d. *as appeareth by the Practice thereof.*

15 Rule. Item. *The like is to be done of any thing that is bought or sould after five score to the hundred, or the Quintall. As for example.*

If 100 pound cost 27 li. 13 s. 4 d. what one pound?

27li. — 13s. — 4d.
2

s. 5 | 53
| 12
1 | 10
5 | 3
d. 6 | 4 | 0
| 10 0. or $\frac{2}{5}$

Maketh 5s. 6d.

I have wrought this at le͂gth for the aid of the young learner, because he should understand how all the multiplication is set down.

But to work it more neatly, it is by a little understanding, ended thus.

27 li. — 13s. — 4d.
20
s, 5 | 53
| 12
d. 6 | 40
| 100

Maketh 5 s. $6\frac{2}{5}$ d.

Item *to the understanding of this & such like questions, the right down line is all the guide, which is pulled down close by* 20. *as you see in the example, where* 27li. 13s. *is reduced all into* s. *& maketh* 553 s.

The 5 *towards the left hand being separated with the hanging or right down line, is the just number of shillings, that answereth the question. Nextly,* 53s. *is multiplied by* 12, *to reduce them to pence, putting to the* 4d. *it yeeldeth for the multiplication of the first figure two* 110. *the one beyond the line towards the left hand, is* 1 *penny towards the rest of the price: then* 53 *also multiplied by* 1 *yeeldeth* 53: *but the* 5 *behinde the line towards the left hand, is also* 5 *pence more, towards the price, which* 1 *and* 5 *I adde together under the line, it maketh* 6 d. *So is there found now as appeareth by the Titles of shillings and pence,* 5 s. 6 *pence.*

Finally. I come now on this side the line towards the

the right hand, and under 12 *I finde first* 10, *and then* 3, *which added together, maketh* 40 . *under which* 40, *you must put the* 100, *and it maketh* $\frac{40}{100}$ *which abbreviated, commeth to* $\frac{2}{5}$. *So the just price of one pound after* 5 *score to the hundred, maketh* 5 s. 6 $\frac{2}{5}$ d. One example more, and so I will leave this Rule.

If 100 cost 10 $\frac{3}{4}$d. what 9874?

6d	246	17	
4d	164	11	4
$\frac{2}{4}$d	20	11	5
$\frac{2}{4}$d	10	5	$8\frac{1}{2}$

	li.	4	42	5	parts of a peny, $\frac{91}{100}$
			20		
Maketh	s.	8	45		
			12		
			45 $\frac{1}{2}$ 91		
	d.	5	100 | 100		

Also the like may bee done of the usuall weights here in *England*, (which is 112, for every hundred weight) in case you know the aliquot parts of a hundred weight, which are these, 56 li. 28 li. 14 li. and 7 li. For 56 li. is the $\frac{1}{2}$ of 112 li. 28 li. is the $\frac{1}{4}$ of 112 li. 14 li. is the $\frac{1}{8}$ and 7 li. is $\frac{1}{16}$ part.

Therefore for 56 li. take the $\frac{1}{2}$ of the summe of the money that 112 li. weight is worth.

For 28 li. take the $\frac{1}{4}$ of the summe of money that 112 li. weight is worth.

For 14 li. take the $\frac{1}{8}$ of the summe that 112 li. is worth.

And for 7li. the $\frac{1}{16}$ of the summe of money that 112 li. is worth.

As

As for example; at 17 li. 19s. the hundreth pounds weight, that is to say, the 112 li. what shall 3 quarters and 7 pound cost?

1. C.	17 li.	19 s.	3 q;	7 li.
2 quarterns	8	19	6	
1 quartern	4	9	9	
7 pounds	1	2	$5\frac{1}{4}$	
Maketh pounds	14	11	$8\frac{1}{4}$	

The second Chapter treateth of the Reduction of divers measures to others value by Rules of Practice.

Now will I shew a few examples of practice in reducing of Measures, as Ells, Yards, Braces, Pawnes of Genee, &c. *Much more would I have touched but that I feare the Booke will rise to too great a Volume.* 18 Rule

In 864 Ells of *Antwerpe*, how many yards of *London*?

864	864
432	216
216	648

maketh 648 yards of *London*.

ITem, *in these and such like questions of* Flemmish *measure, to be brought into yards* English, *first take the* ½ *of the given number as appeareth in the first example*

ample towards your left hand. Then take halfe of that product, or the $\frac{1}{4}$ *of the given number, and adde these two products together, as they shall be yards* English; *as by the example you may perceive.*

The second example toward your right hand is yet briefer then the first, whose worke is this; Take the $\frac{1}{4}$ *of the delivered number, and that product subtract out of the given number, aud the rest sheweth your desire. Of these two wayes use which you thinke best.*

The Proof.

In 648 yards *London*,
How many Ells of *Antwerpe?*

	648	
	216	
maketh	864	Ells of *Antwerpe.*

15 Rule. ITem, *for the understanding of this worke, first take the* $\frac{1}{3}$ *part of the yards of* London, *which found addde that* $\frac{1}{3}$ *part and the yards together, as appeareth by the Practice, and the product sheweth the Ells of* Antwerpe.

Item, in 20 yards of *London*,
How many Ells of *Antwerpe?*

maketh	$426\frac{2}{3}$ Ells,		
	320 yards		Proof,
	$106\frac{2}{3}$		$426\frac{2}{3}$ Ells,
	$426\frac{2}{3}$ Ells	$\frac{2}{3}$	$106\frac{2}{3}$
			320 yards,

Other Reductions,

16 Rule. ITem, *you shall understand, that forasmuch as six braces of* Millain, *make five Ells of* Antwerpe, *whereupon according to the* Rules of Practice, *you may reduce the one into the other, by the like reason aforesaid,*

in

in taking the $\frac{1}{6}$ part & then subtract the same, to make Ells of Antwerpe. *And again by the contrary taking the $\frac{1}{5}$ part with adding the given number, to turn the Ells to Braces. As for example.*

In 876 Braces, how many Ells of *Antwerpe*?

876	The contrary.
146	730 Ells *Flemmish.*
Ells 730 *Antwerpe.*	146 Braces.

Ells 730 *Antwerpe.*

182$\frac{1}{2}$

Yards 547$\frac{1}{2}$ *English.*

Thus appeareth that 876 Braces by Practice make 730 Ells *Flemmish*, which Ells *Flemmish* reduce into *English yards.*

So again upon the same first question of Braces, I would know how many yards *English* they make.

After the rate that 100 Braces are worth 62$\frac{1}{2}$ yards.

876 Braces.

438

109$\frac{1}{2}$

I answer, 547$\frac{1}{2}$ yards.

ITem, *To the understanding of this work, and such like, first take the $\frac{1}{2}$ of the given Braces, and after take the $\frac{1}{4}$ of that halfe, or the $\frac{1}{8}$ of the given number, and adde them together, and the Products are also yards* English.

ITem, *three Ells of* Rochell *make* 5 *Ells at* Lisbone. *So likewise three Ells at* Lions *make* 5 *Ells at* Antwerpe.

To worke these & such like, double the Ells of Lions, *and the Ells of* Rochell, *and from their Product subtract the* $\frac{11}{21}$ *and the rest shall be the Ells of* Antwerpe, *or the Ells of* Lisbone.

I *Example.*

In 63 Ells of *Lions*, how many Ells of *Antwerpe?*

63
63
126
$\frac{1}{2}$ 21

Ans. 105 Ells *Ant.*

In 100 Ells of *Rochel*, how many Ells of *Lisbone?*

110
200
$\frac{1}{2}$ 23 $\frac{2}{3}$.

Ans. 166 $\frac{2}{3}$ Ells of *Lisb.*

Touching the proofe or return of these and such like questions for a generall Rule, you shall first take the $\frac{1}{5}$ of the given number: and adde that $\frac{1}{5}$ and the given number together, and the $\frac{1}{2}$ of that product shall be your desire.

Example.

In 105 Ells of *Antwerpe*, how many Ells of *Lions?*

105
$\frac{1}{5}$) 21
$\frac{1}{2}$) 126

Ans. 63 Ells of *Lions.*

In 166 $\frac{2}{3}$ Ells of *Lisbone*, how many Ells of *Rochell?*

166 $\frac{2}{3}$
$\frac{1}{5}$) 33 $\frac{1}{3}$
$\frac{1}{2}$) 200

Ans. 100 of *Roch.*

Questions

Questions of Factoridge and Interest, briefe and truly resolved by the Rule of Practice without Time.

1 *Question.*

AT 5 shillings *per Centum*, what comes 8860 li. 15 s. 4d. unto ?

Answer. Note that 5s. is the fourth of 20. s. I take the $\frac{1}{4}$ part of 8860 li. 15s. 4d, which makes 2215 li. 3s. 10d. Now the Root is 100, which you should divide by, so cutting the 2 last figures away of the pounds, you have 22 li. then multiply 15 li. by 23s, so adde the 3 unto, you shall have 303s. cut away the two last figures, there resteth 03s. Lastly, there remains 3s. which I multiply by 12 to bring into pence, and so I finde od. and $\frac{46}{100}$ remaining, which being abbreviated make $\frac{23}{50}$ parts of a peny, so I find that there is gained 22 li. 3s. od $\frac{23}{50}$ parts of a peny.

li. 22. 15. 03. 10
20
s. 3 | 30
12
0
o d. | 4
4 23
100|50

2. *Quest*. At 10 s *per centum*, what comes 1448 li. 16s. 8d. unto ?

Answer. Note that 10 s. is the $\frac{1}{2}$ of 20s. I take the $\frac{1}{2}$ 1448 li. 16s. 8d. which makes 724 li. 8s. 4d. cut off the two last figures, and there resteth 7 li. then multiply the 24 li. by 20s. and adde the 8s. and it maketh 488s. cut

li. 7. | 24. 8. 4.
20.
s. 488
12
180
88
d. 10|6 | 0 | 5
100 | 3

the

the two last figures off, and there resteth 4s. then multiply 88s. by 12 d. and take in 4d. and there resteth 1060d. cut off the two last figu es, and there resteth 10d. and $\frac{60}{100}$ which is $\frac{3}{5}$ of a peny : so the whole summe is 7 li. 4s. 10 $\frac{3}{5}$ which is the answer to the question.

3. *Quest.* At 15 shillings per centum, what comes,

1008 l. 12 s. 0d. unto?

Answer. Note 15. that is $\frac{1}{2}$ and $\frac{1}{4}$ of 20. take the $\frac{1}{2}$ of 1008 lib. 12 shillings, there resteth 504 li. 6s. then take the $\frac{1}{4}$ adde them together, the totall will be 756 li. 9s. cut off the 2 last figures, resteth 7 li. then multiply by 20 s. & take in your 9s. it maketh 1129 s. cut off the two last figures, there resteth 11s. then multiply by 12 d. there commeth 348d. cut off the last two figures, there resteth 3d. and $\frac{48}{100}$ which being abbreviated maketh $\frac{12}{25}$ parts of a peny, so shall you find 7 li. 11s. 3d. $\frac{12}{25}$. which is the answer to the question.

```
      1008 l.12s.0d.unto?
       504.6 ——— 0
       252.3 ——— 0
    ------------------
    7 | 56.9 ——— 0
      | 20
      -----
   11 | 29
      | 12
      -----
       58
       29
    ------------------
     3|48|24|12
     100|50|25
```

li. 8|68 li. 13s. 4d. unto?

4. *Quest.* At 1 li. *per centum*, what comes

Answer. Cut away the two last figures, and multiply by 20, and 12, and take in your shillings and pence. And you shall finde 8 li. 13 s. 8 pence $\frac{4}{5}$ as doth appeare by this work.

```
li. 8|68 li. 13s.4d.
     |           unto?
     | 20
s. 13| 73
     | 12
     ------
      150
      73
    ----------------
d. 88 |0 |4|
     10|0 |5|
```

5 *Quest.*

5. *Quest.* At 2 li. *per centum*, what comes 5608 li. 6s. 8d. unto?

Answer. Multiply the whole summe by two lib. thus, then cut off the two last figures of your pounds, as you did before, and you shall finde 112 pound, then multiply by 20 and by 12, taking in your shillings and pence, and you shall finde 112 li. 3s. 4 d. which is either for Factor or Broker, &c.

2
112|16.|13.4
20
3|33
12
70
33
4|00

6. *Question.* At 3 pound, *per centum*, what comes 800 li. 18s. 2d. unto?

Answer. Multiply the sum by 3 pound, thus; then cut off the two last figures, and you shall finde 24 pound, then multiply by 20, and by 12. taking your shillings and d. and you shall finde 0s. 6 d. $\frac{27}{50}$ parts of a peny, which is something above a half-peny.

3
24|02, 14. 6.
20
0 54
12
1 4
54
6|54 27
100 50

7. *Question.* At foure *per centum*, what comes 298 li. 15s. 9d. unto?

Answer. Multiply by 4 li. thus, cut off the two last figures, Multiply by 20, and by 12. taking in your shillings and pence, and you shall finde 11 li. 19 s. 0d. $\frac{9}{25}$ parts of a peny, which is something above a farthing.

4
11|95.3.0.
20
19|03
12
06
03
036 18 9
100 50. 25

8 *Queſt.* At 5 li. $\frac{3}{4}$ *per centum*, what comes

3658 li.	16s.	8d.
	unto	5
18294	03	4
1829	08	4
914	14	2
210|38	05	10
|20		
7|65		
|12		
130		
66		
7|90		
10,0		

Anſwer, Multiply by 5 li. thus, then take the $\frac{1}{2}$ of the whole ſumme and place the figures even, then take the $\frac{1}{2}$ of that $\frac{1}{2}$ & adde all three ſumms together, cut off the two laſt figures, then multiply by 20. and by 12. taking in your ſhillings, and pence, and you ſhall find 210l. 7 s. 7 d. $\frac{9}{10}$ parts of a peny, which is the anſwer to the queſtion.

9 *Queſt.* At 6 l. $\frac{1}{2}$ *per centum*, what comes

5684 li.	12s.	6d.
	unto	6
34107	15	0
2842	06	3
369|50	01	3
|20		
05		
01		
15|3		
100|20		

Anſwer, Multiply by 6li. and then take $\frac{1}{2}$ of the whole ſumme, adde them both together, then multiply by 20, and by 12, taking in your odd ſhillings and pence, and you ſhall find 369 li. 10s. 0d. $\frac{1}{20}$ parts of a peny, which is the anſwer to your queſtion.

10 *Queſt.* At 7 li. $\frac{1}{2}$ *per centum*, what comes

3868 li.	13s.	4d.
unto		7
27080. 13. 4		
1934. 06. 8		
290|150. 0		
|20		
3|00		

Anſwer, Multiply by 7 li. then take the $\frac{1}{2}$ adde them together, cut off the two laſt figures, then multiply by 20, you ſhall finde 290 li. 3s. The anſwer to the queſtion.

11 *Queſt.*

11 *Queſt.* At 8 li. *per Centum*, what comes 2560 l. 17 s. 9 d unto?

Anſwer, Multiply 8 li. cut off the two laſt figures, multiply by 20, and by 12, and you ſhall finde 204 li. 17 s. 5 d. $\frac{1}{25}$ parts of a peny.

```
    2560 l. 17 s. 9 d unto?
          8
204|87.02.0
   |20
 17|42
   |12
    84
   42
   5|04 | 2| 1
   100  50. 25
```

Queſtions of Intereſt with Time, wrought by Practice.

1 *Queſtion.*

AT 6 *per Centum* what comes unto for 1 month 468 li. 16 s. 8 d?

Anſwer, Multiply by 6 lib. there commeth 2813 li. 00 s. 0 d. then take for 1 moneth the $\frac{1}{12}$ of the Totall, and you ſhall finde 234 li. 8. 4 d. of the two laſt figures of the li. Multiply by 20 & by 12, taking in your odde money, and you ſhall finde 2 li. 6. 10 d. $\frac{2}{5}$ parts of a peny, which is the anſwer to the queſtion.

```
      468 li.  16 s.  8 d?
      2813.    00.    0
li. 2|34.  08. 4
     |20
s. 6 |88
     |12
    180
    88
d. 10|16  |0 |3
      10|0  |5
```

2. *Queſt.* At 7 pound $\frac{1}{2}$ *per centum*, what comes unto for 2 months.

Anſwer. Multiply by 7 pound, then take $\frac{1}{2}$, adde them two together, then for your two months take the $\frac{1}{6}$ of the Totall, multiply by 20 and 12, taking in your odde ſhillings and pence, and you ſhall finde 47 pounds 10 ſhillings 1 peny $\frac{2}{10}$ parts of a peny, which is the anſwer to the queſtion.

```
3800 li. 12s. 8d.
               7
-----------------
26604. 08  8
 1900. 06  4
-----------------
28504. 15  0
  47|50 15 10
    |20
  ----
  10|15
    |12
  ----
    30
   1 6
-----------------
  1|9|0|
-----------------
  10|0|
```

3. *Queſtion*, At 8 pound *per centum*, what comes unto for 3 months.

Anſwer, Multiply by 8 pound, then take for your 3 monthes $\frac{1}{4}$ of the Totall, multiply by 20, and by 12, adding in your odde ſhillings and pence, and you ſhall find 197 pound 5 ſhillings 11 pence, $\frac{3}{5}$ parts of a peny, your demand.

```
9864 li. 16s. 4d.
               8
-----------------
78918.  10  8
-----------------
197|29.  12  8
   |20
   ----
  5|92
   |12
   ----
   192
    93
-----------------
 11|12 |6|3
-----------------
  100|50|25
```

4 *Queſt.*

4 *Queſt*. At 6 pound $\frac{1}{2}$ *per centum*, what comes unto for 4 months.

Anſwer, Multiply by 6 li. Then take $\frac{1}{2}$, adde both together, then for your 4 months take $\frac{1}{3}$ part of the whole, cut away your two laſt figures, multiply by 20, and by 12, adde in your odde ſhillings, and pence, and you ſhall finde 131 pounds, 14 ſhillings 11 pence $\frac{12}{50}$ parts of a peny, your demand.

```
6080 li. 13s. 0d.
             6
36483 | 18 0
 3040 | 06 6
39524   04 6
131 | 74    14. 10
    | 20
 14 | 94
    | 12
     188
      94
 11 | 38 |19
    |100 |50
```

5 *Queſtion*, At 8 *per centum*, what comes unto for 5 months.

Anſwer, Multiply by 8 li. then for 5 months take $\frac{1}{4}$ and $\frac{1}{6}$ of the Totall, cut off the 2 laſt figures of your pounds, Multiply by 20 and by 12, adde in your odde ſhillings and pence, and you ſhall find 100 pound 13 ſhillings 4 pence, your demand.

```
3020 li. 00s. 00d.
               8
24160  00  00
 6040  00  00
 4026  13  04
10 | 66  13 04
   | 20
13 | 33
   | 12
    70
    33
   4100
```

6 *Queſt*.

6 *Queſtions*, At 8 *per Centum*, what comes unto for 6 months.

Anſwer, Multiply by 8 li. then for your 6 moneths take the $\frac{1}{2}$ of the Totall, cut off the two laſt figures of your pounds. Multiply by 12. taking in your odde ſhillings and pence, and you ſhall finde 322 lib. 8s. 5d. $\frac{12}{25}$ parts of a peny, your deſire.

8060 li. 12s. od.
8
64484. 16s.
322|42. 08. 0
|20
8|48
|12
|96
|48
5|$\frac{76}{100}$|$\frac{38}{5}$|$\frac{19}{25}$

7 *Queſt*. At 8 li. *per Centum*, what comes unto for 7 moneths.

Anſwer, Multiply by 8 li. then for your 7 months take $\frac{1}{3}$ and $\frac{1}{4}$ of the Totall, cut off the the two laſt figures of your pounds, then multiply by 20 and 12, taking in your odde mony, and you ſhall finde 275 li. 2s. 11 d. $\frac{1}{5}$ your deſire.

5896. 00 od.
8
47168 0 0
15722.0.0
11792.13.4
275|14.13.4.
|20
2|93
|12
190
93
11|2 |0 |1
10|0 |5

8 *Queſt*.

8 *Question*, At 8 *per centum*, what comes unto for 8 months.

Answer, Multiply by 8 li. then for 8 months take $\frac{2}{3}$ of the totall, cut off the two last figures of your pounds, then multiply by 20, and by 12 adde in your odde money, and you shall finde 116 li. 5 shillings, 9 pence, $\frac{3}{25}$ your desire.

3680 li. 08s. 0d.
8
29443. 04 0
9814. 08 0
9814. 08 0
116|28 16 0
20
5|76
12
152
76
9|12 16 |3 1
100|50|25|

9 *Quest*. At 8 li, *per centum*, what comes unto for 9 months.

Answer, Multiply by 8 pound, then for your nine moneths take $\frac{1}{2}$ and $\frac{1}{4}$ of the whole summe, cut off the two last figures of the pounds, then multiply by 29, and by 12. taking in your odd shillings and pence, and you shall find 221 pounds, 1 shilling 11 pence, $\frac{7}{25}$ which is something above a farthing.

3684 li. 19s. 0d.
8
29479, 12 0
14739. 16. 0
7369. 18. 0
221|09. 14 0
20
1|94
12
188
94
11|28 |14|7
100|50|25

10 *Quest*.

10 *Quest*. At $6\frac{1}{4}$. *per centum*, what comes unto for 10 months.

Answer, Multiply by 6 pound, then take the $\frac{1}{2}$ and $\frac{1}{4}$ of 100 pound, adde all three summes together, then for the 10 months take $\frac{1}{2}$ & $\frac{1}{3}$ of the Totall, add them together, cut off the two last figures of the pounds, multiply by 20, and 12, adding in your shillings and pence, cutting off the last figures of your shillings and pence, you shall finde 5 pound 12 shillings, 6 pence, your desire.

100. li.	0s.	0d.
		6
600.	0	0
50.	0	0
25.	0	0
675.	0	0
337.	10	0
225.	00	0
5|62.	10.	0.
|20.		
12|50		
|12		
100		
50		
6|00		

11 *Quest*. At 8 pound *per centum*, what comes unto for 11 months.

Answer, Multiply by 8 pound, then for 11 months take $\frac{2}{3}$ and $\frac{1}{4}$, from the Totall, adde all three summes together; cut off the two last figures of your pounds. Multiply by 20 and by 12, adding in of your shillings and pence, cutting off the two last figures of your shillings and pence, and you shall finde 65 pound 0 shillings 7 d. $\frac{12}{25}$ parts of a peny, your desire.

886 li.	16s.	0d.
		8
7094	08	0
2364	16	0
2364	16	0
1773	12	0
65|03	04	0
|20		
0|64		
|12		
128		
64		
7|68|34|17		
100|50|25		

12 *Quest*.

12 *Quest.* At 8 pound *per centum*, what comes unto for 12 months.

Answer, Multiply by 8 pound, cut off the two last figures of the pounds, Multiply by 20, and by 12, adding in your shillings and pence, cut off the two last figures of your shillings, and the two last of your pence, and you shall finde 726l. 8s. 11d, $\frac{12}{25}$ parts of a peny your desire.

9080 li. 12s. 2d.
8
726 | 44. 17 4
20
8 | 97
12
11 | 68 34 | 17
1005c | 25

The third Chapter teacheth of the Order and work of the Rule of *Three* in broken numbers after the Trade of *Merchants*, digressing something from Master *Record*, which is comprehended in three Rules.

N*Ow, that I have somewhat intreated of the* Rules of Practise, *I will give a few instructions, after my simple order, for the working of the* Rule of Three *in broken numbers, wherein I shall need to say the lesse, because I hope the studious Learner, that hath travelled any thing in the* Grounds of Arts, *is not unfurnished of knowledge capable to understand me.*

But before I deliver any instructions for broken Numbers, I will propose a question which shall be wrought three sundry wayes, thereby to shew as it were, three Degrees of Comparison : how farre the Rule of Three in broken , for more speed of worke , differeth from the whole. which

I

I rather set downe for a view, that the studious herein may be more desirous to attaine broken, leaving any more to discourse in Dialogue forme, but onely to give instructions where need is: and in the rest to put forth the questions, with their answers.

My first question is thus.

If one yard cost 6s. 8d. what are 789 worth at that rate?

The first way.

1 ——— 6s. ——— 8d. ——— 789
12 80
80 63 120d.

Here the Product of the summe are pence, according to the nature of the middle number.

11
1370 1
63120 (5260(263
122212 1220
111

I answer ——— 263 li.

The second way.

1 ——— $6\frac{2}{3}$ s ——— 789
——— 20
3 20 ———
15780 s.

Here the Product of the summes are shillings according to the nature of the middle number.

1
15780 (5260(263
3333 1220
li.

The third way

1 ——— $\frac{1}{3}$ ——— 789
3 1 1
789

Here

Here the Product is pounds, according to the title of the second number.

1
789 (263
333

I answer, 263 li.

Now that you have seen the three former vertues of the Rule of three; whose Products have first brought forth pence, next shillings, and lastly pounds, I will deliver three notes in order following: and with them a dozen questions, that shall shew the work of the Rule of three in broken Numbers or Fractions.

1 The first foure shall be sundry questions of a Fraction comming in the second place. Note these three well

2 The second foure shall be of two Fractions comming in the second or third place.

3 The third foure of Fractions in all three places.

Note, upon the first Rule for a Fraction comming in the second place.

My first Question is this.

If one yard cost me 3 shillings 4 pence, *what are* 756 *worth at that price?* 1 Rule.

In setting down the question to perform the work, The first I turn foure pence into the part of a shilling, which is variety. $\frac{1}{3}$ and then the question standeth thus:

1 ——— ——— $3\frac{1}{3}$ ——— ——— 756.

To the ready working of this question, and all such other like, my first note is this, which take for a generall Rule; That when any one Fraction shall

come,

come, either in the second or third place, that the Denominator of that Fraction or Fractions, must alwayes be brought unto the Number, or Numerator of the first place; and thereby multiply the one into the other.

And this benefit is alwayes gotten by the vertue of bringing the Denominator of the second Numbers Fractions unto the first place: For the Fraction in the middle number is now released: and the Product that cometh of the Multiplication, is of the nature and like the denomination of the whole number in the second place, which here are shillings.

Note this.

Whereupon now to worke the Question, I bring 3, the Denominator of the Fraction in the second place, unto the first Number 1, with a line set under thus $\frac{1}{}$ and the third under it thus, $\frac{1}{3}$ saying, once 3, is 3 my Divisor: that done, reduce $3\frac{1}{3}$ saying, 3 times 3 is 9, and the other 1 over 3 make 10, my second number in the *Rule of Three*, by which 10 I doe multiply my last number 756, as appeareth by the worke thereof, and it yeeldeth 7560 shillings, my Dividend.

Then dividing 7560 by 3, my Divisor, it yeeldeth in quotient 2520 shillings. which maketh 126 pounds, as appeareth here most plainly, both by the Example, and the worke.

At 3s. 4d, the yard, what 756 yards?

1	$3\frac{1}{3}$	756
3	10	10
		7560s.

1	1	
7560	2520	(126.
3333	2220	

I answer 126 li.

Yet

Yet otherwise upon the same question, altering the price now into the proportion it beareth to a pound, for the 3s. 4d. *is* $\frac{1}{6}$ *part of a pound: which Example first standeth thus as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.*

$$1 \text{——} \tfrac{1}{6} \text{——} 756 \qquad \frac{1}{6} \text{———} \frac{\frac{1}{6}}{1} \text{———} \frac{756}{1}$$

126 pounds.

The second variety.

As soon as I have carried 6 the Denominator of my middle number unto my first place, as before hath been taught, I pull down 1, the numerator of 6, with a line under 6, thus. $\frac{6}{1}$, and that one in custome I pull down in sight; being the figure that I will multiply my third or last number by, according to the tenour of the *Rule of Three*. And because one can neither multiply nor yet Divide (though here it is set down in form of multiplication, the rather for your understanding) the Product of the multiplication according to the declaration of this my first Rule or Note, is converted into the Title of my second number, which here are pounds. Now followeth the division performed in my Divisor 6, to make an end of that question.

12
756 (126. *which maketh* 126 li. *as before.*
666

And thus much for the variety in working that question.

And now followeth another.

My second Question.

If one yard of Cotten cost 8 $\frac{1}{4}$d. what 859?

$$\begin{array}{ccc} 1 & 8\frac{1}{4} & 859 \\ \hline 4 & 33 & 33 \\ & \overline{2577} & \\ & \underline{2577} & \\ & 28347 & \end{array}$$

```
  1
  1|321        1|1       li.        s.      d.
28347 (7086 (590    (29 ——— 10 ——— 6¼
 4441  1222    220
```

This Question was also wrought like the first, and bringeth forth 29 li. 10 s. $6\frac{1}{4}$d. *the price of* 859 *yards.*

My third Question.

If seven pounds of any thing cost 3 li.—10s. what comes 987 pounds to?

$$\begin{array}{ccc} & \text{li.} & \\ 7 & 3\frac{1}{2} & 987 \\ 2 & 7 & 7 \\ \hline 14 & & 6909 \end{array}$$

```
 00
141
2347
6909 (493 7/14|1/2
 1444
  11
```

I answer, 483 li.—10s.

Notes

Notes upon my second Rule for two Fractions coming in the second and third place.

My first Question is this.

IF one Ell cost 13 s.———4d. what halfe a quarter or $\frac{1}{8}$ of an Ell ?

Answer, *First bring* 13 s.—4d. *into the parts of a pound, which is* $\frac{2}{3}$, *and then will the question stand thus.*

1 ——— $\frac{2}{3}$ li. ——— $\frac{1}{8}$

ITem, for the performance of this work, doe as before was taught in the first Rule : first *bring* 3 *the Denominator of the second Fraction unto your first number* 1, setting a line under it thus : $\frac{1}{3}$ Saying once 3 is 3, that done, bring 8 the *Denominator of the third Fraction*, setting it under 3, and multiply them together, saying, 3 times 8 maketh 24 which 24 is your *Divisor*. (Now have you done with the *Denominator* 8) therefore you shall put a line under thus, $\underline{3}$. And the like line also under $\underline{8}$. setting or pulling down under them their own *numerators*, that is, 2 under 3, and also 1 under 8, as appeareth in the Example, which *numerators* for a generall rule evermore to be pulled downe of custome in sight, to multiply the one by the other, according to the tenour of the *Rule of Three*. Then I multiply the one by the other, saying, once 2 is 2, which signifieth 2 pound, being of the nature and like denomination of the middle number, which 2 pound is to be reduced into shillings, otherwise it cannot be divided by my first number 24. Then dividing 40, by 24, the quotient bringeth

forth $1\frac{2}{3}$. So much is $\frac{1}{8}$ of an Ell worth after that rate. Otherwise, although 2 pound could not be divided by 24, yet it might have been abbreviated to $\frac{1}{12}$ of a pound: which is worth 1s. 8d. as before.

	li.		
1	$\frac{2}{3}$	$\frac{1}{2}$	(1
3	2	1	2 (6
8	20		40 ($1\frac{2}{3}$s.
24	40		24

Second Question.

I*F one pound of any weight cost* 13 *shillings* 4 *pence, what are* $\frac{7}{8}$ *of the pound worth after that rate?*

Answer, Reduce the 13 shillings 4 pence into the parts of a pound: which is $\frac{2}{3}$. and then will the question stand thus.

li.

1 —— $\frac{2}{3}$ —— $\frac{7}{8}$

ITem, *for the understanding of this, if you mark well the last Example, this and the rest lieth open, and needs small instruction. For as you did last, so again, bring the* Denominator *of the second and third* Fraction, *unto the first figure* 1, *multiplying the one into the other; which maketh also* 24, *your* Divisor.

Note. Then making a line under 3 thus, $\underline{3}$ and a line under 8 thus, $\underline{8}$ and pulling downe their *Numerators* under each figure, that is 2 under 3, and 7 under 8, which as I said before for a generall rule, I pull down of custome in sight, to be the two Numbers, that of duty ought to be multiplied together; which done, I bring 2, being the lesser figure under 7, multi-

multiplying them together, it maketh 14, which are of the nature of the middle number: that is to wit, pounds, which 14 cannot aptly be divided among 4. therefore are reduced into *ſhillings*, as is plainly to be ſeen in the example: then 280 *ſhillings* parted among 24 yeeldeth for his *quotient* 11s. 8d. your deſire and the juſt price of $\frac{7}{8}$ of an *Ell*. Otherwiſe 14. though it could not be divided by 24. might by *Mediation* or *Diviſion* in broken *Numbers* have been divided or abbreviated to $\frac{7}{12}$, which in effect being reduced to his known parts, maketh 11s. 8d. as before. But my good will and meaning is to aid young beginners: therefore have I reduced the 14 *pound* into *ſhillings*, which is the eaſier way.

Now followeth the example.

1 —— 2 —— 7 | 1
3 3 8 2
8 2 7 4 (6
24 3 280 (11$\frac{2}{3}$s.
14 224
20 2

280 s. I anſwer, 11$\frac{2}{3}$s.

The third example.

If one yard coſt me 2s.——6d. what 345 $\frac{1}{4}$ yards?

Anſwer, Firſt put 6d. into the parts of a ſhilling, and then the queſtion ſtandeth thus:

1 —— $2\frac{1}{2}$ —— $345\frac{1}{4}$.

Item, to the ready underſtanding of this, and all

ſuch like, according as before hath been declared, bring the *Denominators* of the ſecond and third *Fractions* unto the firſt place, multiplying them the one into the other, all which make 8 for the common *Diviſor*. Then next reduce your ſecond number: ſaying, two times 2 is 4. and 1 is 5. as was taught in the Example aforeſaid. Laſtly, reduce your third number 345 $\frac{1}{4}$ all into fourths, and they make 1381. which 1381 is to be multiplied by 5. according to the tenour of the *Rule of Three*: which done, maketh 6905 ſhillings, and divided by 8. your *Diviſor* yeeldeth in Quotient 863 $\frac{1}{8}$ ſhillings, which maketh in pounds 43 pound, 3 ſhillings, 1 $\frac{1}{2}$: and ſo much are the 345 $\frac{1}{4}$ yards worth at that price.

The ſame queſtion wrought again by two ſhillings 6 pence, is now converted into the parts of a pound, and ſtandeth thus:

1 —— $\frac{1}{8}$ —— 345$\frac{1}{4}$.

Item, After I have brought here my ſecond and third *denominator* unto my firſt place, and found 32 to be my *diviſor*; having thus finiſhed my firſt place with all things unto him belonging (which is meant of bringing and multiplying the *denominators* of the ſecond and third *Fractions* into him) I then goe in hand to ſee what is to do in my ſecond place, where preſently of cuſtome I pull down my numerator 1 under 8. being the figure in ſight that ſhall multiply my third number.

Then laſtly, I reduce 345 $\frac{1}{4}$ all into fourths as afore was practiſed. which maketh 1381 the which 1381 I am to multiply by 1 my ſecond number, they are nothing increaſed, but by the *Metamorphoſis*

of

of my work they are now 1381. pound, being of the nature of the middle number, as I have often ſhewed you, which divided by 32 my diviſor yeeldeth 43 pound, and $\frac{5}{32}$, which $\frac{5}{32}$ of a pound reduced into known numbers, make 3 ſhillings 1 $\frac{1}{2}$ pence, as before.

Example.

```
                                  1
1 ———————— 1/8 ———————— 345 1/4   105
32          1                     1381   (43  ——— 5/32 ———
                          1381    522
                                  3
```

NOw follow foure other queſtions, which are in all three places broken numbers; or whole and broken together.

Item, Firſt, for the finding out of your *Diviſor*, you ſhall take this for a moſt certaine, and generall Rule: That you muſt multiply the *Numerator* of the firſt *number* in the queſtion, by the *Denominator* of the ſecond: And that *Product* againe, by the *Denominator* of the third: And the totall thereof ſhall be your *Diviſor*.

Secondly, for a generall rule to find out your *Dividend, multiply* the *Denominator* of the firſt number by the *Numerator* of the ſecond, and the whole thereof by the *Numerator* of the third. And the totall thereof ſhall evermore be your *Dividend*.

Now for an example, I propoſe this queſtion, thereby to make my meaning more plaine, and to ſhew you, as I have done in the reſt, the manner and order of the work.

If $\frac{2}{3}$ of any weight or meaſure coſt $\frac{5}{6}$ of a pound, or

203, *what are* $\frac{3}{8}$ *of the like weight or measure worth after that rate?*

Example.

$\frac{2}{3}$ —— $\frac{5}{6}$ —— $\frac{3}{8}$

ITem, for the more plainer understanding hereof, and all other the like in broken Numbers: First, you shall pull down two, the *Numerator* of the first Number or Fraction, with a line under, thus $3\frac{2}{}$: that done, according as you have learned before, bring 6, the *Denominator* of the second Fraction, and set it under two, multiplying the one into the other, which maketh 12. Then lastly, bring 8. the *denominator* of the third *Fraction*, and set it under 12. multiplying that 12 by 8. which amounteth to 96. or else for more brief, multiply 6 by 8, saying, six times 8 makes 48. which 48 set under 2. and multiply the one into the other, it maketh 96. as before. And this 96 is the first number in the *Rule of Three*. That shall alwayes for a most generall Rule be your *divisor*.

Secondly, to work for your *dividend*, you shall, (as it hath beene sufficiently declared before) pull down 5. the *numerator* of your second *Fraction*, and set it under 6, with a line under thus $\underline{6}$.

That done (as you know) you are to pull down 3, the *numerator* of the third *Fraction*, and set it under 8, with a line under it, thus, $\underline{8}$, multiplying the one into the other, according to the Tenour of the *Rule of Three*; which maketh 15. Then according to my Note, forget not to bring the *denominator* of the first *Fraction*, which is three, under 15. and multiply them together, which maketh 45. which 45 is your *dividend*, and are of the nature of

denomi-

denomination of the middle number, as I have taught you before : And therefore are 44 li. which aptly cannot be divided by 96. Therefore you ſhall reduce the 45 li. into s. as you ſee performed in the Example, which amounteth to 90s. which divided by 96 your *diviſor* it yeeldeth 9s. and $\frac{36}{20}$ of a ſhilling, which in leſſer termes is $\frac{3}{8}$: which $\frac{3}{8}$ in mony maketh $4\frac{1}{2}$d. and ſo much will the aforeſaid $\frac{3}{8}$ coſt, as by the work following ſhall appear.

The Example.

2   5   3
3   6   8
2   5   3
6   5 |3
12   15   9|6
8   3   900   ($9\frac{36}{20}$|$\frac{3}{8}$s.
96   45   96
20
900

Otherwiſe though 45 could not be divided by 96. yet by Diviſion in broken numbers it might have been abbreviated to $\frac{15}{32}$ of a pound, which reduced into known parts, will make 9s. $4\frac{1}{4}$d. as before.

Now my ſecond example ſhall be the proof of this queſtion.

If $\frac{3}{8}$ yards coſt $\frac{15}{32}$ of a pound, or 20 ſhillings, what ſhall $\frac{2}{3}$ coſt ?

Anſwer, Work as was taught you before, and you ſhall have your deſire.

Here

$$\frac{3}{8} \text{———} \frac{15}{32} \text{———} \frac{2}{3}$$

3	15	2
32	2	
96.	30	
3	8	
288	240	

Here as appeareth by the work, the Multiplication being ended, 240 is to be divided by 288, which to some perchance may seem hard, yet notwithstanding is the work good. Therefore abbreviate 240. by 288. as you see here is practised: and the end of your abbreviation shall come to $\frac{5}{6}$ your desire, $\frac{240}{288}$ $\frac{30}{36}$ $\frac{5}{6}$.

Otherwise, 240 120 60 30 | 5
288 144 | 72 | 36 | 6

Otherwise, 340 | 40 | 5
288 | 48 | 6

The third Question.

If $\frac{3}{4}$ *ells cost* 13s. —d. *what* 156 $\frac{1}{2}$ *ells ?*

Answer, To work this question the shortest way: reduce 13 shillings 4 pence, into the parts of a pound, which is $\frac{2}{3}$.

Then as you did afore, after you have set down the question, the *Numerator* of the first *Fraction* 3 is pulled down under 4. and *Denominators* of the other two *Fractions* multiplyed into him, which maketh 18. your *Divisor*.

Then the *Numerators* of the second *Fraction* is pulled downe, under 3 in custome now in sight, ready to *multiply* my third number, by which is performed as soon as the last numbers 156 $\frac{1}{2}$ is reduced into halfs.

Then

Then laſtly, I multiply that product by 4 the *Denominator* of the *Fraction* : it yeeldeth 2504. which I divide by 18. and my quotient is 139. pound, and $\frac{2}{18}$ or $\frac{1}{9}$ of a pound remaining, which is worth 2s—2$\frac{2}{3}$d. And ſo much will 156$\frac{1}{2}$ Ells coſt, as by the work following doth appear.

3	2	156$\frac{1}{2}$	1		
4	3	313	74		
3	2	2	1762		li.
6		626	2504	139$\frac{1}{9}$	
18		4	1888		
		2504	11		

The fourth queſtion.

If 2$\frac{1}{2}$ Ells coſt 1$\frac{2}{3}$ pounds, what commeth 29$\frac{1}{4}$ Ells to?

Item, to the workemanſhip of this queſtion, firſt reduce your ſecond *number* in ſaying, three times 1 is 3, and two is 5, Then bring the *multiplication* of the *Denominators* of the ſecond and third *Fractions* which maketh 12. and *multiply* that 12 by 5 your firſt *numerator*, and it maketh 60. which is your *diviſor*.

Then the *Reduction* of the ſecond Number, which is 5, *multiplied* by 117 the Product of the laſt *numbers* reduction, make 585, which 585 yet reſteth to be *multiplied* by 2, the *Denominator* of the Fraction in the firſt place, yeeldeth 1170, which divided by your *Diviſor*, 60. yeeldeth 19. pound, 10 ſhillings, as appeareth by the work thereof.

Thus having now touched the 12 queſtions whereof I firſt pretended, which with diligence and oft practiſe,

practiſe, I truſt are ſufficient to aid the deſirous unto the working of any broken numbers. I will now treat of divers neceſſary rules incident unto traffick as hereafter followeth.

The fourth Chapter teacheth of loſſe, and gaine, in the Trade of Merchandiſe.

IF one yard coſt 6s.——8d. and the ſame is ſold againe for 8s.———6d. the queſtion is what is gained in one hundred pounds laying out on ſuch commodities:

Anſwer, The *Rule of Three* direct, applyed two manner of wayes to doe the ſame: the one is to ſay, If $6\frac{2}{3}$ give $8\frac{1}{2}$, what giveth 100? *Multiply* and Divide, and looke what your quotient bringeth forth, above your laying out, is the neat gaines and ſolution to your queſtion: If you follow the worke, your ſolution will bring forth 127li.———10s. which is 27li.-- 10. more then your principall, and ſo much is gained in the 100 pounds laying out.

Item, to work in the other way, which I take the neareſt, ſeek the difference betwixt the juſt price and the other price, which is one ſhilling ten pence, then ſay by the *Rule of Three*.

If $6\frac{2}{3}$s. gain $1\frac{5}{6}$s. what ſhall a 100 pound gain?

Multiply and divide, and you ſhall find 27li.-10s. and ſo much is gained in 100 li. laying out.

You may uſe which of theſe two wayes you think good.

The proofe.

If a yard of cloth be delivered for 8s. 6d. whereupon was gained after the rate of 27li. 10s. in 100. pounds laying out: The question is, what the yard cost at the first hand?

Answer, Put your gain 27li.——10s. to 100 pounds, all maketh 127li.——10s. Then say, If 127li. 10s. give but a 100 pounds, what giveth $8\frac{1}{2}$s. Work, and you shall finde 6s. 8d. the true solution to your question.

Yet another example or proofe upon the first Question.

If one yard cost 6s.——8d. the question is, at what price the same is to be sold again, for to gain 27li. 10s. in 100 pound laying out?

Answer, *Say by the* Rule of Three, *if a* 100li. *gain* 127li. 10s. *what giveth* $6\frac{2}{3}$s? *Multiply and divide, and you shall find* 8s. 6d. *your true solution.*

If one Ell cost 7s 8d. and be sold again for 8s. 6d. *The question is,* What is gained in 20 pound laying out in such commodities.

Answer, *Seek the difference betwixt the just price, and the other price which is ten pence, and then apply the* Rule of Three, *as before is taught, saying, If* $7\frac{2}{3}$s. *give* $\frac{5}{6}$ *shilling, what giveth* 20li? *Multiply and divide, and you shall finde* 2li. $3\frac{11}{23}$s. *and so much is gained in* 20li. *laying out.*

The proofe also by an example of Losse.

A *Merchant hath bought Holland cloth at* 8s. 6d. *the ell, which proveth not to his expectation, whereupon he is content to lose* 2li. $3\frac{11}{23}$ *in* 20 *pounds laying*

laying out. The question is, what price ought to be made of the Cloth, abating this losse?

Answer, Doe as before in Gains hath been taught, putting 2 li. $3\frac{11}{23}$s. to your 20 pound, all together maketh 22 li.-$3\frac{11}{23}$. Then say by the *Rule of Three.* If 22 li $3\frac{11}{23}$s. give but 20li. what shall come of $8\frac{1}{3}$s? work, and you shall finde 7s.-8d. the just price that the Ell ought to be sold for after the rate of this losse.

Thus it appeareth evidently, as in company the *Rule* is appliable as well to gain as loss.

If, $20\frac{1}{4}$ yards cost 36 li. 10s. *how shall I sell the same again $\frac{2}{3}$ of the Principall, or to make of 3,4. which is all one.*

Answer, By the *Rule of Three*, If 3 doe give 4, what will $36\frac{1}{2}$ give? Multiply and divide, and you shall finde $48\frac{2}{3}$ pounds, Then say again, if $20\frac{1}{4}$ yards do give 48. $\frac{2}{3}$ pounds, as well principall as gain, what will one yard be worth at that price? Multiply and divide, and you shall find 2 li. $\frac{28}{243}$.

If one Ell of Cloth cost me 8s. 8d. *and afterwards I sell* $10\frac{1}{4}$ *Ells thereof for* 5 li. 13s. 4d. *I would know, whether I winne or lose: and how much upon the* 100 *pounds of mony.*

Answer, See first at 8s. 8d. the Ell, what $10\frac{1}{2}$ Ells comes to, and you shall finde 4li. 11s. and I sold the same for 5li.—13s.—4d. so that I did gain upon the $10\frac{1}{2}$ Ells 22 shillings 4d. Then if you would know how much is gained in 100 pounds, I say by the *Rule of Three*, if 4 li.—11s. did gain 22—4d. what will 100 pounds gain? Multiply and divide, and you shall finde 24 li.—10s.—10d. $\frac{10}{91}$ and so much is gained in the 100. pound of mony.

If $12\frac{1}{2}$ *yards cost me* 11 *pound five shillings, and I sell*

sell the yard again for 16 *shillings, the question is whether I doe winne or lose, and how much in or upon the pound of mony?*

Answer, Look what the $12\frac{1}{6}$ yards come to at 16s the yard, and you shall finde ten pound. But they cost 11 pound 5 shillings. So there is lost upon the whole 1 pound 5s. Then to know how much is lost in the pound, say by the *Rule of Three*, if $11\frac{1}{4}$ pound doe lose $1\frac{1}{4}$ pound, what will 1 pound lose? Multiply and divide, and you shall finde 2s.2d $\frac{2}{3}$. and so much is lost in the pound of mony.

If I sell the 100 *weight of any commodity for* 4 *pound, whereupon I doe lose after ten pound in the* 100 *pound, I demand how much I shall lose or gain in the* 100 *pound, if in case I had sold the same for* 4 *pound ten shillings.*

Answer, Say, if 90 pound yeeldeth 100. how much will 4 give? Multiply and divide, and you shall find $4\frac{4}{9}$. Then say again, if $4\frac{4}{9}$ give me $4\frac{1}{2}$ what will 100 come to? Multiply and divide, and you shall finde 101 pound $\frac{1}{4}$ which is more then 100 pound by 1 li. 5 shillings: and so much is gained in the 100. pound.

A Merchant hath sold Currants for the summe of 430 *Pound, and he hath gained therein after ten pound, in the* 100 *Pound. The question is, to know how much is gained in all.*

Answer, Say by the *Rule of Three*. If 100 pound doe gain ten pound, what will 430 pound gain? Multiply and divide, and you shall finde 43, and so much hath hee gained in all.

If one yard be worth $28\frac{1}{2}$s. *for how much shall* 10 *yards be sold to gain after* 8 *pound,* 6 *shillings* 8 *pence in the* 100 *pound?*

Answer,

Answer, First, adde 8li.—6s,—8d. to 100. Then say, if 100li. do give 108 $\frac{1}{3}$s. for principall and gain, what will 28 $\frac{1}{2}$s. principall yeeld? Multiply and divide, and you shall finde 30 $\frac{7}{8}$ s. Then say, again, by the *Rule of Three*, if 1 yard do give 30 $\frac{7}{8}$ s. (which is as well the principall as the gain) what shall ten yards give? Multiply and divide, and you shall finde 15 li. 8 s. 9d. And for the same price shall the ten yards be sold, for to gain after the rate of 8li--6s.--8d. upon the 100.

A branch or Proof out of this Question.

A Merchant hath sold clothes for 15*li.*--8s.--9d. *and he hath gained in the whole the summe of* 1 li—3s.—9d. *The question is to know how much he hath gained in the* 100. *pound?*

Answer, To know this, first rebate the gains from the price, and there will remain 14 li. 5s. 0d. Then say by the *Rule of three* direct: If 14 li. $\frac{1}{4}$ give me 1 li. 3 $\frac{3}{4}$, what will 100li. give? Multiply and divide, and you shall finde 8li. 6s.-8d. the effect desired, the proof is apparent in the question before.

Yet another branch or proofe of the first Question.

If ten yards be delivered for 15 li. 8 s 9 d. *whereupon was gained after the rate of* 8li. 6s. 8d. *upon the* 100. *pound, the question is, what the yard did cost at the first hand?*

Answer, First, say by the *Rule of three*, if ten with principall and gain yeeld 15 li. 8s. $\frac{3}{4}$ shillings, what shall 1 yeeld. Multiply and divide, and you shall find

30

$30\frac{7}{8}$s. Then say again by the *Rule of Three*, if 108 $\frac{1}{3}$ principall and gain give but 100 what shall $30\frac{7}{8}$ of principall and gain yeeld ? Work, and you shall finde $28\frac{1}{2}$ s. And so much did the yard cost at the first peny.

If one yard cost 36 s. how much shall 12 yards be sold for, to gain after the rate of 10. li. *in the* 100?

Answer, First say, If 100. give 110 li. principall and gain, what will 36 s. give ? Multiply and divide, and you shall finde $39\frac{3}{5}$ s. Then say again by the *Rule of Three*: If one yard of principall and gain yeeld $39\frac{3}{5}$ shillings, what shall 12 yards gain ? Multiply and divide, and you shall finde 23 li. 15 $\frac{1}{5}$ s. which $\frac{1}{5}$ s. in knowne number, is $2\frac{2}{5}$ d. And for the same price shall the 12 yards be sold, to gain after the rate of 10 in the 100.

The Proof.

If 12 yards be sold for 23 li. 15s. $2\frac{2}{5}$d. *whereupon is gained after* 10 li. *in the* 100. *the question is, what the yard cost at the first peny?*

Answer, First say, If 12 give 23 li. 15 $\frac{1}{5}$ s. what one yard ? Multiply and divide, and you shall finde $39\frac{3}{5}$ s. Then say againe by the *Rule of Three*, If 110. pounds give but a 100. what shall $39\frac{3}{5}$ s. give ? Work, and you shall find 36s. the just price of the yard at the first hand.

Item, *When one Merchant selleth wares to another, and he giveth to the buyer* 1 li. 6. 8d. *upon the score, or* 20 li. *the question is, How much shall the buyer gain upon the* 100 li. *after that rate ?*

Answer, First, adde 1 li. 6s. 8d. unto 20 li. and they are $21\frac{1}{3}$. Then say, If 20 pound give $21\frac{1}{3}$, what shall

ſhall 100. give? Multiply and divide, and you ſhall finde 106 $\frac{2}{3}$. So the buyer getteth after the rate of $6\frac{2}{3}$ li. upon the 100 li.

Gentle Reader, other neceſſary queſtions appertaining to Loſſe and Gain, you ſhall have in the eighth Chapter of this Treatiſe.

The fifth Chapter entreateth of Loſſe and Gain upon time, wrought by *the double Rule of Three, or by the Rule compoſed* : which is contained in foure ſpeciall ſelected branches, or queſtions of divers formes, *each one of them ſpringing from the firſt Queſtion,* and each one of them alſo being a proof to other, *&c.*

IF one yard coſt me 2s. 8d. *ready money, and after I ſell the ſame again for* 2s. 10d. *to be paid for it at the end of three months: the queſtion is, what I gain upon the* 100 li. *in* 12 *moneths.*

Anſwer, Firſt ſay, if 2 $\frac{2}{3}$ gain $\frac{1}{3}$, what ſhall 100 li. gain? Multiply and divide, and you ſhall finde 6 $\frac{1}{4}$ li. Then ſay again, by the *Rule of Three*, if three months gain 6 $\frac{1}{4}$ pound, what ſhall 12. months gain? Work, and you ſhall finde 25 li. and ſo much ſhall I gain in 12 moneths after that rate.

Item, You may alſo work it all at one working by the firſt part of the *Rule of Three* compoſed, ſaying, if 2 $\frac{2}{3}$ d. in three months doe gain $\frac{1}{6}$ of a ſhilling, (which is 2d.) what will 100 li. gain in 12 months? Which for thy further encouragement, the work of this one example I have here put down, to verifie that I affirme in the firſt part of this *Ground of Arts*, that this

Rule

Rule, and ſo all others, more rejoyceth in *Broken*, then in *Whole*.

```
s.      months        s.          li.        mo.
2 2/3 ——— 3 ——— ——— 1/6 ——— ——— 100 ——— 12
8                     1           20
3 2                               2000
24 72020                          3
6 14444 (500 (25                  6300
144 144 220                       12
      1                           7.000
```

Where the multiplication and the diviſion being ended, maketh 25 li. your deſire.

If a yard be delivered for 2s. 10d. *to be payed at* 3 *months, whereupon was gained after the rate of* 25 li. *in the* 100. *for* 12 *months, the queſtion is now, what the yard coſt at the firſt hand?*

Anſwer, Firſt ſay, if 12 months gain 25 li. what ſhall 3 months gain? Work, and you ſhall finde 6 1/4 li. Then ſay again the ſecond time, if 10 1/4 li. give but 100. what ſhall 2 5/6 s. give? Work and you ſhall finde 2s. 8d. which is the juſt price that the yard coſt at the firſt hand.

If one yard of Cloth coſt me 2s. 8d. *ready mony, for what term ſhall I ſell the ſame again for* 2s. 10d. *ſo that I might gain after the rate of* 25 *pound upon the* 100. *pound in* 12. *moneths?*

Anſwer, Firſt ſay, if 2 2/3 gain 1/6, what ſhall 100 li. gain? Multiply and divide, and you ſhall find 6 1/4 Then ſay again for the ſecond work. If 25. pound be come of twelve months, what ſhall come of 6 1/4? Work, and you ſhall find 3 moneths, the juſt term of time that the Cloth ought to be delivered

at 2s. 10d. to gain 25li. upon the 100li. in 12 moneths.

If one yard cost me 2s. 8d. ready money, for what price shall I sell the same again to be paid at the end of 3 moneths, so that I may gain after the rate of 25 pound in the 100 pound for 12 moneths?

Answer, First say, if 12 gain 25li. what shall 3 moneths gain? Multiply and divide, and you shall finde $6\frac{1}{4}$li. Then say for the second worke, if 100li. give $106\frac{1}{4}$, what giveth $2\frac{2}{3}$s? Work and you shall finde 2s. 10d. and for that price must the yard be sold to gain after 25 pound in the 100 pound for 12 moneths.

Many other of these questions I might here have delivered, but for feare the Booke would rise to too thick a volume, and so to make the price so much the dearer, whereby it might not be so portable to my Countrie-men, as I wish it. But these 4 I have of purpose framed in this order, having relation one to another, assuring you that what question soever may be proposed within the compasse of this Rule you shall finde by one of these 4 to make a solution. And moreover, divers others are yet to be delivered where the Creditor giveth divers dayes of payment, which can never be well wrought, nor yet understood, unlesse you can first finde by Art the just times that all those payments, how different soever they be, ought to be paid at once: whereupon first I think good here to give some instructions unto such a Rule; for it is the onely aid for the finishing of such questions as hereafter shall follow.

The

The sixth Chapter entreateth of Rules of Payment, which is a right necessary Rule, and one of the chiefest handmaids that attendeth upon buying and selling, *&c.*

Example.

A Merchant doth owe a summe of mony, whereof the $\frac{1}{3}$ is to be paid at sixe months, and the $\frac{1}{2}$ at eight moneths, and the rest at a year. If he would pay all at one payment, the question is, what time ought to be given him?

Answer, I have omitted the quantity of the summe, for you shall understand the Rule is appliable, and yeeldeth a true solution to what summ soever shall be proposed: But now for order sake in teaching, I do imagine the summe to be 60 pounds, whereupon the manner of this worke is to multiply the proportionate part of the money by the time, as in company. Then 20. being the first payment, and the $\frac{1}{3}$ of 60 which $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment, maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the Example in the Operation, maketh two moneths: next 30. which is the $\frac{1}{2}$ multiplyed by his terme 8. yeelds 4. moneths, then the rest which is 10 li. must needs be abbreviated into the proportion it beareth to 60. which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 moneths, produceth $\frac{12}{6}$, maketh two moneths. All which added together, as appeareth in the Operati-

$\frac{1}{3}$ by $\frac{6}{1}$ $\frac{6}{3}$	2 Moneths.
$\frac{1}{2}$ by $\frac{8}{1}$ $\frac{8}{2}$	4 Moneths.
$\frac{1}{6}$ by $\frac{12}{1}$ $\frac{12}{6}$	2 Moneths.

on, maketh 8 moneths, which is the just time that all those payments ought to be paid at once.

A Merchant hath 800 li. *to pay, the* $\frac{1}{6}$ *thereof ready money, the* $\frac{1}{4}$ *at two moneths, the* $\frac{1}{2}$ *at* 4 *moneths, and the rest at a yeare. The question is, if he would pay all at one payment, what time ought to be given him?*

Answer, The ready money is never multiplied, then $\frac{1}{4}$ multiplied by two moneths, as you did before, maketh $\frac{1}{2}$, then $\frac{1}{2}$ by 4 produceth 2 moneths, as appeareth here in the operation: But now for the rest of the money you cannot multiply it untill you have sought what proportion it beareth to 800 pounds. Therefore you must subtract the ready mony, the $\frac{1}{4}$ and $\frac{1}{2}$ out of the principall. The rest will be $66\frac{2}{3}$ li. which you must look what part it beareth to the principall, which you shall finde to be $\frac{1}{12}$, the same you must also multiply by his time 12 months and it yeeldeth 1 month, so all make $3\frac{1}{2}$ months, as appeareth in the operation.

$\frac{1}{4}$	$\frac{2}{1}$	
	$\frac{2}{4}$	$0\frac{1}{2}$
$\frac{1}{2}$	$\frac{4}{1}$	
	$\frac{4}{2}$	3
$\frac{1}{12}$	$\frac{12}{1}$	
$\frac{12}{12}$		1

A Merchant is to pay 1200 li. *in three terms, that is to wit,* 400 li. *at two weeks: and* 600 li. *at foure months: Lastly,* 200 li. *at five months: The question is, in what time they ought to be paid at once.*

Answer, Proportionate the parts, and you shall finde that 400 is $\frac{1}{3}$ part, and for 600. you shall finde $\frac{1}{2}$, and likewise 200 is the $\frac{1}{6}$ part which multiply by their times as before & you shall have $\frac{2}{3}$ weeks, more eight weeks: and lastly $3\frac{1}{3}$ weeks, which together maketh 12. weeks, or three months, your desire.

A Merchant is to pay 600 *pound in three terms, whereof* 100 *pound is paid present, more* 300 *pound at twenty dayes, and the rest at five months, accounting thirty*

thirty dayes to a month. The question is, what time ought these payments to be paid at once?

Answer, Worke, and you shall finde 2 months.

The seventh Chapter treateth of buying and selling in the Trade of Merchandize, wherein is taken part ready money, and diverse dayes of Payment given for the rest: and what is wonne or lost in the 100 pound forbearance for twelve months more or lesse, according to the quantity of money, or proportion of time, *&c.*

A Merchant hath bought satins which cost 8 s. *the yard ready money: and he selleth the same again to another man for* 10 s. *the yard, but he giveth 2 dayes for the payment, that is to say, 3 months for the one halfe, and 5 months for the other halfe. The quest. is to know how much the seller doth gain upon the* 100li. *in* 12. *moneths after that rate.*

Ans. Seeke first, by the Rules of payment, at what time those 2 payments ought to be paid at once, and you shall find 4 mōths, at which time the second Merchant ought to have paid the whole entire paymẽt: & therfore say by the first part of the *Rule* of 3. composed, If 8 s. in 4 months do gain 2 s. what will 100. li. gain in 12 months.

s. m. s. li. m.
8 —— 4 —— 12 — 100 — 12
4   20
32   2000
2
4000
12
4800

~~1~~
~~18~~
~~48000~~   ~~1~~s.
~~322~~   (~~1~~500   (75
~~3~~   ~~22~~0

Multiply and divide, and you shall finde 75. pounds, as appeareth in the example, & so much doth the first Merchant gain upon the 100 li. in 12. months.

A Merchant hath sold 50. Clothes, at $9\frac{1}{2}$ li. *the piece, to be paid the one* $\frac{1}{2}$ *at foure moneths, the* $\frac{1}{3}$ *at five moneths, and the* $\frac{1}{6}$ *at 7 moneths, and the sellers minde is to take no more but after* 8 li. *in the* 100. *for* 12. *moneths. The question is now, what the first Merchant gaineth in the sale of these Clothes after that rate.*

Ans. First, looke what the 40 Clothes come to at that price, and you shall find 475 li. Then secondly according to your direction in the Rules of payment, seeke at what time all the payments are to be performed at once. And you shall finde $4\frac{5}{6}$ moneths. Then thirdly say, by the first part of the *Rule of* 3. composed. If 100li. in 12 months gain 8 li. what will 475li. gain in $4\frac{5}{6}$ months? Work, and you shall find 15 li. and $\frac{11}{36}$ of a pound, which is the neat gains that the first Merchant hath after the rate aforesaid.

A Merchant hath bought Holland at 7s. 3d. *the Ell ready money, and he selleth the same again, for* 8s. 4d. *the Ell, to be paid* $\frac{1}{4}$ *part in ready money, more* $\frac{1}{3}$ *part at 2 moneths, and the rest at 4 moneths; The question is now to know how much the first Merchant doth gain upō the* 100 li. *in* 12. *months after that rate?*

Answer, According to the direction delivered you in the Rule of payment, the ready mony is not to be multiplied. Then working for the other 2 payments to find out the true proportion at what time they ought to be paid at once, you shall find for $\frac{1}{3}$ at two moneths, $\frac{2}{3}$ of a month. And the rest of the money which is $\frac{5}{16}$ multiplyed by his term 4 months, yeeldeth $1\frac{2}{3}$ months, both which added together make $2\frac{1}{3}$ months,

the

the juſt time that both the payments ought to be performed at once. And therefore ſay by the firſt part of the *rule of 3* compoſed, if $74\frac{1}{4}$ in $2\frac{1}{3}$ months doe gain $\frac{12}{46}$ of a li. what ſhall 100li. gain in 12 months after that rate? Work, & you ſhall finde $78\frac{172}{603}$ li. And ſo much doth he gain upon 100. pounds in 12 months.

A Merchant hath bought 30. clothes at 6 li. *the peice for ready mony: Afterward he ſelleth* 10 *of them for 7* li. *the peice, for 3 months term: And the other 20. he ſelleth for* 8li. *the piece, for 4 months term, the queſtion is now, what he gaineth upon 100 pounds in 12 months?*

Anſ. Firſt find the value of the 30 clothes, which amount to 180li. Secondly ſeek what the ten pieces come to at 7. li. & what the 20 pieces come to at 80 li. the one comes to 70. and the other to 60, both which together make 230. which is 50li. more thē they coſt. Thirdly, as I have taught you in the rule of payment, proportionate the firſt and 2d prices unto the proportiō they bear unto 230 the product of their 2 prices, & you ſhall find $\frac{7}{23}$ for the firſt, & $\frac{16}{23}$ for the latter. Then fourthly, multiply thoſe parts by their times, & you ſhall have $\frac{21}{23}$ and $\frac{64}{23}$ both which together maketh 3 whole months, & $\frac{16}{27}$ of a month, which is the juſt time that both thoſe payments ought to be paid at once.

Then ſay by the firſt part of the *Rule of* 3. compoſed, If 180li. in $3\frac{16}{23}$ months doe gain 50li. what ſhall 100 gain in 12 mōths? Multiply & divide, & you ſhall find $90\frac{10}{51}$ li. & ſo much doth he gain upon 100 pounds in 12 months.

A Merchāt hath bought Cinnamō which coſt him 9s. the li. *ready money. The queſtion is now, at what price he ought to ſell the 100 weight. To wit* 112li. *to be paid the* $\frac{1}{4}$ *at 2 months, & the reſidue at the end of 3 mōths, ſo that he may gain after the rate of ten* li. *upon* 100li. *for 12 months.*

Anſ. Seek firſt by the Rule of payment at what term both the payments ought to be paid at once, where

the $\frac{1}{4}$ multiplyed by his term two months, maketh $\frac{2}{4}$ months.

Likewise the next payment, which is $\frac{3}{4}$ multiplied by his term three months, maketh 2 $\frac{1}{4}$ moneths, both which added together, maketh 2 $\frac{3}{4}$ months, which is the time that both the payments ought to be paid at once. Then say by the *Rule of Three*, If 12. months do give me ten pounds, what will 2 $\frac{3}{4}$ months give? Multiply and divide, and you shall finde 2 $\frac{7}{24}$ pounds. Then say again by the *Rule of Three*, If one pound cost me 9s. what will 112. pounds cost? Multiply and divide, and you shall finde 50 li. 8s. Then say once again: If 100. pound doe give 102 $\frac{7}{24}$, what will 50 $\frac{2}{5}$ pounds give? Multiply and divide, and you shall finde 51 li. 11 s. 1 $\frac{3}{5}$ d. and for that price ought I to sell 112. pound of Cinnamon to be paid at the two severall payments aforesaid, to gain thereby after the rate of tenne pounds upon the hundred pound in twelve months.

Brief Rules for our hundred weight here at *London*, which is after 112. pound for the 160.

ITem, *Who that multiplieth the pence that one pound weight is worth by 7, and divided the* Product *by 15. shall finde how many pounds in mony 112 pound weight is worth.*

And contrariwise, he that multiplieth the pounds that 112. pounds weight is worth by 15. and divideth the Product *by 7. shall finde how many pence in money the one pounds weight is worth.*

Example.

At 10 pence the pound weight, what is 112. pounds weight worth? *Answer,*

Answer, Multiply 10 by 7. and thereof commeth 70. the which divided by 15. and you finde $4\frac{2}{3}$ pounds. And thus the 112 pounds is worth 4 li. 13s. 4d, after the rate of 10. pence the pound aforesaid.

At 6 pounds the 112 pounds weight, what is one pound worth ?

Answer, Multiply 6 by 15, and thereof commeth 90. the which divide by 7. and you shall finde $12\frac{6}{7}$d. So much is one pound worth when the 112. pounds did cost 6 pounds.

The eighth Chapter intreateth of Tares, and allowances of Merchandize sold by weight, and of Losses and Gains therein, *&c.*

AT 16 pound the 100 Suttle, what shall 895 pound Suttle be worth, in giving 4 pound weight upon every 100. for Treat?

Answer, *Adde* 4 *unto* 100, *and you shall have* 104. *Then say by the Rule of Three, If* 104. *be worth* 16. *pounds, what are* 895 *pounds worth?* *Multiply and divide, and you shall finde* 137li. 13s. $10\frac{2}{11}$ d. *and so much shall the* 895. *pounds weight be worth.*

Item, at 3s, 4d. the pound weight what shall $754\frac{1}{2}$ pound be worth, in giving 4 pounds weight upon every hundred for Treat ?

Answer, See first by the *Rule of three* what the 100. pound is worth, saying, If one cost $3\frac{1}{3}$s. what 100 ? Multiply and divide, and you shall finde $16\frac{2}{3}$ pounds. Then adde 4 unto one 100, and they are 104. Then

104. Then ſay again by the *Rule of Three*, If 104 be ſold for 16 $\frac{2}{3}$ pounds, for how much ſhall 754 $\frac{1}{2}$ be ſold for? Multiply and divide, and you ſhall find 120 li. 18s. 3 $\frac{3}{13}$ d. And for ſo much ſhall the 754 $\frac{1}{2}$ pound be ſold for at 3s.4d. the pound, in giving 4 upon the 100.

Other neceſſary briefe Rules there are for the finding of Treats, or caſting up of Cheſts of Sugar, &c. which for that it is a myſtery, I omit: if any lack inſtruction that way they ſhall finde me ready to pleaſure them.

Item, If 100 pounds be worth 36s.8d. what ſhall 860 pounds be worth in rebating 4 pounds upon every hundred for tare and cloff?

Anſwer, Multiply 860 by 4, and thereof commeth 3440, the which divide by 100, and you ſhall have 34 $\frac{2}{5}$ pounds, abate 34 $\frac{2}{5}$ from 860 and there will remain 825 $\frac{3}{5}$ pounds. Then ſay, by the *Rule of* 3. If 100 li. coſt 36 $\frac{2}{3}$ s. what will 825 $\frac{3}{5}$ coſt after that rate? Multiply and divide and you ſhall finde 15 li. 2s. 8 $\frac{16}{25}$ d. And ſo much ſhall the 860 coſt in rebating 4 li. upon every 100, for tare and cloff.

Item, *Whether doth he loſe more that giveth* 4 li. *upon the* 100, *or he that rebateth* 4 li. *upon the* 100?

Anſwer, Firſt note, that he that giveth 4. pound on 100 giveth 104 for 100. And he which rebateth 4. pounds upon the 100. giveth the 100. for *96*. Therefore ſay by the *Rule of Three*, if 104 be delivered for 100, for how much ſhall the 100 be delivered. Multiply and divide, and you ſhall find *96* $\frac{2}{13}$, and he which rebateth 4 in the 100, maketh but *96* pounds of 100, ſo that he loſeth 4 pounds in the 100, and the other which giveth 4 pounds unto the 100 loſeth but 3 $\frac{11}{13}$ pounds upon the 100. Thus you

you may see, that he which abateth 4 pounds in the 100, loseth more by $\frac{11}{13}$ pound in the 100 pounds, then the other which gave 4 pounds upon the 100, for tare and closse.

If 100 pounds of any thing cost me 23s. 4d. the question is, how I shall sell the pound, to gain after the rate of ten pounds, upon the 100 pound.

Answer, Say by the *Rule of three*, if 100 pounds give 100 pounds, what shall 23 $\frac{1}{5}$ s. give, multiply and divide, and you shall finde 1 $\frac{17}{60}$ pounds, Then say again, if 100 pound be worth 1 $\frac{17}{60}$ pounds, what is one pound worth, multiply and divide and you shall finde 3 d. $\frac{2}{15}$ And so much is the pound worth in gaining ten pounds upon the 100.

Item, *A Grocer hath bought C. weight of commodity for* 6 li. 10 s. *The question is now to know how many* li: *thereof he shall sell for* 33s. 4d. *to gain* 20s *in C. weight*

Answer, Adde 20 s. unto 6 li. 10s, and they make 7 li, 10s, Then say, if 7 $\frac{1}{2}$ pound, yeeld me 112 pound, what shall 1$\frac{2}{3}$ pounds yeeld; multiply and divide, and you shall find 24$\frac{8}{9}$ li. And so many pound ought he to sell to gain 20s. in his C. weight.

Item, *If one pound weight cost* 3s. 4d. *and I sell the same again for* 4s, *what is gained in a hundred pound of mony laid out in that commodity*;

Answer, You may say, If 3 $\frac{1}{3}$ s, give 4 what will 100 pound gain; But then when you have found, you must subtract 100 pounds out of the Product, the rest is your neat gain: or else to produce the neat gain in your work at the first, subtract the just price out of the overprice, as I taught before in the first beginning of Losse and Gain, and your conclusion shall be all one. Multiply and divide by which of

the

the two wayes you think good and you shall finde that he gaineth 20 pounds in the 100 pound.

Item, *If the pound weight which cost* 4 s. *be sold again for* 3. s. 4 d. *I demand what is lost in the* 100 *pounds of money.*

Answer, Say if 4 s. lose $\frac{2}{3}$s what shall 100 lose? Multiply and divide and you shall finde 16 li. 13. s. 4d. and so much is lost upon the 100 of money.

Item, *If C. weight of any commodity cost* 45 li. *and the buyer repenting would lose* 5 *pounds in the* 100 *of money. I demand how the pounds may be sold, his losse to be neither more nor lesse then after the rate aforesaid of five by the hundred.*

Asnwer, By the *Rule of three,* if a 100 lose 5, what shall 45 lose? Work and you shall finde $2\frac{1}{4}$ pound, which rebated from the principall 45, resteth 42 li. 15 s. Lastly say if 112 yeeldeth but 4 li. 15 s. what one pound? Multiply and divide and you shall find 7s. 7d. $\frac{1}{2}\frac{2}{8}$. And so much is the pound worth after that losse.

A Grocer hath bought three pieces of Raisins weighing 175 $\frac{1}{2}$ *pounds,* 182 $\frac{1}{4}$ *pounds,* 191 *pounds: tare for each fraile* $2\frac{1}{4}$ *pounds at* 25 $\frac{1}{2}$s. *the C. weight. The question is, what they amount to in money.*

I answer, 6 li.——— 3 s.——— 4 d $\frac{23}{28}$ d.

A Grocer hath bought three sacks of Almonds weighing 267 $\frac{1}{2}$ *pound, tare two pound,* 257 $\frac{1}{2}$ *pounds, tare* $2\frac{1}{2}$ *pound,* 252 *pound, tare* 3 *pound, at* 2s. $10\frac{1}{2}$ d. *the pound, what amount they to in money.*

I asnwer, 110li.——— 12 s.——— 3 $\frac{3}{4}$ d.

The

The ninth Chapter intreateth of lengths and breadths of Arras and other Cloths, with other questions incident unto length and breadth.

IF a peice of Arras be 7 Elles and $\frac{3}{4}$ long and 5 Elles and $\frac{2}{3}$ broad, how many Elles square doth the same peice containe ?

Answer, Multiply the length by breadth, that is to say, $7\frac{3}{4}$ by $5\frac{2}{3}$ And thereof will come $43\frac{11}{12}$ elles : so many elles square doth the same piece contain.

Item, *more, a piece of Arras doth contain 22 Elles square, and if the same were in length $3\frac{1}{2}$ elles, I demand how many elles in breadth the same peice doth contain.*

Answer, Divide 22 elles by $3\frac{1}{2}$ and thereof commeth 6 $6\frac{2}{7}$ So many elles doth the same contain in breadth.

Item, *more, a Merchant hath $3\frac{1}{4}$ elles of Arras, at $1\frac{2}{3}$ elles broad, which he will change with another man for a peice of Arras, that is $\frac{7}{8}$ elles square. The question is, how many elles of that squarenesse ought the first Merchant to have?*

Answer, Multiply the first merchants peice his length by the breadth, and you shall finde it containeth $5\frac{5}{12}$ elles, which $\frac{5}{12}$ elles, you shall divide by $\frac{7}{8}$ and you shall finde $6\frac{4}{21}$ elles, and so many elles of that squarenesse ought the latter merchant to give the first.

Item, *A Student hath bought $3\frac{1}{2}$ yards of broad Cloth, at 7 quarters broad, to make a gown, and should line the same throughout with Lamb at a foot square each skin, the question is now how many skins he ought to have.*

Answer,

Answer, Seek first the number of yards square that his cloth containeth, which to doe, multiply $3\frac{1}{2}$ his length, by $1\frac{3}{4}$ his breadth, and you shall finde $6\frac{1}{8}$ yards square: then say by the *Rule of three*, if one yard square give 9 foot, what shall $6\frac{1}{8}$? Work, and you shall find $55\frac{1}{8}$ skins.

Item, *more, a Lawyer hath a rich peece of seeling come home which is 24 foot and 3 inches long, and 7 foot and $2\frac{1}{2}$ inches high: the Joyner is to be paid by the yard square: the question is how many yards this containeth.*

Answer, Multiply his length by his breadth, that is to wit, $24\frac{1}{4}$ foot by $7\frac{5}{24}$ foot, and you shall find $174\frac{77}{96}$ foot square, which 174 you shall divide by 9 (for so many foot make a yard square) and you shall finde 19 yards 3 foot and $\frac{77}{96}$ of a foot, and so many yards doth this piece hold.

Item, *I bought a piece of Holland cloth containing 36 Els $\frac{1}{3}$ Flemmish. The question is how many Ells English it makes.*

Answer, You must note, that five ells *Flemmish* doth make but three ells *English*.

Therefore say by the *Rule of three*, if five ells *Flemmish* make but three ells of *English*, how many ells *English* will $36\frac{1}{3}$ ells *Flemmish* make? Multiply and divide and you shall finde $21\frac{4}{5}$ and so many *English* doth $36\frac{1}{3}$ ells *Flemmish* contein. The like is to be done of others.

Item, *more, I have bought 342 Ells Flemmish, of Arras work, at two Ells broad Flemmish, and I would line the same with Ell broad Canvas of English measure. The question is, how many Ells English will serve my turn?*

Answer,

Answer, For as much as three ells *English* are worth five ells *Flemmish*, therefore put three ells *Flemmish* into his square, in multiplying three by himselfe, which maketh nine. Likewise multiply the *English* ell, which is five quarters, every way into himselfe squarely, and you shall finde 25. Then multiply 342 which is the length of the peice, by 2, which is the breadth, and thereof commeth 684, then say by the *Rule of three*, as before : if 25 ells square of *Flemmish* measure, be worth nine ells square of *English* measure, what are 684 of *Flemmish* measure? Multiply and divide, and you shall finde 246 $\frac{6}{25}$ ells *English*.

The same is also wrought by the Backer *Rule of three*, in seeking the squares contained in the *Flemmish* ell of two ells broad (which are 18) and also in seeking the squares contained in the *English* ell (which are 25) then say by the *Rule of three* backward, If 18 quarters require 342 ells, what shall 25 quarters give? Multiply and divide by the *Rule of three* Reverse, and you shall find as before 246 $\frac{6}{25}$ ells *English*?

Item, *more, at three shillings foure pence the Flemmish Ell, what is the English Ell worth after the rate?*

Answer, Say, if three quarters give 3 $\frac{1}{3}$s. what giveth five quarters? Multiply and divide, and you shall finde 5 s. 6 $\frac{2}{3}$ d.

Item, *more*, *at* 8 s. 4 d. *the Flemmish Ell square, what is the English Ell worth after that rate?*

Answer, According to the reason of the last Question, consider that a *Flemmish* ell square is equall to nine quarters of a yard *English*, and an *English* ell square is equall to 25 quarters of a yard. There-

fore ſay by the *Rule of three*, if 9 quarters give 1 $\frac{2}{3}$ s. what 25 quarters? Work and find 23s. 1 $\frac{7}{9}$ pence. And ſo is the *Engliſh* ell worth.

Item, *more, at* 6 s. 8 d. *the ell ſquare : what ſhall a piece of Cloth coſt that is* 7 $\frac{1}{2}$ *ells long, and* 3 $\frac{1}{4}$ *ells broad?*

Anſwer, Multiply the breadth by the length, and you ſhall find 24 $\frac{3}{8}$ ells ſquare coſt 6 $\frac{2}{3}$s. what 24 $\frac{3}{8}$ els? Multiply and divide, and you ſhall find 8 pounds, 2 s. 6 pence, and ſo much the ſame piece of cloth coſt.

Item, *more, a Mercer ſold 3 pices of Silk. To wit* 24 $\frac{1}{4}$, 13 $\frac{2}{3}$ *and* 25 *yards, at* 9 $\frac{3}{4}$ *s. the yard, and was glad to receive in part of payment again, a cloth containing* 34 $\frac{1}{3}$ *yards at* 7 $\frac{2}{3}$ *ſhillings the yard. The queſtion is now, what the Debtor is in the Creditors debt? Work, and you ſhall finde he oweth the Mercer* 17 l. 8 s. 11 d $\frac{1}{4}$.

The tenth Chapter intreateth of reducing of Pawns of Geanes into Engliſh yards.

NOte that 100 *Pawnes doe make* 26 *yards, whereupon three Pawnes* $\frac{11}{13}$, *doe make one yard, and one Pawn after the rate and proportion is* $\frac{13}{50}$ *of a yard.*

In 4563 *Pawnes of Geanes, how many yards engliſh?*

Anſwer, Say by the *Rule of three*, if a hundred Pawns do make 26 yards, what will 4563 Pawns make? Multiply and divide, and you ſhall find 1186 yards $\frac{19}{50}$ So many yards doe 4563 Pawns make.

Otherwiſe take ſome other number at your pleaſure;

ſure, as ten pawns, which is the $\frac{1}{10}$ part of 100, then to find his proportion, take the $\frac{1}{10}$ part of 26, which is 2 $\frac{3}{5}$ & then ſay alſo by the *Rule of three*, if ten pawns give 2 $\frac{3}{5}$ yards. what will 4563 pawns give? Work and you ſhall finde 1186 $\frac{19}{50}$ yards, as before.

More, at 2 s. 6 d. *the Pawns of Geans, what will the Engliſh yard be worth after the rate?*

Anſwer, Say by the *Rule of three, if* $\frac{13}{50}$ of a yard coſt 2 $\frac{1}{2}$ s. what one yard? Multiply and divide and you ſhall finde 9 s. 7 $\frac{5}{13}$ d.

More, if 346 $\frac{1}{2}$ *Pawnes coſt* 30 li. 13 s. 4 d. *ſterling, what is that the Engliſh yard after the rate?*

Anſwer, Say by the *Rule of three*, if 346 $\frac{1}{2}$ Pawns coſt 30 $\frac{2}{3}$ pounds what are 3 $\frac{1}{13}$ pawns worth (for ſo many Pawns make a yard?) Multiply and divide, and you ſhall find $\frac{2200}{27027}$ parts of a pound, which in known numbers is worth 6 s. 9 d. $\frac{6271}{9009}$.

The eleventh Chapter intreateth of Rules of Loan and Intereſt, with certain neceſſary queſtions and proofs incident thereunto, &c.

ITem, *I lent my friend* 326 *pounds for* 5 $\frac{1}{2}$ *months ſimply without any Intereſt, upon condition, to have the like courteſie againe when I need. But when I came to borrow, he could ſpare me but* 149 li. 8 s. 4 d. *The queſtion is now how long time I ought to hvea the uſe thereof, to countervaile my friendſhid before time ſhewed him?*

Anſwer, Say by the backer *Rule of three*, if 326 pounds give 5 $\frac{1}{2}$ months, what time will 149 $\frac{5}{12}$ pound, give?

give? Multiply and divide, and you ſhall finde twelve moneths, and ſo long time ought I to uſe his money.

The Proofe.

Item, *lent my friend* 149 li. 8 s. 4 d. *for twelve months. The queſtion is now how much mony he ought to lend me again for* 5 $\frac{1}{2}$ *moneths to recompence my friendſhip ſhewed him?*

Anſwer, Say by the Backer or Reverſe *Rule of three.* If twelve months give 149 $\frac{5}{12}$ what ſhall 5 $\frac{1}{2}$ months give? Work, and you ſhall finde 326 pounds, and ſo much ought he to lend me to requite my gentleneſſe or good turn.

Two other branches, yet more, for proofe out of the ſame queſtion.

Item, *lent my friend* 149 li. 8 s. 4 d. *for* 12 *months, to have the like friendſhip again when I need. And comming to borrow of him, he very courteouſly took me* 326 *pounds (for that he could well then ſpare the ſame) The queſtion is now, how long I ought to occupy it, not uſurping friendſhip, but in his due time to reſtore it again.*

Anſwer, Say by the *Rule of three* reverſe, if 149 $\frac{5}{12}$ pounds give 12 moneths, what ſhall 326 pounds give? Multiply and divide, and you ſhall finde that at 5 $\frac{1}{2}$ moneths terme, I ought to reſtore it again.

Proofe.

Item, *Lent my friend* 326 *pounds for* 5 $\frac{1}{2}$ *moneths The queſtion is now, how many pounds he ought to lend me for* 12 *moneths to recompence this pleaſure again.*

Scholar. Work by the *Rule of three* reverſe, as you have

have done before, and you ſhall finde 149 li.—8 s. —4 d.

Again four other ſelected queſtions, of Loan and Intereſt, all out of one branch, and each one alſo a neceſſary queſtion, and a particular proofe to other.

ITem, *Lent my friend* 430 *pounds at Intereſt for three moneths, to receive after the rate of* 8 *pounds in the* 100 *pounds for* 12 *moneths. The queſtion is what the Intereſt commeth to.*

You may, if you pleaſe, work it at two workings by the *Rule of three* direct, in ſaying, if 12 moneths give 8 pounds, what giveth three moneths? Multiply and divide; and it giveth 2 pound.

Then for the ſecond work ſay: If a hundred pound yeeld 2 pounds, what yeeldeth 430 li? Multiply and divide, and you ſhall find 8 li. 12 s. and ſo much comes the loane of 430 li. to for 3 moneths after the rate of 8 pound in the hundred pounds of 12 moneths.

Otherwiſe wrought thus by the rule of three at twice alſo.

If 100 pounds give 8 pounds, what giveth 430 pounds? Multiply and divide, & you ſhall find 34 li. $\frac{2}{5}$. Then again for the ſecond work ſay: If 12 moneths give 34 pounds $\frac{2}{5}$ what giveth three moneths? Worke and find 8 li, 12 s. as before.

Otherwiſe yet at one working: By the firſt part of the rule of five numbers forward, in ſaying, if 100 pounds in twelve moneths gain 8 pounds, what ſhall 430 pounds gain in three moneths? Mul-

tiply the firſt by the ſecond for your Diviſor, and the other three, the one into the other for the dividend, and you ſhall finde eight pounds 12 ſhillings, as aforeſaid.

Proofe.

Item, *A friend of mine received of me* 8 *pounds* 12 *ſhillings for the Intereſt and Vſe of* 430 *pounds for three moneths terme: The queſtion is now, what he tooke in the* 100 *pound for* 12 *months after that rate?*

Anſwer, For moſt brief, ſay by the firſt part or rule of five numbers forward: If 430 pounds in three moneths did pay 8 pound 12 ſhillings, what doth 100 pound in 12 moneths take after the rate? Work, and you ſhall finde 8 pounds, and ſo much he took upon the 100 pounds for 12 moneths.

A third Queſtion and proof alſo by the Backer Rule of five Numbers.

Item, *I lent my friend* 430 *pounds to receive for the intereſt thereof, after the rate of* 8 *pounds in the* 100 *for* 12 *moneths. The queſtion is now how long time my friend ought to give the uſe thereof, that it may be returned with* 8 *pounds* 12 *ſhillings gains.*

You may work it, if you pleaſe, by the Rule of three direct at twice, in ſaying: If 100 pounds, yeeld 8 pounds, what yeeldeth 430 pound? Multiply and divide, and find 34 pound and $\frac{2}{5}$.

Then again for the ſecond work ſay, if 34 $\frac{3}{5}$ pounds, give twelve moneths, what giveth 8 $\frac{2}{5}$ pounds? Multiply and divide, and you ſhall finde three moneths, and ſo long time ought my friend to uſe it to return with 8 li. 12 s. gain.

Other-

Otherwise at one working by the Backer Rule of 5 numbers, in saying, if 100 li. in 12 moneths doe gain 8 li. how much time shall 430 li. be a gaining of 8 li. 12 s.? Multipliy the first and the second into the last for your dividend, and the third and fourth multiply together for your Divisor, and then divide, and you shall finde three moneths, the just time that my friend ought to use it to return it with 8 li. 12 s. gain.

A fourth derived question out of this Branch, which is a proof of this last, and also of the other two going before.

ITem, *How much money, ought a Merchant to deliver after* 8 li. *in the* 100 *for twelve moneths, that in three moneths he may gain* 8 *pounds twelve shillings.*

Answer, You may also if you please, work it by the Golden Rule of three at twice: first saying, If three moneths give 8 $\frac{3}{5}$ pound, what 12 moneths gain? You shall finde 34 $\frac{2}{5}$. Then say again, If 8 pounds be come of 100 pounds, what shall come of 34 pounds 8 shillings? Work, and you shall finde the answer to the question, which is 430 pounds, and so much ought the Merchant to deliver.

But most briefly it is answered by the Backer Rule of 5 numbers, where I argue thus, saying: If 100 pounds be 12 moneths a gaining of 8 pound, then but for three moneths terme onely to take 8 pounds 12 s. must needs be a good round summe to work it, set your numbers thus: 100 ——— 12 — 8 ——— 3 ——— 8 $\frac{3}{5}$ multiplying the first into the second, and also by 43 the

the product of the fifth, for your dividend, and the third and fourth together with 5. the Denominator of your fraction for your Divisor: then divide, and you shall find as before 430 pounds: the true solution to your question.

The twelfth Chapter intreateth of the making of Factors, which is taken in two sorts.

THe first is, when the estimation of the Factor is taken upon the sending of the Merchant, as if the estimation of his person be $\frac{1}{4}$ it is understood that he shall have $\frac{1}{4}$ of the gain, the Merchant the other $\frac{1}{4}$.

The other sort is, when the estimation of his making is out of the sending of the Merchant, as if the order and agreement between them were such, that the Merchant shall put in 800li. and the Factor for his making shall have $\frac{1}{4}$: neverthelesse he shall have but $\frac{1}{5}$ of the gain or profit, for the $\frac{1}{4}$ of 800 is 200 (for the estimation of his making) which with the 800 pounds in all make 1000 pounds, whereof the 200 li. is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, under such condition, that the said Factor shall have the $\frac{1}{4}$. And after certain time they finde in profit 124 li. 6 s. 8 d. I demand how much the Merchant shall have hereof, and how much ought the Factor to have?

Answer. *When the estimation of the Factor is out of the sending of the Merchant, it maketh*,

li.

li.	s.	d.		
99	9	4	for the	Merchant.
24	17	4		Factor.

But if that his estimation be at the sending of the Merchant, then it maketh but,

li.	s.	d.		
93	5	0	for the	Merchant.
31	1	3		Factor.

For the Merchant is then to have $\frac{1}{4}$*, and the Factor* $\frac{1}{4}$*.*

A Merchant doth put into the hands of his Factor 800 pounds, and the Factor 400 li. to have the $\frac{1}{5}$ part of the profit: I demand now for how much his person is estemed, when the same is counted upon the sending of the Merchant.

Answer. *According to the Tenour and order before prescribed in the first Rule, that is, if his estimate be* $\frac{1}{4}$ *he shall have the* $\frac{1}{4}$ *of the gain. Therefore say by the* Rule of three direct: *If* $\frac{1}{4}$ *taken put in 400 pound, what is the estimate, or putting in of* $\frac{1}{5}$ *taking? Multiply and divide, and you shall finde 320 pounds, and so much is the person of the Factor estimated.*

Otherwise.

To finde the estimation of the person of the Factor, you shall consider, that seeing it was agreed between them, that the Factor should take the $\frac{1}{5}$, then the Merchant shall have the residue, which are $\frac{4}{5}$: wherefore the gain of the Merchant unto that of the Factor is in such proportion as 5 unto 4. Then if you will know the estimation of the person of

of the Factor, say, If 5 give 4, what will 400 give? Multiply and divide and you shall finde 320 pound. And so much is the person of the Factor esteemed to be worth.

Other conditions then these aforesaid, may also be between Merchants and Factors, without respect eigher of sending, or not sending of the Merchant, where most commoly the estimation of the body of the Factor is in such proportion of the stock which the Merchant layeth in, as the gain of the said Factor is unto the gain of the merchant. As thus, if a merchant doe deliver into the hands of his Factor 400 pound, and he to have half the profit, the person of the said Factor shall be esteemed to be worth 400 pound: and if the Factor do take but $\frac{1}{3}$ of the gain, he should have but $\frac{1}{2}$ so much of the gain as the merchant taketh, which must have $\frac{2}{3}$, wherefore the person of the Factor is esteemed but the $\frac{1}{3}$ of that which the merchant layeth in, that is to say, two hundred pound.

And if the Factor did take the $\frac{2}{5}$ of the gain, then the merchant shall take the residue which are $\frac{3}{5}$, wherefore the gain of the merchants unto the Factor is then in such proportion as 3 unto 2: whereupon if you will then know the estimation of the person of the Factor, say, if 3 give 2, what shall 400 give? Work, and you shall finde 266 $\frac{2}{3}$ pounds. And so much is the person of the Factor esteemed to be worth.

And if the merchant should deliver unto his Factor 400 pound, and the Factor would lay in 80, and his person, to the end he might have the $\frac{1}{2}$ of the gain, I demand how much shall his person be esteemed?

Answer, *Abate* 80 *from* 400, *and there will remain* 320. *And at so much shall his person be esteemed.*

A

A Merchant hath delivered unto his Factor 900 pounds to govern in the Trade of Merchandize, upon condition that he shall have the $\frac{1}{3}$ of the gain if any thing be gained, and also to bear the $\frac{1}{3}$ of the losse, if any thing be lost. Now I demand how much his person was esteemed at?

Answer, *Seeing that the Factor taketh the $\frac{1}{3}$ of the gain, his Person ought to be esteemed as much as $\frac{1}{2}$ of the stocke which the Merchant layeth in: that is to say, the $\frac{1}{2}$ of 900 pound, which is 450. The reason is, because $\frac{1}{3}$ of the gain that the Factor taketh is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gaine that the Merchant taketh, & so the Factor his person is esteemed to be worth 450 pounds.*

A Merchant hath delivered unto his Factor 600 pounds, and the factor layeth in 250 pound, and his person. Now because he layeth in 250 pounds, and his person, it is agreed between them, that he shall take the $\frac{2}{5}$ of the gain. I demand for how much his person was esteemed?

Answer, *For as much as the Factor taketh $\frac{2}{5}$ of the gain, he taketh $\frac{2}{3}$ of that which the Merchant taketh, for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therefore the Factors laying in ought to be 400 pound, which is $\frac{2}{3}$ of 600 pound that the Merchant laid in. Then subtract 250, which the Factor did lay in, from 400 pound which should have been his whole stock, and there remaineth 150 pound for the estimation of his person.*

More, a Merchant hath delivered unto his factor 800 li. upon condition that the factor shall have the gain of 160 li. as thought he laid in so much ready mony: I demand what portion of the gain the factor shall take?

Answer, *See what part the 160 (which the Factor laid in) is of 960, which is the whole stock of their company,*

company, and you shall find $\frac{1}{6}$: And such part of the gain shall the Factor take.

But in case that in making their Covenants it were so agreed between them, that the Factor should have the gain of 160 pound of the whole stock which the Merchant layeth in, that is to say, of the 800 pound: then should the Factor take $\frac{1}{5}$ of the gains, for 160 is $\frac{1}{5}$ of 800 pound.

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandize, which is distinct into seven Rules, with divers other necessary questions incident thereunto.

The first Rule.

TWo Merchants, willing to change their Merchandize the one with the other: The one hath 24 broad clothes at 10 li. 10 s. the piece: The other hath Mace at 12 shillings the pound. The question is, how many pounds of Mace he ought to give for his Cloth, to save himselfe harmlesse, and be no loser?

Answer, *Seek first by the* Rule of three *what the 24 Cloths cost at* 10 *pound* 10 *shillings the piece, and you shall finde* 252 *pound: Then to finde the quantity of Mace, say again by the* Rule of three, *If* 12 *shillings buy one pound, what shall* 252 *pound buy me? Worke, and you shall finde* 420 *pound of Mace: And so many pound ought he to give for his Clothes.*

The Proof.

Two barter. The one hath 420 pounds of Mace, at

at 12 s. the pound, to barter or change broad Clothes, at 10 pounds 10 shillings the piece. The question is, how many broad Clothes he ought to give for all his Mace?

Answer, *First say, If one cost 12 shillings, what 420? you shall finde 5040 shillings. Then say again, If* $10\frac{1}{2}$ *pounds give one cloth, what shall 5040 shillings give? Worke, and you shall finde 24 Clothes, your desire.*

The second Rule.

Two change merchandize for merchandize: The one hath Pepper at two shillings foure pence the pound, to sell for ready money. But in barter he will have no lesse then three shillings the pound. And the other hath Holland at five shillings six peace the Ell ready money. The question is now, at what price he ought to deliver the Ell in the barter to save himself harmlesse.

Answer, *Say by the* Rule of Three direct: *If* $2\frac{1}{3}$ *ready money give 3 shillings in barter, what shall* $5\frac{1}{2}$ *give in barter? You shall finde* $7\frac{1}{14}$ *shillings, and at that price ought the second merchant to sell his Holland in barter.*

The Proof.

Two barter. The one hath Holland at 5 s. 6 pence the Ell to sell for ready money. And in barter he will have $7\frac{1}{14}$ shillings. The other hath Pepper at 2 s. 4 pence the pound, to sell for ready money. The question is now how he ought to sell in barter.

Answer, *Say by the* Rule of Three direct, *If* $5\frac{1}{2}$ *ready mony give* $7\frac{1}{14}$ *shillings, in barter; what ought* $2\frac{1}{3}$ *to take in barter? multiply and divide and you shall finde 3 shillings your desire.*

The

The third Rule.

Two barter. The one hath cloth of Arras at 30 s. the Ell ready money, but in barter he will have 35 $\frac{1}{2}$ s. And the other hath white Wines which he delivered in barter for 16 pounds the Tun. The question is now: what his Wines cost the Tun in ready mony.

Answer, *Say by the* Rule of Three direct, *If 35 $\frac{1}{2}$ shillings in barter, give but 30 shillings ready money, what did 16 pound in barter cost? Worke, and you shall finde 13 pound 10 shillings $\frac{30}{71}$. And so much cost his Wines for a Tun ready money.*

The proofe.

Two barter Merchandize for Merchandize: The one hath white Wines at 13 pounds 10 s. $\frac{30}{71}$ s the Tun to sell for ready mony: But in barter he delivered it for 16 li. The other, to make his match good and save himself harmlesse, delivereth Arras at 35 $\frac{1}{2}$ s. the ell. The question is now, what an ell of his Arras cost in ready mony?

Answer, *Say by the* Rule of Three direct: *If 16 pounds in barter give but 13 pounds 10 $\frac{30}{71}$ shillings, in ready money, what shall 35 $\frac{1}{2}$ shillings yeeld in barter? Work, and you shall finde 30 shillings your desire.*

The fourth Rule.

Two barter: The one hath Kerseyes at 14 pounds the piece ready money: But in barter he will will have 18 pounds: and yet he will have the $\frac{1}{3}$ part of his overprice in ready money. And the other hath Ginger at eight groats the pound to sell for ready money. The question is, how he ought to deliver the Ginger by the pound in barter to save himselfe harmlesse, and make the barter equall.

Answer,

Answer, *Item*, for the working of this question, and such other the like, you must understand, if the party over-selling his wares, require to have also some portion in ready mony, as $\frac{1}{2}\frac{2}{3}\frac{3}{4}$. &c. Then shall you first rebate the same demanded part, whatsoever it be, from the over price, and also from the just price. And those two numbers that shall remain after the subtraction is made, shall be the two first numbers in the *Rule of Three*. And the just price of the same Merchandize shall be the third number, which by the operation of the *Rule of Three direct*, shall yeeld you a true solution, how, and at what price you shall over-sell that your merchandize, to have your self harmlesse, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighteen) which is the over-price of his Cloth, which $\frac{1}{3}$ of eighteen is six, which you must subtract from 18, there rest 12. And also abate it from 14, which is the just price of the Cloth and there remaineth 8, which 8 and 12 are the two first numbers in the *Rule of Three*. Then take eight groats, or 2 $\frac{2}{3}$ shillings for the third number. Then say by the *Rule of Three direct*. If eight pounds give 12 pounds, what shall 2 $\frac{2}{3}$ s. give? Multiply and divide, and you shall finde 4 shillings. And for so much shall the second Merchant sell his Ginger, or his commodity in barter, to ballance the same equall.

14 ——	18
6	6
8	12

The Proof.

Two barter; the one hath fine Kerseys, at 14 *pounds the piece ready money: But in barter he will have* 18 *pounds:*

pounds: and yet he will have the $\frac{1}{3}$ part of his overprice in ready money. And the other hath Ginger, which he having cunning enough to make the barter equall, delivered in barter for 4 shillings the pound. The question is now, what his Ginger cost him in ready money?

Answer, After you have made the subtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14 (as before was taught you:) then will there remain 8 and 12 for your two first numbers in the *Rule of three*. Then say, If 12 give 8, what shall come of 4 the over-price of the pound of Ginger? Multiply and divide, and you shall find 2 s. 8 pence, your desire.

Two Merchants barter Merchandize for Merchandize. The one hath Devonshire *whites at 7 pound 13 shillings 4 pence the peice ready mony: but in barter he doth them away for 8 pound, 3 shillings 4 pence, and yet he will have the $\frac{1}{3}$ part of his price in ready money. And the other hath Cottens at three pounds the piece ready money. The question is now, at what price he ought to sell or exchange his Cottens in barter to save himselfe harmlesse, and make the barter equall?*

li.	s.	d.	li.	s	d.
7	13	4	8	3	4
2	14	$5\frac{1}{3}$	2	14	$5\frac{1}{3}$
4	18	$10\frac{2}{3}$	5	8	$10\frac{2}{3}$

Answer, First seek the $\frac{1}{3}$ part of 8 li. 3 s. 4 d. which is 2 li. 14 s. 5 $\frac{1}{3}$ d. which rebated from 8 li. 3 s. 4 d. there resteth as appeareth by the Example abovesaid, 5 li. 8 s. 10 $\frac{2}{3}$ d. which is $\frac{2}{3}$ parts of 8 li. 3 s. 4 d. also rebated from 7 lib. 13 s. 4 d. there resteth 4 li. 18 s. 10 $\frac{2}{3}$ d. the two first numbers in the *Rule of Three*, and the three pounds, which is the neat price of

of the piece of Cotten, is the third number : Then say by the *Rule of Three direct*, as was taught before. If 4 li. 18 s. 10 $\frac{2}{3}$ d. give 5 li. 8 s. 10 $\frac{2}{3}$ d. what shall three pounds give ? Multiply and divide, and you shall find three pounds 6 $\frac{6}{8}$ s. the just price that he ought to deliver his Cottens in barter.

The fifth Rule.

Two Merchants will change Merchandize for Merchandize. The one hath Kerseys at 40 s. the piece to sell them for ready money. And in barter he will sell them for 56 s. 8 d. and he will gain after ten pound upon the 100 pound. And yet he will have the $\frac{1}{2}$ of his over-price in ready money. The other hath Flax at 3 d. the pound ready money, The question is now, how he shall sell the pound of his Flax in barter ?

Answer, See first at 10 pound upon the 100 pounds what the 56 $\frac{2}{3}$ s. commeth to, in saying (by the *Rule of Three direct*) If 100 pounds give 110 pounds, what 56 $\frac{2}{3}$ s ? Multiply and divide, and you shall finde 3 pound 2 shillings 4 pence, of which the $\frac{1}{2}$ that he demandeth in ready money, is 1 pound 11 shillings 2 pence; the same 31 s. 2 d. abated from 40 shillings, and also from 56 s. 8 d. there will remaine 8 s. 10 d. and 25 s. 6 d. for the two first numbers in the *Rule of Three*, and 3 pence the price of the pound of Flax for the third number. Then multiply and divide, and you shall find 8 $\frac{3}{5}\frac{1}{3}$ d. And for so much shall he sell the pound of Flax in barter.

The sixt Rule.

Two are willing to exchange Merchandize : The one hath Norwich *Grograns at 25 s. the piece ready*

money: and in barter he will have 30 s. and he will have the $\frac{1}{4}$ part of his over-price in ready money. The other hath Norwich *Stockins, at* 40 s. *the dozen to sell for ready money. But in as much as the first Merchants Grogranes are no better, he would deliver them so to ballance the barter, that he may gain* 10 *pounds in the* 100 *pounds. The question is now, how he shall sell his hose the dozen in barter, according to his request.*

Answer, Say, if 100 give 110 li. what shall 40 s. give, which is the just price of the dozen of stockins? Multiply and divide, and you shall finde 44 s. Then take the $\frac{1}{4}$ of 30 s. which is 7 s. 6 d. and subtract it from 25 s. and also from 30 s. and there will remaine 17 s. 6 d. and 22 s. 6 d. for the two first numbers in the *Rule of three*, and 44 shillings, which is the just price (with his gain in the dozen of Stockins) for the third number. Then multiply and divide, and you shall finde 56 s. 6$\frac{6}{7}$ d. and for so much he is to sell his dozen of stockins in barter.

The seventh Rule.

Two Merchants will change their Merchandize one with the other: The one hath 720 *Ells of Cambricke at* 5 s. *the Ell to sell for ready money, but in barter he requireth* 6 s. 8 d. *And yet notwithstanding, he loseth by it after* 10 *pounds upon the hundred pounds, whereupon he requireth one halfe of his over-price in ready money: and the other Merchant having skill enough to make the barter equall, delivered* English *Saffrons at* 30 s. *the pound, The question is now, what his Saffrons cost the pound in ready money.*

Ans. You must first seeke what is lost upon the 100 li. which to do, you may say (if you please) if 100 pound

pound lose 10, what shall $6\frac{2}{3}$ lose? Work, and you shall finde $\frac{2}{3}$ s. (or 8 d.) which must be rebated from 6 s. 8 d. so resteth 6 s. still. Or you may say, If 100 pound give me but 90 pounds, what shall 6 s. 8 d. give? Work this way either, and you shall find also as before directly in your quotient 6 s. your desire. Then are you next to cast up what the 720 Ells of Cambrick commeth to at 6 s. 8 d. the Ell, and you shall find 240 pounds: the $\frac{1}{2}$ whereof the Cambrick Merchant will have in ready money (which is 120 pounds:) Nextly, you must cast what the Cambrick commeth to after his losse in the 100 pound, which as you found, is but 6 s. an Ell, and you shall find 216 pounds: Now must you subtract his ready money (which is 120 pounds in all) out of 240 pound; and also out of 216 pound, and there will remain 120 pounds, and 96 pounds for your two first numbers in the *Rule of three*, and 30 shillings is the over-price of your Saffron for the third number: Then multiply and divide, and you shall finde 24 shillings. And so much did his Saffron cost in ready money.

Two Merchants barter; the one hath fifty Clothes to put away for ready money at 11 *pounds the Cloth, and in barter putteth them away for* 12 *pounds, taking Holland Cloth at* 20 d. *the Flemmish Ell, which was worth no more but* 18 d. *The question is now, what Holland payeth for the Cloth, and what he winneth or loseth by the bargain?*

Answer, Fifty Clothes at 11 pounds the Cloth commeth to 550 pounds, and put away at 12 pounds the piece, maketh 600 pound. Then to finde what Holland payeth for the Cloth, say by the *Rule of three direct*, If 20 d. buy one Ell, what 600 pounds?

Worke, and you shall finde 7200 Ells. Now to finde the Estate of his gain or losse, you must seek what his 7200 Ells commeth to at 18 d. the Ell: Work by the Rule of proportion direct, and you shall finde 540 pounds, which is not so much as his Clothes were worth in ready money by tenne pounds: and so much lost the first Merchant by his Exchange.

A Venetian *hath in* London 100 *peices of silk, to put away for ready money at* 3 li. *the piece. But in barter he delivered them for* 4 li. *the piece, taking wools of a Felmonger at* 7 li. 10 s. *the* C. *weight, which was worth no more but six pounds the* C. *ready money. The question is now, what wools payeth for the silkes, & which of them winneth or loseth by the barter?*

Ans. A hundred pieces of Silke at 3 li. is in all 300 li. and at 4 li. is 400li. Then to find what Wools pay for the Silke, say by the *Rule of three direct*: If $7\frac{1}{2}$ li. buy me 1 C. weight, what 400? Work, and finde $53\frac{1}{3}$ C. weight of wooll. Now to find the Estate of their gaine and losse, cast up his wooll at 6 lib. the C. (for so much they were worth ready money) and you shall finde 320 pound which is 20 pound more then the Silks were to be sold for ready money, whereby the *Venetian* gained 20 pounds by the Barter.

A Merchant hath $53\frac{1}{3}$ *weight of Wooll at* 6 *pounds the* C. *to sell for ready money, but in barter he will have* 7 *pounds* 10 s. *and another doth barter with him for Silks, which are worth three pounds a piece ready money. The question is now, how he ought to deliver his Silks the peice in barter, and how many payeth for the Wools.*

Answer, Say by the Rule of proportion, (or by the *Rule of three direct*) If, 6 pounds for C. weight ready money,

money, yeeld me 7 li. 10 s. what will 3 li. yeeld, which is the just price of a piece of Silk in barter, to make the Truck equall? Work, and find 3 li. 15 s. the price of a piece of Silk, in barter: then say, If 3 li. 15 s. require one piece of Silk, how many pieces of Silk are bought with 400 pound, which is the value of 53 $\frac{1}{3}$ C. weight of wooll, at 7 li. 10 s? Worke by the *Rule of three direct*, and you shall finde 160 pieces of Silk and $\frac{2}{3}$ of a piece, and so many of Silk pay for the wooll, and neither party hath advantage of other.

Two men will change Merchandize, the one with the other. The one of them hath beer at 6 s. 8 d. *the barrell, to sell for ready money, but in barter he will sell the barrell for* 8s. *and yet he will gain moreover after* 10 *pound upon the* 100 *pounds. And the other hath white* Spanish *Wooll at* 20 s. *the Rove, to sell for ready money. The question is now, how he shall deliver the Rove of Wooll in barter to save himself harmlesse.*

Answer, Say, if 6 $\frac{2}{3}$ s. which is the just price of the barrell of Beer, be sold in barter for 8 shillings: for how much shall 20 shillings (which is the just price of the Rove of Wooll) be sold in barter? Work by the Rule of Three direct, and you shall finde 24 s. Then for because the first Merchant will gain after 10 pounds upon the 100 pounds, he maketh his 100 pounds, 110 pounds. And therefore say by the Rule of Three, If the second Merchant of 110 pounds doe make but 100 pounds, how much shall he make of 24 s? Multiply and divide, and you shall finde 21 s. 9 d. $\frac{9}{11}$ of a peny. And for so much shall he sell the Rove of Wooll to be delivered in barter, to the end the first Merchant may give 10 in the 100.

Two Merchants will change their Commodities the one

one with the other. The one of them hath white Paper at 4 s. the Reame, to sell for ready money. And in barter he will doe it away for 5 s. and yet he will gain moreover after the rate of 10 pounds upon the 100 pounds. And the other hath Mace at 14 s. 6 d. the pound weight to sell in barter. Now I demand what the pound did cost in ready money.

Answer, Say, if 5 s. (which is the over-price of the Paper in barter) be come of 4 s. the just price, of how much shall come $14\frac{1}{2}$ shillings, which is the surprize of the pound of Mace in barter? Multiply and divide, and you shall find $11\frac{3}{5}$ s. Then for because the first Merchant of Paper will gain after 10 upon the 100, Say if 100 doe give 110, what shall $11\frac{3}{5}$ shillings give? Worke and you shall find 12 s. $9\frac{3}{25}$ d. and so much did the pound of Mace cost in ready money.

The fourteenth Chapter intreateth of exchanging of money from one place to the other.

EXchange is no other thing, then to take or receive money in one City, to render or pay the value thereof in another City, or else to give money in one place, and receive the value thereof in another, as term of certain dayes, moneths, or fairs, according to the diversity of the place.

But this practice chiefly consisteth in the knowledge of the money or Coynes in divers places, of which for the benefit (after a few examples given to the Introduction of this work) I will set down certain notes of the diversity

diversity of the common and usuall Coynes in most places in Christendome for trafficke.

And first I will begin at *Antwerp*, where they use to make their accounts by *Deniers de grosse*, that is to say, pence *Flemmish*, whereof 12 doe make 1 s. *Flemmish*, and 20 s. doe make one pound *de grosse*.

Item, *A Merchant delivered at* Antwerp, 400 *pound* Flemmish *to receive in* London 20 s. *sterling, for every* 23 s.——4 d. Flemmish : *The question is now, how much sterling money is to be received at* London *for* 400 *pounds* Flemmish ?

Answer, Say by the Rule of Three, If 23 $\frac{1}{3}$ *Flemmish* give 20 s. sterling, what 400 pounds *Flemmish*? Worke, and you shall finde 342 l——17 s.——1 $\frac{5}{7}$ pence, and so much sterling shall I receive in *London* for the said 400 pounds *Flemmish*.

Otherwise also wrought by Rules of Practice in taking the $\frac{1}{7}$ of the *Flemmish* money delivered, and abating the same from the principall, the rest is *English* money, as before.

400 li———	0 s.———	0 d.
57———	2———	10 $\frac{2}{7}$
342———	17———	1 $\frac{5}{7}$ sterling.

A Merchant at London *delivered* 200 lib. *sterling for* Antwerp, at 23 s.——5 d. Flemmish *the pounds sterling : the question is, how much he must receive at* Antwerp.

Answer, Say by the Rule of Three, if 1 pound sterling give 23 s. 5 d. *Flemmish*, what 200 li. sterling? Work, and thou shalt finde 234 li.——3 s.——4 d. So many pounds *Flemmish* shall he receive at *Antwerp* for the said 200 pounds sterling.

Otherwiſe by Practice.

	1 —— 3 —— 5 —— 200	
3 s. - 4 d.		33 —— 6 —— 8
1 d.		—— 16 —— 8
maketh ſterling		234 li. - 3 s. - 4 d

In London 20 *pound ſterling is delivered by Exchange for* Antwerp, *at* 23 s. 9 d. Flemmiſh *the pound ſterling: the queſtion is, at what rate the* Flemmiſh *money ought to be returned to gain foure pounds upon the* 100 *pound ſterling at* London.

Anſwer, Firſt, ſay by the Rule of three direct: If 1 pound ſterling give 23 $\frac{3}{4}$ Flemmiſh, what 200 pounds ſterling? Multiply and divide and you ſhall finde 237 pounds 10 ſhillings. The which to return to gain 8 pounds ſterling in *London*, ſay by the backer Rule, If 200 pounds ſterling require the exchange 23 s. 9 d. Flemmiſh, what the exchange to make 208 li. ſterling? Work by the Rule, and finde 22 s. 10 $\frac{1}{26}$ d. Flemmiſh, the effect in the queſtion required.

If I take up money at Antwerp *after* 19 s. 4 d. Flemmiſh, *to pay for the ſame at* London, 20 s. *ſterling, and when the day of payment is come, I am forced to return the ſame money again in* London, *to pay my Bill of Exchange: ſo that for* 20 *ſhillings which I take up here at* London, *I muſt pay* 19 s. 6 d. *at* Antwerp, *I demand whether I doe win or loſe, and how much in or upon the* 100 *pounds of money?*

Anſwer, Say by the Rule of three: If 19 $\frac{1}{2}$ give 19 $\frac{1}{3}$ what will 100 pounds give? Multiply and divide, and you ſhall finde 99 li. 2 $\frac{16}{117}$ s. which being abated from 100 pounds, there will remain 17 s. $\frac{11}{117}$, and ſo much I do loſe upon the 100 pounds of money.

If

If I take up at London *20 ſhillings ſterling to pay at* Antwerp 22 s. 4d. *and when the day of payment is come, my Factor is conſtrained to take up money again at* Antwerp, *wherewith to pay the aforeſaid ſumme, and there he doth receive* 23 s. 4 d. Flemmiſh, *for the which I muſt pay* 20 s. *at* London : *The queſtion is now, whether I doe win or loſe, and how much upon the* 100 lib. *of money after that rate.*

Anſwer, Say by the Rule of Proportion, If 22 $\frac{1}{3}$ s. give 23 $\frac{1}{3}$ s. what will 100 pounds give ? Multiply and divide, and you ſhall finde 104 pounds 9 ſhillings $\frac{12}{67}$, from the which abate 100 pounds, and there will remain 4 pounds 9 ſhillings $\frac{12}{67}$, and ſo much is there gained upon the 100 pounds of money.

In Antwerp *is delivered* 200 *pounds* Flemmiſh *by exchange for* London *at* 20 *ſhillings ſterling for every* 23 *ſhillings* 4 d. Flemmiſh, *The queſtion is, at what rate the ſame is to be returned to gain* 10 *pounds upon the* 100 *pounds* Flemmiſh *in* Antwerp?

Anſwer, Firſt, ſay by the Rule of three, if 23 $\frac{1}{3}$ Flemmiſh give 20 s. what ſhall 200 pounds gain ? Work, and you ſhall finde 171 pounds 8 s. 6 $\frac{6}{7}$ d. then ſay again by the Rule of three direct, if 171 pounds 8 s. 6 $\frac{6}{7}$ d. ſterling, give me 210 pounds Flemmiſh, what ſhall 20 s. ſterling give ? Work, and you ſhall finde 24 s. 6 d. Flemmiſh. And at the ſame rate ought the ſame to be returned at *Antwerp*, to gain 10 pounds upon the 100 Flemmiſh.

A Merchant of Antwerp *delivereth* 234 lib. 3 s. 4 d. Flemmiſh, *to receive at* London 200 li. *ſterling* : *The queſtion is now, how the exchange goeth after this rate?*

Anſwer, Say by the Rule of three direct, if 200 give 20, what 234 $\frac{1}{6}$? Multiply and divide, and you ſhall

shall find 23 s. — 5 d. And for so much goeth the exchange.

Item, *the exchange from* London *into* France, *is not like as it is to* Flanders, *but it is delivered by the* French *Crown, which is worth* 50 Soulx Turnois *the piece.*

Whereupon also you must note that in *France* they make their accounts by Franks, Soulx, and Deniers Turnois, whereof 12 Deniers make one Soulx Turnois, and 20 Soulx maketh one pound Turnois, which they call a Liure or Frank. But the Merchants to make their Accounts, doe use French Crowns, which is currant among them for 51 Soulx Turnois. But by exchange it is otherwise, for it is delivered but for 50 Soulx Turnois the Crown, or as the taker up of the money can agree with the deliverer. And note that this △ Character representeth the Crown by exchange, and is ever 50 Soulx Turnois or *French* money.

A Merchant delivereth at London 240 *pounds sterling, after five shillings sixe pence the Crown, to receive at* Paris 50 *Soulx Turnois for every Crowne. I demand how much* Turnois *or* French *money payeth the Bills for the said* 240 *pounds sterling.*

Answer, Say by the Rule of Three. If $5\frac{1}{2}$ s. sterling give me 50 s. Turnois, what shall 240 pounds sterling give? Reduce the pounds into shillings, then multiply and divide, and you shall finde 2181 Liures, 16 Soulx, 4 Deniers, and $\frac{4}{11}$ Turnois, and so much payeth the Bills at *Paris*, for the 240 pound sterling.

A Merchant delivereth at Roan, *or elsewhere in* France, 1430 *pounds or franks, the which franke or pound is* 20 *Soulx, or a pound* Turnois *to receive in* London

London 6 s. 4 d. *sterling for every* △ *of* 50 *Soulx* Turnois. *The question is, how much sterling money I ought to receive at* London *for my* 1430 *pound* Turnois.

Answer, Say, if 2 $\frac{1}{2}$ pounds give me 6 $\frac{1}{3}$ s. what will 1430 give me? Worke, and you shall find 3622 $\frac{2}{3}$ shillings sterling, which maketh 181 li. 2 s. 8 d. and so much money is to be received at *London*, for the said 1430 Liure Turnois, after 6 s. 4 d. for every △ of 50 Soulx.

In London *is delivered* 200 *pound sterling by exchange for* Paris, *at* 5 s. 9 d. *the* △ *of* 50 *Soulx* Turnois. *The question is, at what price the said* △ *is to be returned to gain* 6 *pounds upon the* 100 *pounds sterling at* London.

Answer, First, say (by the Rule of Three direct) if 5 $\frac{3}{4}$ s. sterling give 50 Soulx Turnois; what shall 200 pound sterling give? Work, and you shall find 1739 Franks of Liures, 2 $\frac{14}{23}$ Soulx. Then the which to return and gaine 6 pounds upon the hundred pounds in *London*, say by the Rule of Three direct, if 1739 Franks 2 $\frac{14}{23}$ Soulx yeeld 212 pound, what the △ of 50 Soulx? work and finde 6 s. 1 $\frac{2}{50}$ d. the effect required in the question.

A Merchant delivered in London 160 li. sterling *to receive in* Biskay *for every* 5 s. 6 d. *one Ducat of* 374 Marvides. *The question is, how many* Marvides *ought I to receive at* Biskay?

Answer, Say, if 5 $\frac{1}{2}$ s. sterling give 374 Marvides: what shall 160 pounds sterling give? Multiply and divide, and you shall finde 217600 Marvides, and so many I ought to receive at *Biskay* for my 160 pounds sterling.

A

A Merchant delivered in Baion, 40000 Marvides *to receive in* London 5 s. 8 d. *sterling for every Ducat of* 374 Marvides. *The question is now, how much sterling money payeth the Bills of Exchange for the said* 40000 Marvides?

Answer, Say, if 374 Marvides make one Ducat, what 40000 Marvides? Multiply and divide, and finde, 106 $\frac{178}{187}$.

Then say again, if 1 Ducat give 5 $\frac{2}{3}$ s. what giveth 106 $\frac{178}{187}$ Ducats? Work, and find 30 l. 6 s. $\frac{34}{561}$ Otherwise it is wrought more briefe at one working, as in the last question before, in considering that 5 s. 8 d. containeth one Ducat, or 374 Marvides. Therefore say by the Rule of Three, if 374 Marvides give 5 $\frac{2}{3}$ s. what 40000 Marvides? Work, and you shall also finde in your quotient 30 l. 6 s. $\frac{34}{561}$ s. And so many pounds sterling is to be received for the 40000 Marvides.

In London 200 *pounds delivered by exchange for* Vigo, 374 *Marvides the Ducat of* 5 s. 10 d. *sterling, maketh* 256457 $\frac{1}{7}$ *Marvides: the which to return and gain* 10 li. *upon the* 100 *pounds in* London, *say by the* Rule of three direct, *if* 220 li. *require* 256457 $\frac{1}{7}$ *Marvides: what* 5 s. 10 d? *Work, and finde* 340 *Marvides, the price of every Ducat in return, which is the effect in the question required.*

These may seem sufficient for instructions.

NOtwithstanding *for thy further aid and benefit, hereafter follow six speciall and most briefe* Rules of Practice, *for* English, French, *and* Flemmish *money.*

How

1 How to turn Flemmish to English sterling.
2 How to turn English sterling to Flemmish.
3 teacheth How to turn Flemmish to French.
4 How to turn French into Flemmish.
5 How to turn Sterling into French.
6 How to turn French into Sterling.

The fifteenth Chapter intreateth of the said six Rules of brevity, and of valuation of *English, Flemmish* and *French* money, and how each of them may easily bee brought to others value.

How briefly to reduce pounds, shillings, and pence Flemmish *into pounds, shillings, and pence* English *sterling.*

IT is to be noted, that 7 pounds *Flemmish* maketh but 6 pounds sterling: 7 s *Flemmish* maketh 6 s sterling, and 7 d. *Flemmish* 6 d. sterling so that 7 yeeldeth but 6. Wherein is evident that then is lost $\frac{1}{7}$, (if it may be so called) when it is reduced into *English* money: wherefore to know how much 233 l.— 13 s.— 4 d. *Flemmish* maketh *English*, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which resteth after this subtraction, is the summe required: so that 233 li.— 13 s.— 4 d. *Flemmish*, maketh 200 li. 5 s. 8 $\frac{4}{7}$ d. sterling. 1 Rule.

Example

Example.			Another Example		
li.	s.	d.	li.	s.	d.
233	13	4	311	0	0
$\frac{1}{7}$ 33	7	$7\frac{2}{7}$	$\frac{1}{7}$ 44	8	$6\frac{6}{7}$
200	5	$8\frac{4}{7}$ Ster.	266	11	$5\frac{1}{7}$

Rule 2. *To reduce pounds, shillings, and pence sterling, into pounds, shillings, and pence* Flemmish.

Note that a pound sterling maketh 1 li. 3 s. 4 d. *Flemmish*: that is, 1 $\frac{1}{6}$ li: 1 s. sterling maketh 1 $\frac{1}{6}$ s. *Flemmish*, and 1 d. sterling maketh 1 $\frac{1}{6}$ d. *Flemmish.* So that there is gained (if it may be so called) $\frac{1}{6}$ of the summe being thus reduced to *Flemmish*, for of 1 $\frac{1}{6}$ is made $\frac{7}{6}$ which is one whole and $\frac{1}{6}$. Then to know how much 237 li. 7 s. 6 d. sterling maketh *Flemmish*, Subtract from your sterling the $\frac{1}{6}$ of the whole summe, & adde it to the same summe, & it maketh 276 li. 18 s. 4 d. which is the summe required.

Example.			Another Example.		
li.	s.	d.	li.	s.	d.
237	7	6 Ster.	337		
$\frac{1}{6}$ 39	11	3	$\frac{1}{6}$ 56	3	4
276	18	9 Flem.	393	3	4

Rule 3. *To reduce pounds, shillings, and pence* Flemmish, *into pounds, shillings, and pence* French.

Ye shall note, that the equality of Flemmish and French money is this, that is to say, the pound Flemmish, maketh 7 pound $\frac{1}{5}$ French, or Turnois. 1 s. Flemmish maketh 7 $\frac{1}{5}$ s. French, and a groat Flemmish maketh 7 $\frac{1}{5}$ d. French.

Where-

Wherefore to know how much 143 li. 4 s. 9 d. Flemmish maketh French, ye must multiply the whole number twice by 6, beginning at pence, and so forward, and the Product of your second multiplication divide by 5, so the work is finished. Or multiply the said summe by 7, and take out of it $\frac{1}{5}$ adding it to the Product of your multiplication by 7. and that is your number required. So that as well by the one as by the other, 143 li. 4-s.--9--d. Flemmish, maketh 1031 li.--6 s.--$2\frac{2}{5}$ d. French or Turnois.

Example.

li.	s.	d.	
143	4	9	Flem.
		6	
859	8	6	
		6	
5156	11	0	Fren.
$\frac{1}{5}$ 1031	6	$2\frac{2}{5}$	Fren.

The same otherwise.

li.	s.	d.
143	4	9
		7
1002	13	3
$\frac{1}{5}$ 28	12	$11\frac{2}{5}$
1031	6	$2\frac{2}{5}$

Another Example.

143 l. Flem.
6

858
6

$\frac{1}{5}$ 5148 French.

1029 li. $\frac{1}{5}$ *or* 12 s. French.

Or thus:

143
7

1001
$\frac{1}{5}$ 28 —— 12

1029 li. —— 12

To

Rule 4. *To reduce pounds, shillings, and pence,* French, *into pounds, shillings, and pence,* Flemmish.

Multiply 233 li.—8 s.—4 d French by 5, and divide the Product twice by 6, that is, the said number by 6, and the Product or Quotient again by 6, and the quotient of this second Division is the thing required. So that 233 li.--8 s.--4 d. French, maketh 32 li.-- 8s.--4 $\frac{5}{9}$ d. Flemmish.

Example.				Another Example.			
li.	s.	d.		li.	s.	d.	
233—	8—	4	Fren.	753			French.
		5		5			
1167—	1—	8		3765—			
$\frac{1}{6}$ 194—	10—	3 $\frac{1}{3}$		$\frac{1}{6}$ 627—	10—		
$\frac{1}{6}$ 32—	8—	4 $\frac{5}{9}$	Flem.	$\frac{1}{6}$ 104—	11-	8	Flem.

Rule 5. *To reduce pounds, shillings, and pence Sterling, into pounds, shillings, and pence,* French, *or Turnois.*

The pound Sterling maketh 8 li. 8 s. *French,* that is to say, 8 $\frac{2}{5}$ pounds: the shillings maketh 8 $\frac{2}{5}$ shillings, and the peny 8 $\frac{2}{5}$ d. *French.* Wherefore to know what 231 li. 13 s. 4 d. Sterling maketh *French,* ye must multiply the whole summe by 42, that is, by 7. and the Product of it by 6. and divide this second Product by 5. and that is the summe required.

Otherwise, multiply the summe Sterling by 8, and adde twice to the Product $\frac{1}{5}$. and it shall produce the summe required. So that both waies 231 li.--13 s. --4 d. Sterling, maketh 1946 pound *French,* as here under followeth.

Example

Example.			*Ster.*	*The same otherwise.*			Sterling:
li.	s.	d.		li.	s.	d.	
231	13	4		331	13	4	
		6				8	
1390	0	0		1853	6	8	
		7		46	6	8	
$\frac{1}{5}$ 9730	0	0		46	6	8	
				1946	0	0	
$\frac{1}{5}$ 1946	0	0	*Fren.*				French.

Another example.		The same.		Sterling.
753	ster.	753		
6		8		
4518		6024		
7	$\frac{1}{2}$	150	12	
31626	$\frac{1}{2}$	150	12	French.
$\frac{1}{5}$ 6325 — 4 *Fren.*		6325	4	Rule 6.

To reduce pounds, shillings, and pence, French, *into pounds, shillings, and pence, sterling.*

To know how much 1256 li. 12 s. 6 d. *French* maketh in sterling money: multiply the summe by 5, and divide the Product by 7 and 6 at twice, and the last Quotient shall be the thing required, that is to say, 1256 li. 12 s. 6 d. maketh 149 pounds, 11 s. 11$\frac{4}{7}$ d. sterling.

Example,				*Another example.*			
li.	s.	d.	*French.*	li.	s.	d.	
1256	12	6		2531	0	0	*French.*
		5		5			
6283	2	6		12655			
$\frac{1}{6}$ 1047	3	9		$\frac{1}{6}$ 2109	3	4	
$\frac{1}{7}$ 149	11	$11\frac{4}{7}$ Ster.		$\frac{1}{7}$ 301	6	$2\frac{2}{7}$ Ster.	

Note, that when any money is given by exchange at *London* for *Roan* at $71\frac{1}{2}$ d. or rather $71\frac{1}{7}$ for the Crown of 50 s. French, there is neither gain nor losse: for it is one money for another, accounting 8 li. 8 s. French, for one pound sterling. So the giver loseth the time of payment, which is about 15 dayes, and he that taketh it, hath the gain of the same.

They of *Roan*, that put forth or take money by exchange for *London*, ought to have like consideration.

Item, When any man giveth at *London* 64 pence $\frac{1}{3}$, or rather $65\frac{2}{7}$ d. to have at one of the fairs of *Lions* a Crown *de Marc*, he that so giveth the money, loseth the time, and he that taketh it, gaineth the same: for 62 pence $\frac{3}{7}$ is equall in value to 45 s. French. He that putteth or taketh mony at *Lions* for *London*, ought to consider the same.

Item, when any deliver in *Antwerp* 75 pence, to receive at *Lions* a Crowne *de Marc*, he that putteth it forth, loseth the time, and he that take it gaineth the same. For 75 groats Flemmish is equall in value to 45 s. French,

Thus for this time I make an end of the Practice of Exchange,

Exchange, and the instructions thereunto belonging, and according to my promise: yet further to gratifie such as are desirous to know the common Coynes used for trafficke among merchants in these Cities following, a briefe declaration of their moneyes, and the reckonings, and account of them I will here set downe.

The sixteenth Chapter containeth a Declaration of the valuation and diuersity of Coyns of most places of Christendome for traffick; And the manner of exchange in those places from one City or Town to another: which known, is right necessary for Merchants, by means whereof they do finde the gain or losse upon the exchange.

ITem, for as much as the greatest diversity of money of exchange is at *Lions*; therefore I will begin duely of the money of that place.

At *Lions* they use Franks, Soulx, and Deniers Turnois. A Frank maketh 20 Soulx, and one Soulx 12 Deniers; but the Merchants to keep their Bookes of accounts, doe use French crownes of the marke at 45 Soulx the piece, and doe divide it into 20 Soulx, one Suolx is 12 Deniers.

Item, a mark of gold maketh 65 △ of the mark, which serveth for exchange, and divide it into 8 ounces, the ounce into 24 pence or deniers, the denier into 24 grains, and so the summe or whole by imagination or guesse.

△ This mark standeth for a Crown.

Also at *Lions* there are four Fairs in a year, at the which they doe commonly exchange, which are

from 3 moneths to three moneths.

At *Geans* they use the Soulx : one Ducat maketh 3 pound.

At *Naples* they use Ducats, Taries, and Grains; the Ducat maketh five Taries, and one Tary 20 Grains : but they take 6 Ducats which maketh 30 Taries for the ounce.

A Ducat maketh ten Carlins, and a Carlin ten grains, so that 2 Carlins make a Tary, and 100 Grains make a Ducat.

At *Rome* they use the Ducats of the Chamber; one Ducat is worth 12 Guillis, and one Guillis ten Soulx.

At *Venice* they use Ducats currant at 124 Soulx a piece, or 24 Deniers, & one Denier maketh 32 Picolis.

At *Palerme* and *Messina* they write, after ounce, tary, and grains, and one ounce is worth 6 Ducats of 30 Taries, and 1 tary is 20 Grains, and 1 grain 6 Picolis, 1 Ducat is also worth 24 Carlins.

At *Millan* they use li. s d, of Ducat Imperials, and △ of exchange is worth 4 li.

At *Lucques*, *Florence*, and *Ancone*, they use the △ of Gold : in Gold the *French* Crown is worth 7 li. but at *Boloigne* 3 li. 10 s.

At *Barcelone* they use the Soulx : the Ducat of exchange is worth 22 Soulx.

At *Valence* and *Saragosse* they use the Liver, Soulx, and Denier : the *French* Crowne of exchange is worth 20 Soulx, and one Soulx is 12 Deniers.

At the Fairs of *Castile* they use the Marvides, the Ducat is worth 375 Marvides.

At *Lisbone* they use the Rayes, one Ducat of exchange is worth 400 Rayes.

At *Norenburg*, *Frankeford*, and *August* in *Germany*

many, they uſe the *Krantzers*, whereof 60 make a Floren.

At *Antwerp* they uſe li. s. d. de Gros, and they exchange into the Denier de Gros, to wit, our Engliſh peny.

At *London* they uſe the li. the s. and d. ſterling, and they exchange in pence ſterling.

The exchange of Lions *at ſundry places.*

Item, at *Lions* there is exchange in three ſorts, at the Cities and towns following.

Firſt, they deliver at Lions one mark to have or receive at Naples almoſt 41 ½ Ducats, at Venice 70 ducats currant, at Rome 63 ducats of the Chamber, at Lucques and Florence 65 △ of Gold, at Millan 82 △.

And contrariwiſe, at the ſaid Cities aforeſaid they doe give ſo much of money, to have a mark of Lions.

Secondly, they give at Lisbone one of △ of Mark of 45 Soulx Turnois a piece, to have at Geans almoſt 68 Soulx, at Palerme and Meſſine almoſt 24 Carlins, at Barcelone 22 Soulx, at Valence or Saragoſſe 20 Soulx, at the Faire at Caſtile 350 Marveides, at Lisbone 360 Rayes, in Antwerp 57 Deniers de Gros, and at London, 70 d. ſterling.

And contrariwiſe, they give in the ſaid Cities almoſt as much of their money to have a French Crown of the mark at *Lions*.

Thirdly, they do give at *Lions* a △ of the Sun to have almoſt 93 Krentzars at *Frankeford*, *Augſburg*, *Noremberge*, or other Cities in *Almain*.

Alſo at *Lions* onely they do pay, they change the ⅔ in Gold, and ⅓ in money, or elſe all in money, in giving 1 ½ for the hundreth.

Changes at Naples *and other Towns.*

Item, at *Naples* they give or deliver almost 112 Ducats, to receive at *Rome* 100 Ducats of the Chamber at the old value.

Through *Lucques* and *Florence* they deliver 100 Ducats Carlins, to receive there almost 86 △ of Gold.

Through *Palerm* and *Messine*, one Ducat of 5 Tary, to receive there almost 164 grains.

Through *Millain*, one Ducat to receive there almost 90 Soulx.

Through *Geans*, one ducat to receive there almost 65 Soulx. The whole summe to be paid within ten dayes after the sight of the Bill of exchange.

Also at *Naples*, they deliver one ducat to receive in *Antwerp* almost 67 d. or Deniers *de Gros*, within two moneths. At *London* almost 60 d. sterling in three moneths. At *Barselone* almost 20 Soulx within two moneths.

At *Valence* almost 18 Soulx within two moneths. At *Lisbone* 333 Rayes within three moneths: and at the Fair at *Castile* almost 340 Marvides.

Change of Venice *to other places.*

At *Venice* they deliver 100 Ducats currant to receive in *Almain* almost 140 Florens at 60 Krentzers the piece.

At *Lucques* and *Florence* almost 108 △ of Gold in 10 dayes.

Likewise at *Venice* they deliver a Ducat currant to receive at *Palerm* and *Messine* almost 21 Carlins; at *Millan* almost 93 Soulx: at *Geans* almost 69 Soulx, the whole at 10 dayes end.

Of

Of the Pair *or* Pari.

As touching the exchange, it is neceſſary to underſtand or know the *Pair*, which the *Italians* call *Pari*, which is no other thing then to make the money of the change of one City or Town, to or with the money of another, by means whereof they doe finde the gains or loſſe upon the exchange.

Example.

Item, having received Letters of credit of one of *Antwerp*, that the △ of the ſunne is there worth 7 Soulx: The queſtion is, what the ſame is worth at *London*, when the *Pair* of exchange goeth for 23 ſhillings?

Anſwer, *Say, if 23 give but 20, what giveth* 7? *Worke, and finde* 81. 1. $\frac{1}{3}$ d. *and ſo much is the* △ *of the Sun worth at* London.

The ſeventeenth Chapter containeth alſo a Declaration of the diverſity of the Weights and Meaſures of moſt places of Chriſtendome for traffique. At the end of which diſcourſe are two Tables, the one for Weight, and the other for Meaſure, proportionate and reduced to an equality of our *Engliſh* Meaſure and Weight, by the aide whereof, the ingenious may eaſily by the *Rule of Three*, convert the one into the other at pleaſure, *&c.*

AT *London*, and ſo all *England* thorow, are uſed 2 kindes of Weights and Meaſures, as the *Troy* weight, & the *Haberdepoiſe*. From the *Troy* weight is derived the proportion and quantity of all kinde of dry and liquid Meaſures, as Pecks, Buſhels, Quarters, &c.

wherewith is bought and ſold all kinde of grain and other Commodities meet by the Buſhell. And in liquid, Ale, Beere, Wine, Oyle, Butter, Honey, &c, Upon theſe grounds and Statutes is bread made, and ſold by the *Troy* weight: and ſo is Gold, Silver, Pearle, precious Stones, and Jewels. The leaſt quantity of this *Troy* weight is a grain: twenty four of theſe graines make a penny weight, twenty penny-weights an Ounce, and twelve ounces a pound, two pounds or pints of this weight maketh a quart. And ſo aſcending into bigger quantities, is produced the Meaſures whereby are ſold our other naturall ſuſtenance: *viz.* Ale or Beere, with all other neceſſary commodities, as Butter, Hony, Herrings, Eeles, Sope, &c. All which laſt before rehearſed, though their Meaſures (wherein they are contained) bee framed and derived from the *Troy* weight, yet are they in traffique with divers Commodities, as Lead, Tinne, Flax, Wax, with all other commodities, both of this Realme, and of other forraine Countries whatſoever, bought and ſold by the *Haberdepoiſe* weight after ſixteen ounces to the pound, and 112 pound to the hundred weight. And to every hundred is allowed but 12 pound weight at the common-beam. From hence is alſo derived the weight of *Suffolke* Cheeſe, which containeth thirty two Cloves, 8 pound to a Clove, and weigheth in all 256 pounds. And alſo the Barrell of *Suffolke* Butter is, or ſhould be of like weight with the weight of Cheeſe, *viz.* 256 pounds. More 14 of theſe pounds make a Stone, and 26 Stone containeth a Sacke of *English* Woll: Forraine Wool, to wit, *French*, *Spaniſh*, and *Eſtrich*, is alſo ſold by the pound, or C. weight, but moſt Commonly by the

See further of theſe Weights and meaſures in Reduction. beginning pag. 111.

Rove,

Rove, 25 pound to a Rove: other commodities of Tale, are bought and sold by the C fivescore to the C. except headed ware, to wit, Cattell, Nails, & Fish, which are sold after sixscore to the C. There are also two other sort of measures, to wit, the Ell and the yard. By the Ell is usually meete Linnen cloth, as Canvas, *&c.* And by the yard Silks, woollen clothes, *&c.*

Antwerp.

At *Antwerp* are also two sorts of weights, their Gold and Silver weight and their common weight. Gold and Silver is weighed by the mark, the mark is 8 ounces, the ounce 20 esterlings, and the esterling 32, as our grains. The Goldsmiths divide that into smaller, but not the Merchants. The proof of Gold is made by Karects, whereof 24 maketh a marke of fine Gold, the Karect is 24 grains: the proof of the money is made by Deniers: 12 Deniers is 1 s. fine, that is, a mark of fine silver: the Denier also is divided into 24 grains, & the grain into foure quarters.

Item, 100 marks in *Antwerp* Troy weight, maketh at *Lions* 103 marks, 2 ½ ounces, and 20 grains 23 d. At *Noremberg* 103 marks 2 ½ ounces, 2 Quints, 3 Deniers; at *Frankeford* 105 marks: at *Amburg* 104 marks, 3 ounces, 1 Quint: at *Venice* 103 marks, 1 ounce, 7. Deniers. 18 grains: at *London* 66 pounds.

The Mark of gold or silver at Antwerp, *Troy weight, which is 8 ounces, maketh 7 ½ ounces common weight, with which all other Merchandize is weighed. So that the Troy weight is greater then the common weight by 6 ¼ in the* C. *By this weight of* Troy, *they also weigh* Musk, Amber, Pearl, *&c.*

All silks are bought at Antwerp, *by the* Barges Ell, *which*

which is greater then the common measure, by which they retaile by 2 in the hundred. Their common Ell is $\frac{3}{4}$ of our yard, and $\frac{3}{5}$ of our Ell.

Lions.

At *Lions* is used 3 sorts of weight, whereof the first is the common Towne weight, with which they weigh all kind of Spicery, and divers other Merchandize. The second is called *Geneva* weight, which is 8 in the C. greater then the common weight, with which they weigh Silks, *&c.* The third is *French* weight, called commonly the mark weight, and 100 pounds thereof maketh 106 $\frac{1}{4}$ li. *Geneva,* and 114 $\frac{1}{4}$ of their common weight: with which *French* weight, is weighed all things that paid Custome or Toll.

At *Lions* is also used two sorts of Ells or Aulnes. The one wherewith they measure grosse clothes, as Canvasse, and such like. The other is called the *French* Ell or Aulne, with which they measure all other kind of Merchandize, whereof 7 common Towne Ells maketh 11 ordinary *French ells.*

Roan.

At *Roan*, 6 $\frac{1}{2}$ Muides of Salt, being the measure of the place, make a C. at *Armviden* in *Zeland*, and the C. of *Bronage* measure of *Armviden*, maketh at *Roan* 11 Muides. 30 Muides make a Last of Corn and 16 a Last of Oats, 100 pound weight there, maketh at *London* 114 $\frac{1}{2}$, and 190 $\frac{1}{4}$ at *Antwerp*. And 200 Ells make at *London*, 115 $\frac{1}{4}$.

Noremburge.

A 100 pound weight at *Noremburge* maketh at *London*

London 111 $\frac{3}{4}$; at *Antwerp* 107 $\frac{1}{7}$. and 100 Ells at *Noremburge* make at *London* 75 $\frac{1}{5}$, at *Antwerp* 95 $\frac{3}{5}$, &c.

Lisbone

The C. weight at *Lisbone* maketh 4 Roves, every Rove 32 pounds, so that their C. weight is 128 pounds, and their pound containeth 14 Ounces, and 100 pounds of their weight maketh at *London* 113 $\frac{1}{8}$.

Their Silke, Cloth of Gold, and Woollen is measured with a measure which they call a cubit, containing about $\frac{3}{4}$ of a Vare of *Castile*. Howbeit their common Measure is called a Varre, which maketh five Palms, and containeth 1 $\frac{1}{4}$ of a Varre of *Castile*, our Ell of *London* is equall with the Varre of *Lisbone*.

All kinde of Merchandize brought from *Flanders*, *Roan*, or *Brittain*, payeth at *Lisbone*, as a duty or custome to the King, 20 in the C. which they call the tenth in Merchandize, and the other tenth in money.

Note also, that all kinde of Merchandize comming to *Lisbone* by land, payeth lesse in custome, then that that commeth by water.

Sivill.

The Rove of *Sivill* is 30 pound, 4 Roves make their C. weight, which is 120 pounds. The 100 pounds of *Sivill* maketh at *London* 102 pounds. Their other common measure is a Varre, whereof 100 maketh at *London* 74 Ells, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* bee two sorts of weight, the one called *la Grosse*, the other *la Suttle*; with the grosse is weighed

weighed all kinde of great wares, and with the small all kind of spicery, and such like: *96* pounds of grosse weight there maketh at *London* 100 pound, and 100 pounds of spicery there, without any tare or allowance, make at *London 94*, and with tare *65*.

Their owne common Measure are Braces, whereof 100 make at *London* 55 1/2 ells, at *Antwerp* 92 1/2, &c.

Florence.

At *Florence* the 100 lib. weight maketh at *Aquila*, for Saffron, 110, and 145 pounds of *Florence*, make at *Roan* but 100 pounds; the weight of *Florence*, and that of *Lucque* is all one.

Their other measures are Braces, whereof 100 maketh at *Antwerp*, *Burges* measure, 81 2/3 els, 100 Braces there, make at *London* 49 els, &c.

Lucque.

The *Lucque* Sattens commonly sold at *Lions* by weight, and 133 1/3 pounds maketh at *Lions* 100 pound, so that 1 pound 1/3 maketh at *Lions* but one pound.

Their other measures are Braces whereof 100 of them make at *London* 50 ells, at *Antwerp* 83 1/3 ells, &c.

Aquila.

At *Aquila*, their 100 pounds maketh at *London* 71 1/4, their 136 2/3 pounds of saffron maketh at *Geneva* but 100, and 11 li. of *Geneva*, maketh 15 li. at *Aquila*.

Valentia.

At *Valentia* be two sorts of weights, a great and

a ſmall. The C. weight or great weight containeth four Roves, the Rove 36 li. ſo the C. great weight is 144 li. and the C. weight ſmall containeth but 120 pounds, and is alſo parted into four Roves, which is 30 pounds to a Rove. By the ſmall is ſold the ſcarlet grain, with all other kind of ſpicery, and by the great is ſold wooll, with all ſuch like groſſe wares. The 1 $\frac{1}{5}$ pounds of Silke at *Valentia*, maketh at *Lions* one pound *Geneva* weight. The charge of great Merchandize at *Valentia* containeth 432 pounds, and in ſmall wares 360 pounds.

The weight here and at *Barſellone* is all one.

Their 100 pound weight maketh at *London* 78 pound, at *Antwerp* 75.

Danſicke.

At *Danſick* or *Spruceland* the rule is, that whoſoever buyeth any merchandize there, buyeth it by the ſhip-pound, which, is 320 li. 20 Liſpounds make a ſhip-pound, and the Liſpound containeth 16 pound, which ſhip-pound of *Danſick* maketh at *Antwerp* 266$\frac{2}{3}$ li. Their 100 li. weight maketh at *London* 86, *&c.*

Their other common meaſures are ells, whereof 100 maketh at *London* 72$\frac{1}{4}$, and at *Antwerp* 120 $\frac{1}{2}$ ells.

Toulhouſe.

At *Toulhouſe* 6 cabes of Woad maketh a charge, two ciſterns of corn-meaſure, and all kinde of grain maketh a charge, the Ciſtern weigheth 160 l. weight of that place. Their 100 in weight maketh at *London* but 91$\frac{1}{4}$ pound.

Geans.

Geans.

At *Genua* or *Geans*, 100 li. of their weight maketh at *London* 71 $\frac{1}{4}$, and at *Antwerp* 68 $\frac{1}{5}$. 100 li: weight at *Genua*, maketh at *Venice*, to Suttle wit, 106 li.

Their other common Meaſures are *Palmes*, whereof 100 make at *London* 20 $\frac{3}{5}$ Ells, and at *Antwerp* 34 $\frac{3}{5}$.

The reſt are ſupplied in two Tables, which hereafter followeth: whereby the ingenious may gather his deſire.

The Table of the agreement of the weights of divers Countries, the one with the other, being reduced to an equality, as followeth.

112 pounds weight at *London*, make at

Antwerp	107 $\frac{1}{5}$
Frankeford	099
Colen and *Ausburgh*	102 $\frac{1}{4}$
Noremburg.	100 $\frac{1}{3}$
Roan	098
Paris	102 $\frac{1}{4}$
Lions	118 $\frac{1}{5}$
Deep	100 $\frac{1}{4}$
Geneva	090 $\frac{1}{8}$
Toulouſe	122 $\frac{1}{4}$
Rochell	124 $\frac{1}{4}$
Marſellis	124 $\frac{3}{4}$
Sivill, &c.	109 $\frac{1}{4}$

112 pounds weight at *London* make at

Venice groſs weight.	105 $\frac{3}{8}$
Venice ſuttle weight.	166 $\frac{2}{7}$
Aquina	157 $\frac{1}{7}$
Vienna	089 $\frac{3}{8}$
Preſlaw	134 $\frac{2}{6}$
Leipſig	101 $\frac{1}{4}$
Danſick	129 $\frac{1}{4}$
Lubeck	097 $\frac{3}{8}$
Barcellon	144 $\frac{1}{2}$
Lisbone	099
Geans	157 $\frac{1}{5}$

The other Table of agreement of Meaſures of divers Countries reduced unto an equality, by the aid whereof you may with the uſe of the Rule of three, convert either more or leſſe of any one Meaſure unto the other.

Antwerp

60 Ells, or 75 Yards at London make at

Antwerp	100	*Ells.*
Noremburg	$104\frac{1}{2}$	*Ells.*
Frankeford	125	*Ells.*
Leipsig	125	*Ells.*
Preslaw	125	*Ells.*
Dansick	183	*Ells.*
Vienna in *Austria*	87	*Aulnes.*
Lions in *France*	$60\frac{3}{4}$	*Aulnes.*
Paris in *France*	57	*Aulnes.*
Roan in *Normandy*	52	*Aulnes.*
Lisbone	60	*Varres.*
Sivil in *Spain*	81	*Varres.*
Castile in *Spain*	81	*Varres.*
Methera Isles	62	*Varres.*
Venice	108	*Braces.*
Lucques	120	*Braces.*
Florence	$122\frac{1}{2}$	*Braces.*
Millain	138	*Braces.*
Rome	90	*Canes.*
Geans	$288\frac{6}{13}$	*Palms.*

The eighteenth Chapter treateth of Sports, and Pastimes, done by number.

IF you would know the number that any man doth think or imagine in his minde, as though you could divine, bid them triple it, or put twice so much more to it as it is, which done, aske him whether it be even or odd; if he say odde, bid him take one to it, to make it even, and for that one, keepe one in your minde. Now after he hath taken one to it, to make it even,

even, bid him give away half, and keep the other half for himself, which when he hath done, bid him triple that half, and again, after he hath tripled it; aske him whether it be even or odde: if hee say odde, then bid him take one to make it even again, and for that last one, keep two in your minde: now after hee hath made his number even, bid him cast away the one halfe, and keep the other still, from which halfe that he keepeth, cause him subtilly to put away or give you nine out of his number, as oft as he can, and for each 9 that he giveth you, keep in mind, and thereunto joyn the 3 which I bad you keep, and you shall have your desire.

Example.

Imagine he thought 7, the triple whereof is 21, and because it is odde, he is to take 1 to make it even, which first 1 given, is for you to keep in mind. Then the halfe of his 22 being cast away, he reserveth still 11, which after you have bid him triple, it maketh 33: then in giving of him one again to make it even, upon that last 1 reserve 2 in your minde, then his halfe of 34 maketh 17; from whence he can give you 9 but once. Therefore that yeelding to you 4, and the 3 that you keep, make 7 your desire.

Another kind of Divination, to tell your friend how many pence of single pieces, reckoning them one with another, he hath in his purse, or should think in his mind.

Which to doe, first bid him double the pieces he hath in his purse, or the number he thinketh (if he participate his number or secrecie unto some one friend that sitteth by him that can but multiplie, and adde never so

ſo little, if their number be great, then ſhall they worke as you bid them ſo much the ſurer.)

Now after he hath doubled his number, bid him adde thereunto 5 more; which done, bid him multiply that his number by 5 alſo; which done, bid him tell you the juſt ſumme of his laſt multiplication, which ſumme the giver thinketh it nothing available, becauſe it is ſo great above his pretended imagination: yet thereby ſhall you preſently with the help of Subtraction tell his propoſed mumber.

The Rule is this.

Imagine he thought 17, double 17, and it maketh 34, whereunto if you adde 5, it maketh 39: which multiplied by 5, as here is practiſed, it yieldeth 195, which 195 is the ſumme delivered you in the worke: then for a generall Rule you ſhall evermore cut off the laſt figure toward your right hand, with a daſh of your pen, as here is performed, as a figure nothing available unto your work, and then rebate 2 from your firſt figure, after 5 is cut off, and the reſt ſhall evermore be your deſire, as by this example doth appeare.

$$\begin{array}{r} 17 \\ 2 \\ \hline 34 \\ 5 \\ \hline 39 \\ 5 \\ \hline 195 \\ 2 \\ \hline 17 \end{array}$$

Another of a Ring.

If in any company you are diſpoſed to make them merry by manner of divining, in delivering a Ring unto any one of them, which after you have delivered it unto them, that you will abſent your ſelfe from them, and they to deviſe after you are gone, which of them ſhall have the keeping thereof, and that you at your return will tell them what

person hath it, upon what hand, upon what finger, and what joynt: Which to doe, cause the persons to sit down all in a row, and to keep likewise an order of their fingers: now, after ye are gone out from them to some other place, say unto one of the lookers on, that he double the numbers of him that hath the Ring, and unto the double bid him adde 5 and then cause him to multiply the Addition by 5, and unto the product bid him adde the number of the finger of the person that hath the Ring. And lastly, to end the work, beyond that number, towards his right hand, let him set down a figure signifiying upon which of the joynts he hath the Ring, as if it be upon the second joynt, let him put down 2. Then demand of him what number he keepeth, from the which you shall abate 250, and you shall have three figures remaining at the least. The first towards your left hand, shall signifie the number of the person which hath the Ring, the second or middle number shall declare the number of the finger, and the last figure towards your right hand shall betoken the number of the joynt.

Example.

Imagine the seventh person is determined to keep the Ring upon the fifth finger, and the third joynt: first double 7, *it maketh* 14, *thereto adde* 5, *it maketh* 19, *which multiplyed by* 5, *yeeldeth* 95, *unto which* 95, *adde the number of the finger, and it maketh* 100: *and beyond* 100 *toward the right hand, I set down* 3 *the number of the joynt, all maketh* 1003, *which is the number that is to be delivered you, from which abating* 205, *there resteth* 753, *which prefigureth unto you the seventh person, the fifth finger, and the third joynt.*

But

But note, that when you have made your subtraction, if there doe remain 0, in the place of tens, that is to say, in the second place, you must then abate 1, from the figure which is in the place of the hundreds, that is to wit, from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tenth finger, as if there should remain 803, you must say, that the seventh person upon his tenth finger, and upon his third joynt, hath the Ring.

Another of three Dice.

If a man doe cast 3 Dice, you may know the points of one of every of them. For if you cause him to double the points of one Die, and to the double to adde 5, and the same summe to multiply by 5, and unto the product adde the points of one of the other Dice; and behind the number towards the right hand, to put the figure which signifieth the points of the last Die, and then aske what number he keepeth, from which abate 250, and there will remaine 3 figures, which doe note unto you the points of every Die.

Another of things hidden.

If three divers things are to be hidden of three divers persons, and you to divine which of the three persons hath the three divers things, doe thus: imagine the three things to be represented. *A*, *B*, *C*. Then secondly, keep well in your mind which of the persons you mean to be the first, second, and third. Then take 24 Counters or Stones, and your three things, and give *A* to the party whom you imagine to be your first man, and therewithall give him one of your 24 Counters in his hand, & *B*, unto your second man,

 and

and therewithall 2 Counters, and *C*, unto your third man, and therewithall 3 Counters: and leave the reſt, which are 18, ſtill among them: which done, ſeparate your ſelfe from them, and afterwards bid them change the things among them as they ſhall think good: which done, after they are agreed, bid him that hath ſuch a thing, as before you have repreſented by *A*, for every Counter that he hath in his hand, to take up as many more. And for him that hath *B*, for every one in his hand to take up 2. And for him that hath *C*, for every one in his hand to take up 4, and the reſt of them to leave ſtill upon the board. Theſe 3 things, and the three perſons being fully printed in your minde, come to the Table, and you ſhall evermore finde out of theſe 6 numbers, 1, 2, 3, 4, 5, 6, or 7. If therefore one remaine ſtill upon the board, then have they made no exchange, but keep them ſtill as they were delivered unto them. So that the firſt man hath *A*, the ſecond *B*, and the third man *C*. But if 2 remain, then the firſt man hath *B*, your ſecond man *A*, and your third man *C*. The reſt of the work and the order thereof are here apparent by the Table following.

	1	*A*		1	*B*
1	2	*B*	5	2	*C*
	3	*C*		3	*A*
	1	*A*		1	*C*
2	2	*B*	6	2	*A*
	3	*C*		3	*B*
	1	*C*		1	*C*
2	2	*B*	7	2	*B*
	3	*A*		3	*A*

A

Another divination of a number upon the Casting of two Dice.

First let the Caster cast both the Dice, and mark well the number: then let him take up one of them, it maketh no matter which, and look what number it hath in the bottom, and adde all together: then cast the Die again, and keep in his mind what all together maketh: then let the Dice stand, and bring seven with you, and thereunto adde the rest of the pits that you see upon the upper side of the Dice, and so many did the Caster cast in all.

FINIS.

An Appendix concerning the Resolution of the *Square* and *Cube* in Numbers, to the finding of their sides by *Ro. Hartwel.*

A Figurate Number is a number made by the multiplication of one number or more by another. A figurate number what.

The sides of a figurative number, are the numbers by whose multiplication it is made. The sides of a figurate number what.

A Figurate number is two-fold, a { *Plain.* / *Solid.* }

And it is { *Of one Multiplication.* / *Or consequently of many.* } *as a* { *Plain.* / *Solid.* }

And in each { *Both Æquilater.* / *And Inæquilater.* }

A figurate number made of one multiplication by two sides or numbers multiplyed together, is called a plain figurate number.

A plain figurate number.

For every number made by the mutuall multiplication of two numbers, may be called a *Plaine*, because it bringeth forth a rightangled parallelogramme, according to his unites disposed in *length* and *beadth*, the *sides* whereof are the two multiplying numbers. As the number 20, made by the mutuall multiplication of 4 and 5 is called a *Plaine*, and the *sides* thereof are 4 and 5 as here

* * * * *
* * * * *
* * * * *
* * * * *

Because the unites thereof disposed in *length* and *breadth*, as the *sides* do expresse, do bring forth an *inaequilater Parallelogram*, for that the numbers, or *sides* are inequall.

By like reason 36 made by multiplication of 6 by 6, is called an *Æquilater plain*, for the *sides* thereof 6 and 6 are equall.

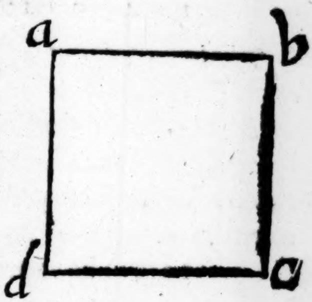
Moreover one and the same *plain number* may have many *sides*, as the *plain number* 24, hath sides 4 and six: 3 and eight: 2 and 12. For it is produced from the mutuall multiplication of these numbers: whereupon for the invention of the sides, to wit, in *inaequilater Plains*, it is needfull to give one of the *sides* by which, the *plain* it selfe divided, the other *side* is made known. As the *plain* 48 being divided by the *side* 8, the *quotient* 6 is the remaining *side*. Notwithstanding another resolution and inquisition doth happen in the *sides* of the *Æquilater plains*.

An Equilater Plain or quadrat what.

An Æquilater plain is a number made by two equall sides, or by any number multiplyed by it selfe. It is vulgarly called a square or quadrat; by the Arabians

Arabians Zenſus, *it is commonly expreſſed by this note z, by us q.*

A quadrat or *ſquare* in *Geometry* is called a right lined *plain figure*, made by four equall right lines, and ſo many right Angles, & every one of the lines is called the *ſide* of the *quadrat*, as this *figure a b c d*, whoſe *ſide* is *a b*, or *bc*, as alſo *cd*, and *a d*.



To the ſimilitude hereof, that number is called a *Quadrate*, which is made by the multiplication of two equall numbers, or of one in it ſelf, of which manner 36 is made, by 6 multiplyed in it ſelfe, or by the mutuall multiplication of 6 and 6. For if 36 unites be placed in plain forme, it bringeth forth a perfect *Geometricall Quadrat*, having in every ſide ſixe unities as here.

```
* * * * * *
* * * * * *
* * * * * *
* * * * * *
* * * * * *
* * * * * *
```

The number whereof the Quadrat is produced by multiplication in it ſelfe, is called the ſide or root of the Quadrat. The ſide or root of a number what.

Concerning the extraction of the Quadrat or ſquare Root.

THerefore to find the *Quadrat Root*, or the *ſide* of any *Quadrat number*, is to ſearch a number, which brought or multiplyed in it ſelf, maketh the number propounded: concerning the finding whereof, as it is requiſite that the *ſides* (being leſſer then 10) of the

the *squares* under an hundred should be gathered by the Table of Multiplication: so the *sides* of the *greater squares* are to be sought out by *Art*. First, the *squares* whose *sides* are simple numbers, are here set down as you see.

The roots	1	2	3	4	5	6	7	8	9
The squares.	1	4	9	16	25	36	49	64	81

The knowledge of a square is by finding out his side expressed by a whole number.

Although the finding out of the *side* of a *square* be applyed to each number given, as to a *square*, yet *square numbers* only have a *side* to be expressed by a certain number of unites, or by rationall numbers, the other are to be expressed but onely in power. The *sides* are commonly called *Roots* by a *Metaphoricall phrase*.

The Root or side of a square is to be found by the Theorem following.

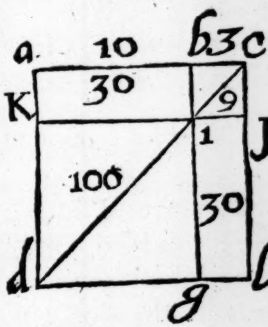
1 If the odde degrees of a *square number* being marked from the right toward the left hand with points, you subduct from the number given, the particular *square* of the last *period*, setting the *side* thereof alone by it selfe.

2 Then going on, if you divide the remainder (if there be any) with the figure going before it, by the double of the side set alone by it selfe.

3 And multiply the *quotient*, found out (being placed by the side, which was first set alone by it selfe, and also before the doubled number on the right hand) by both the numbers (namely by the double number, & the Figure set by it selfe) being counted as one *divisor*, subducting the products from the given number

ber, and then renue this last work of division so many times as there are pricks remaining, the *side* of the *square* shall be found out.

This artificiall device is taken out of the 4 *Pro.* 2 l. *of* Euclide. *Where by demonstration it is proved, that if a right line be cut into two* segments, *howsoever the* square *of the whole line is equall to the* squares *of the* segments, *and the two* right-angled figures *made of the* segments *as in* the figure *annexed, the two* Diagonals, *k g, and b f, are the* squares *of the* segments, *a b, and b c, Also the* complements *b k, and f g, are the* right-angled figures *made by multiplying the line a b, by b c.*



To extract the square roo

The selfe same parts are to be found in any *square number*. As for *example*, let the *number* be 169, whose *side* is 13. This *side* being divided into two pieces, 10 and 3, multiplying each pieces, by it selfe once, namely, 10 by 10, and 3 by 3: then multiply one by another, as 10 by 3, and 3 by 10, so shall you have 4 *plain numbers*, whereof 2 are *square*, as here you see.

The first example.

Therefore as the *square* 169 is made by adding together of these 4 *plain numbers*, so by subducting them severally it is resolved.

10	3
10	3
100	
30	
30	
9	
169	

First therefore I mark each odde place with points, because the *particular squares* are to be found in the odde places. Then for so much as the unity standing under the first point next the left hand, and representing the last *period*, is both a *square* and the *side* of a

square:

ſquare: that *figure* therefore being ſet alone in the *quotient*, and being ſubducted from the unity ſtanding over the point, there remaineth nothing.

This unity ſet alone by it ſelf in the *quotient*, ſhall ſignify 10, when another *figure* is ſet by it, repreſenting the ſide of ſome other particular *ſquare*. Whereupon I ſay, that the greater *Diagonall k g*; is now ſubducted from the whole *ſquare*, and the ſide of it *k i*, or *a b*, (for they are equall one to another) and alſo the *ſide* of the *complement* is found out.

This is the firſt ſtep to this Reſolution.

Moreover, I double the *figure* found out, becauſe being doubled, it is the ſide of both the *complements* taken jointly together, namely, *k i*, and *g i*. Then ſetting 2 the doubled number under 6, I divide 6 (which in this place is as much as 60, and repreſenteth both the *complements*) by 2, the *quotient* is 3, repreſenting the other *ſide* remaining of the *complement*; namely, *i f* for *b c*, which number I ſet in the *quotient*, & count it for the *ſegment remaining* of the right line given. Wherefore becauſe this number 3 is the ſide of the remaining *Diagonall*, that is to ſay, of the leſſer *ſquare b f*, therefore being ſet by the *diviſor* on the right hand, and multiplyed by it ſelfe, and alſo by the *diviſor*, it bringeth forth three *plain numbers*, name- the *ſquare b f*, and the two *complements a i*, and *i l*, which being ſubducted from the numbers ſtanding over them, there remaineth nothing.

The

The example is thus.

169 (12	*Which is all one,*	169	
122	*as if you had put*	100	*The greater Diagonall.*
2	*downe the numbers found out in*	60	*The complements twofold.*
69	*this manner.*	9	*The leſſor Diagonall.*
		169	

The ſubtilty of this invention is illuſtrated by many examples. The ſecond example.

Let the *ſquare* given be 1764. This number being marked with two points, telleth us, that the ſide thereof is to be written with two Figures.

Firſt therefore beginning at the point on the left hand, I ſeek the ſide of the laſt period, namely, 17. But for ſo much as it is no ſquare number, I take 4 the ſide of the next leſſer ſquare, which I ſet alone by it ſelf in the quotient, and then multiply it by it ſelf, the *Product* is 16, which being ſubducted from 17, there reſteth 1. Moreover I double the ſide found out, the product is 8, I place this doubled number under 6, and by it divide 16 ſtanding above it, the *quotient* is 2, which muſt be ſet by 4. This *quotient* 2, muſt be ſet before the *Diviſor* 8, on the right hand under the point, and then muſt it be multiplyed both by it ſelfe, and into 8, the product is 164, which being ſubducted from the figures ſtanding over them, there remaineth nothing: whereby I gather that the number given is a juſt *ſquare*.

The

The Example ſtandeth thus.

```
  1
 1764 (42
1682
   2
 164
1764   The Collection.
```

The ſame manner of working is to be followed in greater ſquare numbers given, ſaving that the former part of the worke is to be uſed but once, but the later part is to be followed ſo many times as there are points remaining excepting the laſt.

The third example. As in 5 47 56, I ſay, that the *ſide* of the *ſquare* next unto 5 is 2 : therefore 2 being ſet in the *quotient*, and multiplyed by it ſelf, makes 4, and taken from 5, the remainder is 1. Moreover I double the *quotient* the product is 4, which I ſet under the next *figure* toward the right hand, and thereby divide 14, the *quotient* is 3, which three being ſet both in the *quotient*, and alſo before the *Diviſor* toward the right hand, I multiply both the numbers by it, the *product* is 129: this being ſubducted from 147 ſtanding above it, the remainder is 18. But becauſe there is yet one point remaining, with which I have not medled ; I therefore again double all the whole *quotient*, for in this caſe I muſt take 23 for the ſide of one former *ſquare*, and generally in great numbers, when I light upon more particular *ſquares* then two, I muſt eſteeme them but as two, and take the ſides which are firſt found out, but as the *ſides* of one only *ſquare*. Therefore twice 23 is 46 : by this I divide 185, the number to be ſet in the *quotient* is foure, which

Note.

number

number alſo muſt be ſet before the *Diviſor* on the right hand: then muſt 464 be multiplyed by 4: the *product* is 1856, this *product* being ſubtracted from the numbers ſtanding over it, there remaineth nothing. The example ſtandeth thus.

118

54756 (234.

44364

129

4

1856

54756 *The Collection.*

See alſo the Example following.

10942864 (3308.

Therefore out of this invention is this *conſectarie.* *The number whoſe ſide cannot be expreſſed by whole numbers, is not a ſquare number.* 4 Example of a ſurd number.

Such are all *prime numbers*, and (the *ſquares* themſelves excepted) all other *compound numbers.* For if in them you deſire to find out the *ſquare ſide*, you ſhall labour in vain, becauſe they are not *ſquares*, for to the whole numbers ariſing in the *quotient*, there will be ſome *fraction* adjoyned, whereby it commeth to paſſe, that the number of the *ſide* is not to be expreſſed by a true number, and it is commonly called a *ſurd number.*

Notwithſtanding, if you adjoyne to the *ſide* found out, the number remaining, taking his *denomination* from the double of the *ſide augmented* by an unity, you

you ſhall finde the next ſide that may be like to the *ſide of a ſquare.*

As if from 40 you take the neereſt *ſquare*, to wit, 36, the remainder is 4. Here therefore the ſide ſought for of the *ſquare* exceedeth not the *ſide* found out by an unity, but either by one, or more parts of ſome whole number: wherefore I double 6, the ſide found out, and adde an unity to it being doubled, the totall is 13, this number I ſet under 4 the remainder, and ſay that the ſide of 40 demanded as neere as may be is $6\frac{4}{13}$: the *Denominator* of the *Fraction* being added to the greateſt *ſquare* in the number given, namely unto 36, maketh the next geateſt *ſquare* above it, namely 49, whoſe ſide is 7. But this ſurd ſide, to wit, $6\frac{4}{13}$, multiplyed by it ſelfe, maketh $39\frac{121}{169}$, which are not juſt equall to 40, the given number.

Judge the like concerning the reſt which are not *ſquares.*

Thus much concerning plain figurate numbers, but eſpecially ſuch as are ſquare numbers.

Concerning ſolid figurate Numbers.

A Solid figurate Number is made of two multiplications by three numbers, or ſides, multiplyed together, admitting length, breadth, and thickneſſe.

A ſolid figurate number.

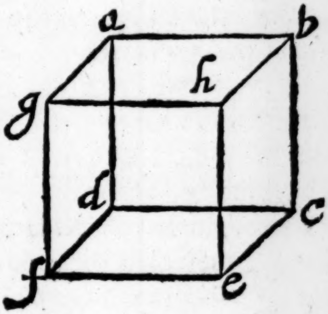
Therefore every *number* made by the mutuall multiplication of three numbers, may be called a *ſolid*, becauſe it bringeth forth *a right angled Parallelepipedon*, diſpoſed according to his unities in *length*, *breadth*, *and thickneſſe*, the *ſides* whereof are the three multiplying *numbers*. As the *number* 30 made by the mutuall

tuall multiplication of 2, 3, and 5, is called an *Inequilater ſolid number*, and the ſides thereof are 2, 3, and 5; becauſe the unities thereof diſpoſed by a certain diſtance one from another, in *length*, *breadth*, and *depth*, as the ſides doe expreſſe, do bring forth in reſemblance an *Inequilater Parallelepipedon*, for that the *numbers* or ſides are inequall.

By like reaſon 216 made by multiplication of 6 by 6, and the product thereof by 6, is called an *Equaliter ſolid*, for the ſides thereof 6, 6, and 6 are equall.

An Equilater, is a number made by three equall ſides, or by any number multiplyed by it ſelfe, and that product againe by the foreſaid number. And it is called an Equilater and Equiangled Parallelepipedon or Cube, and is commonly repreſented by us thus C. An Equilater Solid or Cube.

A Cube in Geometry is a right-angled Parallelepipedon, having ſix equall *ſurfaces* : and 8 *ſoild angles*, and 12 ſides, as this figure *a. b. c. d. e. f. g. h.* whoſe ſide is *a b*, or *a d*, alſo *b c*, or *c d*, either *c e*, or *e f*, likewiſe *e h*, or *h g*, alſo *g f*, or *d f*, or *d a*, and *g a*.



The number whereof the Cube is produced by Multiplication in it ſelfe twice, is called the ſide or root of the Cube, which being found out in whole numbers, the Cube is knowne. The ſide or root of the Cube.

Con-

Concerning the extraction of the Cubick Root.

THerefore every *Cube* in numbers hath such a side as may be expressed in whole numbers, but in magnitudes it is not alwayes so, as indeed in magnitudes there are many things not to be expessed in *whole number*. Now for as much as the side of any *Cube* under 1000, is a simple figure, it is necessary, before we undertake to finde out the side of any great number, to know what *Cube* is made of each simple figure, and what is the side of any *Cube* lesser then 1000, as I have here set them down.

Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	216	343	512	729

But in searching out the *sides* of greater *Cubes* we are to proceed as the *Theorem* following teacheth us.

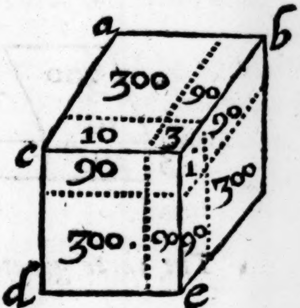
If you distinguish with *points* as it were into periods, the given *Cube*, beginning at the first figure on the right hand, and omitting each two figures continually, and first of all subduct the *particular Cube* of the last *period* from the given number, setting the *side* thereof in the *quotient* : and then set triple of the *quotient* under the *figure* next following the former *point*, on the right hand, and the *square* of the *quotient* being tripled beneath it one *degree* more toward the left hand : and afterward divide the number above written by the triple of the *square*, setting the *quotient* by it self, and then multiply the *divisor* by the *quotient* found out, and the triple *square* by the *square*

square of the *quotient*, and the *quotient cubically*, subducting the *products* (so orderly added together, that each *figure* may answer the *numbers* whereof it was multiplied) from the *number* given, and renue this last manner of division so many times as there are *points* remayning, the *side* of the *Cube* shal be found out.

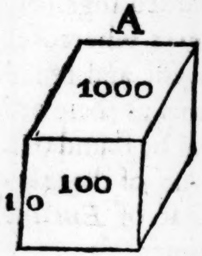
This artificiall device is drawne out of that *theorem*, which *Ramus* made, imitating that of *Euclide* concerning *square numbers* in this manner.

If a right line be cut into two segments, the Cube of the whole line shall be equall to the Cubes of the segments, and the two solid figures comprehended three times under the square of his segment, and the segment remayning. The extraction of the Cubick side or root.

As the line *c d.* which is 13, is cut into two *segments*, 10 and 3, therefore the *Cube* of the whole line, namely 2197, is equall unto the *Cubes* of the *Segments*, namely unto 100, and 27 also to *the two-fold Solids* or *Parallelepipedons thrice* taken whereof three have like *soliditie*, the *soliditie* of each of the three

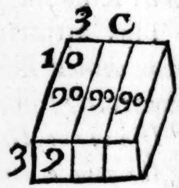


lesser is 90, being made of the *Square of the Segment* 3, that is to say of 9. multiplied by the other *Segment* 10. These three *Parallelepipedons* joyntly taken together, make 270. But of the three greater *Parallelepipedons* each containing 300, being made of 100, the *Square* of the greater *Segment* 10, multiplyed by the *lesser Segment* 3, and they being taken joyntly together, make 900.

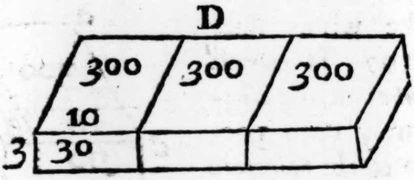


The Cube of the lesser segment 3.

The Cube of the greater segment 10.



The 3 *lesser Parallelepipedons.*



The three greater Parallelepipedons.

The *Cube* therefore hath eight *particular solids in number*, which are made of the parts of the number given, namely of 10 and 3 in this manner. First, let there be foure *plain numbers* made, each part being multiplied by it self, and one by another.

10	3
10	3
	9
	30
	30
	100

If

If againe I multiply the *Plaines* by the same parts, there will arise 8 *solids*, as you see here,

9	9	
30	30	
30	30	
100	100	
3	10	
27	90	*All these being added*
90	300	*together, are equall to*
90	300	*the* Cube *of the whole, to*
300	1000	*wit,* 2197.

Therefore the same way that is kept in making the Cube, *is also to be followed in resolving the* Cube.

As for example, I marke the *Cube* given with points in this manner, 2197. The first example to extract the Cubick root.

Then I subduct the particular *Cube* of the number set under the last point: but for so much as that number is no *Cube*, I take the neerest to it, namely, an unity, which also I set in the *quotient*. This unity in the number given, is 100, but in the *quotient* it is but 10, the unity subducted from 2, the remainder is 1, which must be written over the number given. So that the greater *Cube A*, is to be supposed to be subducted from the number given.

This is the first step of this work,

1
2197 (1
1

After I triple the *quotient* found out (that is to say, I multiply it by 3) this triple representeth the three *sides* (joyntly taken together) of the three lesser *solids* marked with *C*, I place the tripled number under 9, Again, I multiply the *quotient* square wise, and triple the *product*, which

maketh likewise 3. This *product* resembleth the three *square sides* (taken joyntly together) of the three *greater solids*, marked with *D*, I place the *product* on a degree lower toward the left hand underneath 1. With it I divide 11, which written above it, the quotient is 3. This *segment* or *quotient* 3, being multiplyed by 3, the *Divisor* maketh 9, which in respect of the place wherein it standeth, is 900, and representeth the three greater *solids* marked with *D*, taken joyntly together. Furthermore, the same *quotient* being multiplyed square wise, maketh 9, and multiplyed afterward by the *triple number* standing under 9, it maketh 27, which in respect of the place wherein it standeth, is 270, and representeth the 3 *lesser solids* marked with *C*. Last of all, the same *quotient* multiplyed *cubically*, breedeth the lesser *Cube B*. These 3 *products* therefore being added together, and the totall subducted from the *numbers* standing over it, there remayneth nothing, which importeth the given number is a *Cube*.

The example is as you see.

```
1                   2197 (13
2197 (13            1000 The greater Cube.
1 3                    3
  3                    3
-----------         -----------
 9  Or thus :       9 . 0 The 3 greater Parallelepipe.
  27                  270 The lesser Parallelepipedons,
   27                  27 The lesser Cube.
-----------         -----------
2197                2197
```

The matter may be explained by many examples.

The second example of the Cubick.

Let the side of the given *Cube* 16387064, be fought out, contrive it therefore (as it were) into certain periods

periods with points. Then firſt of all, ſearch out the *ſide* of the *Cube* next to the left hand : But for as much as 16 is no *Cube*, take 2 the *ſide* of the next *Cube* under it, that is to ſay, of 8, and ſet in the *quotient*, and ſubduct 8 the *Cube* thereof from 16, there remaineth 8. The firſt work is not to be renued throughout the whole *number*, but the *rules* following muſt be repeated as often as there are *points* remaining.

The firſt ſtep to finde out the root, is in this manner,

8
16387064(2
8

Moreover, triple the *quotient* now found out and the *product* is 6, which is to be placed under 8, namely, under the figure following the next prick toward the right hand. Then multiply the *quotient* by this tripled *number* (or which is all to one purpoſe, ſquare the *quotient* and then triple the *product*) it maketh 12, ſet that number in a lower place one degree neerer the left hand, & make it the *diviſor*: divide 83 by 12, obſerving this rule in chooſing your *quotient*, that it be no greater, then that the numbers afterward produced by multiplication may not exceed the numbers ſtanding over it. So that here you ſhall take 1 in 8, but 5 times, Afterward by this number 5, multiply the *diviſor* 12, & by the ſquare of 5, multiply the tripled number 6, & laſt of all multiply 5 cubically : ſo ſhall you produce three numbers, namely, 60, 150, 125, to be deſcribed in ſuch ſort as you ſee. Theſe numbers added together, and ſubducted from 8387, the remainder is 762.

The second step to find out the root, in this manner.

```
  8762
  16387064 (25
  ------------
       6
      12
  ------------
      60
      150
       125
  ------------
      7625
```

And because there is yet one point remayning, this last manner of Division must be wrought again.

First, therefore I triple the *quotient*, the product is 75. which must be so placed, that the first figure thereof, namely 5, may stand under 6, the second under the 0. Again, multiply the *quotient* by this *tripled number* (or which is all one, square the quotient, and triple the product) it maketh 1875, which must be the *Divisor*, whose first figure namely 5, must be placed under 7, the *last figure* of the *tripled number*. Then see that 1 may be contained in 7, many times, but I can take it but 4 times, I set 4 in the quotient and multiply the *Divisor* by 4, the product is 7500, afterward I square 4, it maketh 16, which I multiply by the tripled number 75, the product is 1200. Last of all I multiply 4 *cubically*, it maketh 64, these products added all together, make 762064, which number being subducted from the *Cube* given, there remayneth nothing, whereby I gather that the number given is exactly *cubicall*.

The

The third step to finde out the side is in this manner.

762
16387064 (254
75
1875
7500
1200
64
762064

Behold also the example following.

614125000 (850

The third example of the Cubick root

Another manner of working.

Hitherto the Princely high-way to find out the side of the Cube *hath been declared.*

But there are moreover certain other wayes also bending thereto, and leaning to the same principles, whereof this is one.

The second form.

Having found out in the Table of simple *Cubes*, the first figure representing the side of the *cube* contained in the number standing under the first *point* on the left hand, set it in the *quotient*, and subduct the particular *Cube* of that figure as you did before: then *square* that *figure*, and triple that *square*, the *product* shall be the *Divisor*, the first *figure* whereof shall be set under that *figure* which is on the right hand next of all to the point (now examined) before going.

See how many times the *Divisor* is contained in the number written over it, and multiply the *Divisor* in the *quotient*, and subduct the *product* from the *dividend*: yet here you must take heed, that you take not a greater *quotient* then that the *products* made

afterward thereby may be ſubducted from the *number* given.

The ſubduction being done, triple the firſt *figure* which was ſet in the *quotient*, and adde to the triple the laſt *number* which was ſet in the *quotient* on the right hand of the *product*.

This totall multiplyed by the ſquare of the *figure* laſt found out, and ſet down the *product* ſo, that the firſt *figure* thereof toward the right hand may ſtand under the *point* next before going on the ſame hand, and finaly, ſubduct the ſame from the *number* given.

The fourth example of the Cubick root.

As in 804357. The particular *Cube*, namely, 729 being taken from the number ſtanding under the laſt *period* upon the left hand, there remaineth 75357, the ſide of that particular *Cube* being 9, I ſet in the *quotient*. Then I *ſquare* that *ſide*, it maketh 81, and triple the *ſquare*, the *product* 243 is my *Diviſor*, which I ſet under the given number, ſo that 3 may ſtand under 3 with this *Diviſor*, divide the number ſtanding over it, you ſhall find 2 to be contained in 7 three times. Therefore I ſet 3 in the quotient, and multiply the *Diviſor* by it, the *product* is 729, which being ſubducted from 753, the remainder is 24.

The Induction is thus:

2
754.
804357 (93
243
729

Moreover I triple 9, the product is 27, by which on the right hand I ſet 3 the quotient laſt found out, the totall is 273.

This

This *Number* I multiply by 9 the ſquare of the quotient laſt found out, the product ſhall be 2457, which being ſubducted from the ſuperiour number, there remaineth nothing.

The Induction is thus.

```
  24
80437(93
  732
    9
———————
 2457
```

Another manner.

THE ſelfe-ſame work may be diſpatched another way, a little differing from the former in this manner.

The figure in the *quotient*, being found out by ſubducting the particular *Cube*, and alſo the ſecond figure in the quotient being found by diviſion, let the totall quotient be tripled, and let the tripled number be multiplyed by the former figure in the quotient. Then let the product be multiplyed againe, by the latter figure found out, and let a *cipher* be ſet on the right hand of that *product*. Laſt of all let the *Cube* of the latter figure found out, be added to this *product*, and let the totall ſumme be ſubducted from the number given. As in 373248. The third forme.

The

The first induction is in this manner.

30
373 248 (7

343

The fifth example

Moreover I *square* the side found out, it maketh 49, and triple the square, the *product* is 147, which shall be the *divisor*, by this I divide 302, the number written over it, the *quotient* is 2. Now I triple the *totall quotient* 72, it maketh 216, and multiply this triple by 7, the former *figure* in the *quotient*, the product is 1512. I multiply this *product* also by 2, the latter *figure* of the *quotient*, and set a *cipher* on the right hand of it, so as it maketh 30240, unto this number last of all I adde 8, the *Cube* of the latter *figure* found out, the total is 30248, which being subducted from this *figure* above it, there remaineth nothing.

The second Induction is thus.

30
373 248 (72

147

30248

All the *points* of the *number* given being examined, if any thing remaine, it signifieth the *number* given is no *Cube*: wherefore the true *side* of it cannot be exactly given in *numbers*. Yet if it please you to sift out the neerest *side* that may be, by the first kinde of reduction of *mixt numbers*, you shall reduce the *num-*

ber

ber given unto a *cubicall* fraction of a greater *denomination*, and afterward seeke out the *cubicall side* of that *fraction*.

For example sake, because 120 is no *Cube*, therefore let it be reduced into sixty *cubicall* parts, after this manner. Multiply 60 *cubically* in it self, it maketh 216000, by this being taken for the *denominator* of the *fraction*, multiply 120 the *number* given, the *product* is 25920000 whose *cubicall side* is $\frac{295}{60}$ that is $4\,\frac{11}{12}$ the nearest to the true *side* that can be.

To finde the neerest Cubick root in a surd number

For the extraction of all sorts of roots, the tables of Logarithmes set forth by Mr. Briggs *are most excellent, and ready.*

FINIS.

A Table of Board and Timber measure more perfect then ever hath been made; shewing also the Squares between 4 and 37 from quarter to quarter, calculated by Robert Hartwell.

Board measure.	Inches & quarters.	Squares.	Timber measure.	Board measure.	Inches & quarters.	Squares.	Timber measure.
36. 0 0	4	16	108.0.0	16. 0. 0	9	81	21. 3. 3
33. 8. 8	1	18	96. 0. 0	15. 5. 7	1	85	20. 3. 3
32. 0. 0	2	20	86. 4. 0	15. 1. 6	2	90	19. 2. 0
30. 3. 1	3	22	78. 5. 4	14. 7. 7	3	95	18. 1. 9
28. 8. 0	5	25	69. 1. 2	14. 4. 0	10	100	17. 2. 8
27. 4. 3	1	27	64. 0. 0	14. 0. 2	1	105	16. 4. 6
26. 1. 8	2	30	57. 6. 0	13. 7. 1	2	110	15. 7. 1
25. 0. 4	3	33	52. 3. 6	13. 3. 9	3	115	15. 0. 3
24. 0. 0	6	36	48. 0. 0	13. 0. 9	11	121	14. 2. 8
23. 0. 4	1	39	44 3. 0	12. 8. 0	1	126	13. 7. 1
22. 1. 5	2	42	41. 1. 4	12. 5. 2	2	132	13. 0. 9
21. 3. 3	3	45	38. 4. 0	12. 2. 5	3	138	12. 5. 2
20. 5. 7	7	49	35. 2. 6	12. 0. 0	12	144	12. 0. 0
19. 8. 6	1	52	33. 2. 3	11. 7. 5	1	150	11. 5. 2
19. 2. 0	2	56	30. 8. 6	11. 5. 2	2	156	11. 0. 8
18. 5. 8	3	60	28. 8. 0	11. 2. 9	3	162	10. 6. 7
18. 0. 0	8	64	27. 0. 0	11. 0. 7	13	169	10. 2. 2
17. 4. 5	1	68	25. 4. 1	10. 8. 7	1	175	9. 8. 7
16. 9. 4	2	72	24. 0. 0	10. 6. 7	2	182	9. 4. 9
16. 4. 6	3	76	22. 7. 4	10. 4. 7	3	189	9. 1. 4

Board measure	Inches & quarters.	Squares.	Timber measure.	Board measure	Inches & quarters.	Squares	Timber measure.
10.2.8	14	169	8.8.1	6.8.6	21	441	3.9.2
10.1.0	1	203	8.5.1	6.7.7	1	451	3.8.3
9.9.3	2	210	8.2.3	6.6.9	2	462	3.7.4
9.7.6	3	217	7.9.6	6.6.2	3	473	3.6.5
9.6.0	15	225	7.6.8	6.5.4	22	484	3.5.7
9.4.4	1	232	7.4.4	6.4.7	1	495	3.4.9
9.2.9	2	240	7.2.0	6.4.0	2	506	3.4.1
9.1.4	3	248	6.9.7	6.3.3	3	517	3.3.4
9.0.0	16	256	6.7.5	6.2.6	23	529	3.2.7
8.8.6	1	264	6.5.4	6.1.9	1	540	3.2.0
8.7.3	2	272	6.3.5	6.1.2	2	552	3.1.3
8.6.0	3	280	6.1.6	6.0.6	3	564	3.0.6
8.4.7	17	289	5.9.8	6.0.0	24	576	3.0.0
8.3.5	1	297	5.8.1	5.9.4	1	588	2.9.4
8.2.3	2	306	5.6.4	5.8.8	2	600	3.8.8
8.1.1	3	315	5.4.8	5.8.2	3	612	2.8.2
8.0.0	18	324	5.3.3	5.7.6	25	625	2.7.6
7.8.9	1	333	5.1.9	5.7.0	1	637	2.7.1
7.7.8	2	342	5.0.5	5.6.5	2	650	2.6.5
7.6.8	3	351	4.9.2	5.5.9	3	662	2.6.1
7.5.8	19	361	4.7.9	5.5.4	26	676	2.5.5
7.4.8	1	270	4.6.7	5.4.8	1	689	2.5.1
7.3.9	2	380	4.5.5	5.4.3	2	702	2.4.6
7.2.9	3	390	4.4.3	5.3.8	3	715	2.4.2
7.2.0	20	400	4.3.2	5.3.3	27	729	2.3.7
7.1.1	1	410	4.2.1	5.2.8	1	749	2.3.2
7.0.2	2	420	4.1.1	5.2.3	2	756	2.2.8
6.9.4	3	431	4.0.1	5.1.9	3	767	2.2.5

Board

Board measure	Inches & quarters	Squares	Timber measure	Board measure	Inches & quarters	Squares	Timber measure
5.1.4	28	784	2.2.0	4.3.6	33	1089	1.5.9
5.0.9	1	798	2.1.6	4.3.3	1	1104	1.5.6
5.0.5	2	812	2.1.2	4.3.0	2	1122	1.5.4
5.0.0	3	826	2.0.9	4.2.7	3	1139	1.5.2
4.9.6	29	841	2.0.5	4.2.3	34	1156	1.4.9
4.9.2	1	855	2.0.2	4.2.0	1	1173	1.4.7
4.8.8	2	870	1.9.8	4.1.8	2	1190	1.4.5
4.8.4	3	885	1.9.5	4.1.4	3	1208	1.4.3
4.8.0	30	900	1.9.2	4.1.1	35	1225	1.4.1
4.7.6	1	915	1.8.9	4.0.8	1	1242	1.3.9
4.7.2	2	930	1.8.6	4.0.5	2	1260	1.3.7
4.6.8	3	945	1.8.3	4.0.3	3	1278	1.3.5
4.6.4	31	961	1.7.9	4.0.0	36	1296	1.3.3
4.6.1	1	976	1.7.7	3.9.8	1	1313	1.3.1
4.5.7	2	992	1.7.4	3.9.4	2	1331	1.2.9
4.5.3	3	1008	1.7.1	3.9.1	3	1350	1.2.8
4.5.0	32	1024	1.6.9	3.8.9	37	1369	1.2.6
4.4.6	1	1040	1.6.6	3.8.7	1	1388	1.2.4
4.4.3	2	1056	1.6.4	3.8.4	2	1406	1.2.2
4.4.0	3	1072	1.6.1	3.8.1	3	1425	1.2.1

The use of this former Table.

IF upon a *Scale or Ruler* you divide one *inch* into ten *equall parts or primes*, and againe by *diagonals*, and *parallell-lines*, you ſubdivide each of them into ten *equall parts or ſeconds*, with your *compaſſes*, you may take a more exact running meaſure for *board and timber*, then by any other meanes whatſoever, and ſo

ſo place the ſame, or this *Table* if you will, upon any *Ruler.*

Alſo by meanes of the *columnes of ſquares*, you may readily finde a *ſquare* equall to any *Parallelepipedon*, or piece of *timber*, which is thicker then it is broad. As for example, ſuppoſe a piece of *timber* to be ten *inches thicke*, and 9 *inches broad*: if I multiply thoſe *ſides* one by another, they will produce 290, then ſeeking the *columne of ſquares* for 290, which I finde not, but I finde 289 the neereſt number to 290, to ſtand againſt 17: thererore I ſay 17 *inches ferè*, will make a *ſquare* equall to ſuch an unlike *ſquared piece*, then looking in the *columne of timber meaſure* againſt 17, you ſhall finde that 5 *inches*, 9 *primes*, or $\frac{8}{10}$ and 8 *ſeconds*, or $\frac{28}{100}$ of an *inch in length*, of that piece will make a *foot of timber.*

Likewiſe for *board meaſure*, you may finde how much in *length* or *breadth* of *board* muſt be in one *foot.*

By the like meanes, ſuppoſe for example that a *board*, appointed to be meaſured, is 15 *inches* $\frac{3}{4}$ *broad*, if I deſire to know how much in *length* thereof will make a foot; I ſeeke in the *columns* that ſtand under *unites* and *quarters*, for 15 $\frac{3}{4}$, and alſo againſt the ſame in the *columne* under the title of *board meaſure*, where I finde 9 *inches*, 1 *prime*, or *tenth* of an *inch*, and 4 *ſeconds*, or *hundreds* of an *inch* will make a *foot* at that *breadth*: The like may be practiſed for any other *breadth of board* whatſoever.

Cer-

Certain *Tables* ſhewing the Intereſt of any *ſumme of mony* whatſoever unto 40 years; how much *Annuities* reſpited or forborn commeth unto. And for *buying* or *ſelling* of *Annuities* for the ſaid time; and alſo the ſame in *reverſion* after any *number of years* unto 30. What they may be worth in preſent ready money, by *R. C.* and now diligently corrected and amended by *Robert Hartwell.*

Definition of Intereſt.

PRincipall, *is the ſumme from which the* Intereſt *is reckoned.*

2 Intereſt *is the ſumme reckoned for the lending or forbearance of the* Principall *for any termes or time.*

3 Intereſt ſimple *is that which is counted from the* Principall onely.

4 Intereſt compound *is that which is counted for the* Principall, *together with the* Arrerage.

5 Intereſt profitable *is that which is added to the* Principall.

6 Intereſt Damageable *is that which is to be ſubtracted from the* Principall.

The uſe *per annum* of	1 li.	is	2 s.
	10 s.		12 d.
	5 s.		6 d.
	2 s. 6 d.		3 d.
	1 s.		1, $\frac{1}{5}$ of a peny.

A Table shewing what 1 li. *with interest, and interest upon interest after* 10 *in the* 100 *comes to every yeer under* 41 *yeers. As followeth.*

yeers	l.	s.	d.	li.	s	d.	yeers.
1	1	2	0	7	8	0	21
2	1	4	2	8		9	22
3	1	6	7	8	19	1	23
4	1	9	3	9	16	11	24
5	1	12	2	10	16	8	25
6	1	15	5	11	18	4	26
7	1	18	11	13	2	2	27
8	2	2	10	14	8	5	28
9	2	7	1	15	17	3	29
10	2	11	10	17	8	11	30
11	2	17	1	19	3	10	31
12	3	2	9	21	2	3	32
13	3	9	0	23	4	6	33
14	3	15	11	25	10	11	34
15	4	3	6	28	2	0	35
16	4	11	10	30	18	0	36
17	5	1	1	34	0	0	37
18	5	11	2	37	8	1	38
19	6	2	3	41	2	10	39
20	6	14	6	45	5	2	40

By the former Table, if you desire to know what 1 li. commeth to with *interest*, and *interest* upon *interest* after 10 in the 100, for any *number* of years unto 40. Look in the *row*, or *margent* (over which is written *years*) and against it on the right hand close unto it in the *row or margent of pounds, shillings, and pence* (which is titled thus, li. s. d.) you shall finde your desire.

Example.

I would know what 1 li. *with interest, and interest upon interest commeth to in* 7 *yeares ?*

I looke in the *row* of yeares for the *number* 7. And against it on the right hand I finde 1 li. 18 s. 11 d. Also what it commeth unto in 13 years. I seeke among the years for 13, and against it I finde 3 li. 9 s.

Again, for 21 years. I look for 21 among the years, and I finde 7 li. 8 s. 0 d. But if you would know for a greater summe then 1 li. Then multiply your summe by that summe of 1 li. in the Table for any of those years, and you shall easily finde it. As thus, I would know what 10 li. commeth to for 7 years with interest, &c. I see that 1 li. commeth to 1 li. 18 s. 11 d. in that time. Then say I that 10 li. must bee 10 times as much in that space, which is 19 li. 9 s. 2 d. Also of 10 li. in 13 years, I see that 1 li. in that time commeth unto 3 li 9 s. Then must 10 li. be ten times as much in that space, which is 34 li. 10 s. Also what 10 li. commeth to in 21 years. I finde first that 1 li. in that space commeth to 7 li. 8 s. Then I say 10 must be 10 times as much, which is 74 li. Lastly, I would know what 100 li. commeth to in 7 years, I see it must bee 100 times as much as 1 li. commeth to in that

that ſpace, which is 194 li. 11 s. 8. d Hereby you ſee the common ſaying is not true, that 100 li. doth double it ſelfe in 7 years, for it wants thereof 5 li. 8 s. 4 d. But in 8 years 100 l. commeth to 210 li. 8 s. 4 d. which you ſee is more then double it ſelfe by 10 li. 8 s. 4 d. And in this ſort may any that can but caſt with Counters, or indeed by memory finde the increaſe of any ſumme whatſoever for any of the number of years in the foreſaid Table, after they have found what 1 li. commeth unto for that time, as before is ſpecified.

A Table ſhewing if 1 li. *annuity to endure for any number of years, under* 41, *be all reſpited or forborne, untill the laſt payment grow due, and then all be received together; with intereſt, and intereſt upon intereſt after* 10 *in the* 100 per annum, *what they will amount unto by any of the ſaid number of yeers. As followeth.*

yeers.	li.	s.	d.	li.	s.	d.	yeers.
1	1	0	0	64	0	0	21
2	2	2	0	71	8	0	22
3	3	6	2	79	10	10	23
4	4	10	10	88	9	11	24
5	6	2	1	98	6	11	25
6	7	14	3	109	3	7	26
7	9	9	8	121	1	11	27
8	11	8	8	134	4	2	28
9	13	11	7	148	12	7	29
10	15	18	8	164	9	10	30
11	18	10	7	181	18	10	31
12	21	7	8	201	2	9	32
13	24	10	5	221	5	0	33
14	27	19	5	245	9	6	34
15	31	5	6	271	0	5	35
16	35	18	11	299	2	6	36
17	40	10	10	330	0	9	37
18	45	1	11	364	0	10	38
19	51	3	2	401	8	11	39
20	57	0	6	442	11	10	40

By this Table you may know what any Annuity being respited or forborn for any number of yeers unto 41. with interest upon interest, after 10 in the 100 will come unto: first seeking in the *Table*, what 1 li. will come unto, in that time, and that being found to multiply it by the summe you desire to know.

Example.

Example.

First, I would know what 1 li. *Annuity being forborn or respited for* 14 *yeers commeth unto.*

I look in this last Table (which is for the purpose) and I finde 27 li. 19 s. 5 d.

Again, what 1 li. *Annuity* respited for 21 *yeers* commeth to, I looke in the said Table for 21 *years*, and I find 64 li. Also the like for 1 li. for 30 *yeers* respited. I look, and find it to be 164 li. 9 s. 10 d. as by the said Table may appeare. Now for greater *Annuities*, as 30 li. *per annum*, respited or forborn, what it amounteth to in 16 *yeers*. I seek first for 1 li. in this last *Table* before for 16 *yeers*, and against it I find 35 li. 18 s. 11 d. Then say I, that 30 li. *per annum* being respited for that time, will come to 30 times as much, which is 1078 li. 7 s. 6 d. Also if there be an *Annuity* of 45 li. due and unpayed for 12 *yeers*, I look in the said Table what 1 li. commeth to, 12 *yeers* being respited, and I find it is 21 li. 7 s. 8 d. Then I conclude that 5 li. must be 45 times as much, which is 962 li. 5 s.

Lastly, I have an Annuity of 50 li. per annum, *which hath been behind for* 16 yeers, *and must be answered unto me with interest, and interest upon interest, all at one payment, what shall or ought I to receive in all, at the* 16 yeers *end?*

I seek what 1 li. comes unto in that time (as before taught) and I finde 35 li. 18 s. 11 d. Then must my 50 li. *per annum* forborne for that time, come to 50 times as much, which is 1797 li. 5 s. 10 d. And thus may you find any other *summe* great or small, for any *number of yeers* contained in the aforesaid *Table*, without the helpe of *Arithmeticke*, if

you can but uſe your Counters, or by memory count well.

A Table ſhewing if 1 li. *Annuity (to indure for any number of years unto* 41*) bee to be ſold for preſent ready money, how much ought that ready money to bee, reckoning* 10 per 100 per annum *abating intereſt, and intereſt upon intereſt. As followeth.*

yeers.	li.	s.	d.	li.	s.	d.	yeers.
1	0	18	2	8	12	11	21
2	1	14	8	8	15	5	22
3	2	9	8	8	17	7	23
4	3	3	4	8	19	8	24
5	3	15	9	9	1	6	25
6	4	7	1	9	3	2	26
7	4	18	4	9	4	8	27
8	5	6	8	9	6	1	28
9	5	15	2	9	7	4	29
10	6	2	10	9	8	6	30
11	6	9	9	9	9	7	31
12	6	16	3	9	10	6	32
13	7	2	0	9	11	4	33
14	7	7	4	9	12	2	34
15	7	12	1	9	12	10	35
16	7	16	5	9	13	6	36
17	8	0	5	9	14	1	37
18	8	4	0	9	14	7	38
19	8	7	3	9	15	1	39
20	8	10	3	9	15	6	40

This

This Table before laſt ſpecified is very neceſſary and commodious for all *Gentlemen* or others, that ſhall have cauſe to buy or ſell *Annuities* or ſuch like, for by this they ſhall know what they doe, whether they demand, or take too little or too much, after the rate of ten in the 100, by which proportion all theſe *Tables* are ruled.

As for example, I am to buy an Annuity of 16 li. per annum, *for* 12 *yeers, and am demanded for it ready mony* 120 li. *I would know, if I give this rate, whether I give too much or too little, according to the proportion of ten in the* 100 per annum, &c.

I look in the *Table* laſt before what 1 li. is worth for 12 *yeers*, and I finde againſt 12 this *ſumme* 6 li. 16 s. 3 d. Now I ſay that 16 li. *Annuity* for that time, and after that proportion commeth to 16 times as much, which is 109 li. So that I ſee the party demanded of me 11 li. too much after the rate of ten in the 100 *per annum*, and therefore I muſt draw him to a lower price or leave it.

Again, I am offered an Annuity of 20 li. per annum *of* 14 *yeers for* 130 li. *I would know if I give it whether I give too much or too little, according to the proportion aforeſaid.*

I ſeek firſt what 1 li. *Annuity* is worth for 14 *yeers*, and I find in the ſaid laſt *Table* 7 li. 7 s. 4 d. Then ſay that the *Annuity* of 20 li. *per annum*, will come to 20 times as much, and will be worth 147 li. 6 s. 8 d. according to the proportion before mentioned: and is more then his demand by 17 li. 6 s. 8 d. So that I ſee if I accept of it, I ſhall have a good bargaine. And thus may you know readily by looking in your Table, and finding what 1 li. is worth for

any time therein contained, how much any greater ſumme will come unto, if you multiply it by that ſumme of 1 li. as before is ſufficiently ſhewed.

But ſuppoſe this, I have 300 *li. ready mony, and would beſtow the ſame for a valuable Annuity anſwerable thereunto according to the proportion aforeſaid. I would know what Annuity to endure* 21 *yeers this* 300 li. *will buy?*

I look in the former Table what 1 li. Annuity will coſt for that time, and I find 8 li. 12 s. 11 d. Then I ſay by the *Rule of proportion.* If 8 li. 12. s. 11 d. will buy 1 li. Annuity for 21 yeeres: what Annuity ſhall 300 li. buy or be worth for that time? I reduce the *ſummes* to the leaſt denomination (which is pence) and I find 34 li. 13 s. 11 d. And after this manner (by the help of this *rule*) may you find all other ſummes for any time contained in the foreſaid laſt *Table.*

A Table shewing what 1 li *in reversion for any number of years under* 31 *is worth in ready money, the buyer staying untill the thing be falne in hand.*

yeers.	li.	s.	d.	li.	s.	d.	yeers.
1	0	18	2	0	4	4	16
2	0	16	6	0	3	11	17
3	0	15	0	0	3	7	18
4	0	13	7	0	3	3	19
5	0	12	5	0	2	11	20
6	0	11	3	0	2	8	21
7	0	10	3	0	2	5	22
8	0	9	3	0	2	2	23
9	0	8	5	0	2	0	24
10	0	7	8	0	1	10	25
11	0	7	0	0	1	8	26
12	0	6	4	0	1	6	27
13	0	5	9	0	1	4	28
14	0	5	3	0	1	3	29
15	0	4	9	0	1	1	30

This last *Table* differeth, and is contrary to the other three before mentioned: For whereas the others increased more and more according to the *number of yeers* specified, this doth grow and diminish lesse and lesse, as the *number of yeers* increaseth. As for example.

There

There is a Tenement, the fee simple whereof after 7 yeers will be worth 40 li. *what am I to give for it in ready mony, now staying untill it fall in hand?*

To know this I look in this last Table for 7 years, and against it I find 10s. 3d. So that a thing that after 7 years will be worth 1 li. is worth now in ready mony but 10 s. 3 d. Then say I, that the foresaid Tenement (which after 7 years will be worth 40 li.) is now worth 40 times 10s. 3d. which is 20li. 10s.

Again there is a Farm which after 9 yeers will be worth the Fee-simple 420 li. *what is it now worth in ready mony, staying untill it fall in hand?*

I look in the said Table what 1 li. is worth in reversion after 9 years, and I find 8s. 5d. Then say I, that the *Farm* of 420 li. so long in Reversion, will be now worth in ready mony, 420 times as much, which is 176li. 15s.

Lastly there is a Lordship to be sold, the Feesimple whereof after 14 *yeers will be worth* 7500 li. *I would know what the same is now worth in ready mony for the Reversion.*

I look in this last *Table* for 14 yeers, and against it I find 5s. 3d. so much 1 li. is worth in reversion after 14 yeers. Then say I, that 7500 li. is worth no more in reversion for that time then 7500 times 5s. 3d. which is 1968 li. 15s. And after this manner may you finde out any other summe whatsoever. And though some men of their own experience can ayme (as they think) neer enough the mark to serve their own turns: yet I dare undertake they shall never so exactly doe it, nor justify what they doe, as if they did it by Art.

FINIS.

New *Tables of Interest* at 8 *per centum per annum*, exactly calculated for 30 yeers by *Robert Hartwell*, with necessary *questions* for the use of them.

The first Table expressing the increase of one pound principall, put out and forborn for any number of yeers under 31 *at* 8 per centum, per annum.

yeers	li.	s.	d.	q.	li.	s.	d.	q.	yeers
1	1	1	7	0	3	8	6	0	16
2	1	3	3	3	3	14	0	0	17
3	1	5	2	1	3	19	6	0	18
4	1	7	2	2	4	6	3	3	19
5	1	9	4	2	4	13	2	2	20
6	1	11	8	3	5	0	8	0	21
7	1	14	3	1	5	8	8	3	22
8	1	17	0	0	5	17	0	0	23
9	1	19	11	3	6	6	9	3	24
10	2	3	2	0	6	16	1	2	25
11	2	6	7	2	7	7	11	0	26
12	2	10	4	1	7	19	9	0	27
13	2	14	4	2	8	12	6	2	28
14	2	18	8	3	9	6	4	0	29
15	3	3	5	1	10	1	3	0	30

The

The deſcription and uſe of the Tables *of Intereſt at* 8 per annum, *being profitable.*

The firſt of them.

THeſe Tables conſiſt of foure *Columns*, in the firſt and fourth, whereof is written over the head, yeeres, and under the firſt number of yeeres deſcending from 1 to 15, likewiſe in the fourth the number of yeeres deſcending from 16 to 30. And againſt every yeere in the ſecond Columne, toward the right hand the pounds, ſhillings, pence and farthings, which one pound, or 20s. principall will amount unto, being put forth and forborn for the number of yeeres ſet againſt it ; (but the pounds, ſhillings, pence, &c. in the third Columne belong to the yeeres ſet in the laſt Columne.)

1. Example.

Let it be required what one pound or 20 ſhillings, being put forth and forborn for 12 yeers, ariſeth to at 8 per centum, per annum, *intereſt upon intereſt.*

Seek in the firſt Columne under the title of yeeres, for 12 the number of yeers propoſed in the queſtion, & right againſt it toward the right hand in the ſecond Columne, you ſhall find 2li——10s——4d——1q. which is the principall and increaſe thereof due for the time required.

2 Example.

If 100 li. *be put forth for 17 yeeres according to the ſame intereſt, I demand what it will amount to in that time?*

Look in the Column under the title of yeers for 17, and right againſt it towards the left hand in the Table is found 3li——14s——0d——0q. which is the

increaſe

increaſe of 1li. by which you may thus gather the increaſe of 100li. or any other *ſumme*; a hundred times 3li. is 300 li. then 100 times 14 *ſhillings* is 70. li. both which added together doe make 370li——os——od which is the *increaſe* of 100li. put forth and forborn 17. *yeeres*, the ſolution to the queſtion.

li	s	d.	q
300	o	o	o
70	o	o	o
370	o	o	o

3 Example.

Suppoſe 60li. *be put forth for* 19 *yeeres according to that rate, what will it increaſe to in that time?*

Seek 19 under the *title* of yeeres, and againſt it toward the left hand is found 4li——6s——3d——3q. now ſay 60 times 4li. is 240, and 60 times 6 *ſhillings* is 360 *ſhillings*, or 18li. and 60 times 3d. is 180 d. or 15 *ſhillings*, & 60 times 3 *farthings* is 3 *ſhillings* 9d. all which added together make 258li. 18 s. 9d. the increaſe thereof demanded.

li	s	d	q
240	o	o	o
18	o	o	o
o	15	o	o
	3	9	o
258	18	9	o

The

The ſecond Table *ſhewing what one pound annuity or yeerly rent is worth at the end of any number of Yeers under* 31, *being forborn, at* 8 per centum per annum.

yeers.	li.	s.	d.	q.	li.	s.	d.	q.	yeers.
1	1	0	0	0	30	6	5	3	16
2	2	1	7	0	33	15	0	0	17
3	3	4	6	0	37	9	0	0	18
4	4	10	1	1	41	8	11	0	19
5	5	12	2	3	45	15	2	3	20
6	7	6	8	0	50	8	5	2	21
7	8	18	5	1	55	9	1	2	22
8	10	12	8	3	60	17	10	1	23
9	12	9	9	0	66	15	3	2	24
10	14	9	8	3	73	2	1	1	25
11	16	12	10	3	79	19	1	0	26
12	18	19	0	2	87	7	0	0	27
13	21	9	10	3	95	6	1	2	28
14	24	4	3	2	103	19	3	3	29
15	27	3	0	2	113	5	7	3	30

The uſe of the ſecond Table, *whoſe diſpoſition is altogether like the former according to the title thereof, being profitable.*

1 Example

1 Example.

There is a Lease worth 28. li. per annum, *to endure* 14 *years, I demand what it will rise unto at the end of those yeares being all forborne with the interest upon interest at the rate prescribed in this* Table.

Look in the third Table for 14 yeares, against which toward the right hand, you shall find 24li——4s——3d——2q. Now Multiply 28 li. by 24, there ariseth 672li. then 28li. by 4 s. yeeldeth 112s. or 5 li. 12 s.

Again 28li. by 3d. produceth 84d. or 7s. finally, 28. by 2 farthings yeeldeth 56 farthings or 1s. 2d. Al which added together make 678li. 0s. 2d. to be received at the end of 14 yeers, the same rent or annuity being respited.

li.	s	d	q
672	0	0	0
5	12	0	0
	7	0	0
	1	2	0
678	0	2	0

2 Example.

If 60 li. *yeerly rent or annuity be forborn* 20 *years: I demand how much it will increase at the end of the said term?*

In the Table I find that 1 pound in 20 yeers will arise to 45li——15s——2d——3q. therefore 60li. in the like terme wil yeeld 60 times as much; which I will reckon thus: 60 times 45 li. is 2700 li, 60 times 15 s. is 900 s. or 45l. 60 times

li	s	d	q
2700	0	0	0
45	0	0	0
	10	0	0
	3	9	0
2745	13	9	0

times 2 d. is 120 d. or 10 s. laſt of all, 60 times 3 q. is 180 farthings, or 3 s.——9 d. all which together amount unto 2745 li——13 s——9 d. the value thereof to be received at the end of the term.

3 Example.

The yeerly rent of 6 li.——13 s——4.*being behind and unpaid the ſpace of 7 yeeres at the end of which term the Tenant is compelled to pay the ſame with the intereſt thereof according to the above named rate ; I demand what the payment ought to be.*

The increaſe of 1 li. *yeerly rent* anſwering to 7 *yeeres*, is 8 li. 18 s. 5 d. 1 d which for 6 li. *rent* taken 6 times, ariſeth to 53 li. 10 s. 7 d. 2 q. now becauſe 13 s. 4 d. is two third parts of 1 li. therefore I take $\frac{2}{3}$ of 8 li. 18 s. 5 d. 1 q. which is the increaſe of 1 l. forborn for 7 *yeers*, that is 5 li. 18. 11 d. 1 q. which together make 59 li. 9 s. 7 d. 0 q. the *ſumme* to be received, as was required.

li.	s.	d.	q.
53	10	7	2
5	18	11	2
59	9	7	0

The

The third Table declaring what one pound due at the end of any number of yeers under 31 *is worth ready mony at* 8 per centum, per annum.

yeers.	li.	s.	d.	q.	li.	s.	d.	q.	yeers.
1	0	18	7	0	0	5	10	0	16
2	0	17	10	3	0	5	4	3	17
3	0	15	1	2	0	5	0	0	18
4	0	14	8	1	0	4	7	2	19
5	0	13	7	1	0	4	3	1	20
6	0	12	7	0	0	3	11	2	21
7	0	11	8	1	0	3	8	0	22
8	0	10	9	2	0	3	4	3	23
9	0	10	0	0	0	3	1	3	24
10	0	9	3	0	0	2	11	0	25
11	0	8	6	3	0	2	8	1	26
12	0	7	11	1	0	2	5	0	27
13	0	7	4	0	0	2	2	3	28
14	0	6	9	2	0	2	1	3	29
15	0	6	3	3	0	1	11	3	30

The third Table *is diſpoſed as the firſt, the uſe according to the Title thereof, being damageable.*

1 Example.

Suppoſe there is 750 li. *due to be payed at the end of 9 yeeres, the Creditor would ſell this debt for preſent money, what ought that mony to be at the rate deſcribed in the* Table?

Seek in this third Table for 9 yeers at the left ſide of the Table, and right againſt it toward the right hand, you ſhall find 10 ſhillings, which multiplyed or taken 750 times, yeeldeth 7500 ſhillings, which is 375 li. the value of that debt in preſent mony.

2 Example.

There is a Leaſe worth 500 li. *after the end of 7 yeers; what is it worth preſent mony, according to the rate deſcribed in the Table ſtaying till it fall?*

I ſeek in the Table for the 7 yeers, and right againſt it I find 11 s—8 d; now I multiply 500 by 11, it yeeldeth 5500 ſhillings, or 275 li. then 500 times 8 d. maketh 4000 d, which is 16 li. 13 s. 4 d. which added together is 291 li. 13 s. 4 d. the value of the Leaſe to be paid before it fall in hand.

li	ſ	d	q
275	0	0	0
16	13	4	0
291	13	4	0

The

The fourth Table *expressing what one pound yeerly rent or annuity for any number of yeeres not exceeding* 30 *is worth ready mony at* 8 per centum per annum.

yeers	li.	s.	d.	q.	li.	s.	d.	q.	yeers
1	0	18	6	0	8	17	0	1	16
2	1	15	7	3	9	2	5	0	17
3	2	11	6	2	9	7	5	1	18
4	3	6	2	3	9	12	0	3	19
5	3	19	10	1	9	16	4	1	20
6	4	12	5	1	10	0	4	0	21
7	5	4	1	2	10	4	0	0	22
8	5	14	11	0	10	7	5	0	23
9	6	4	11	1	10	10	6	3	24
10	6	14	2	1	10	13	5	3	25
11	7	2	9	1	10	16	2	1	26
12	7	10	8	2	10	18	8	1	27
13	7	18	0	3	11	1	0	0	28
14	8	4	10	2	11	3	2	0	29
15	8	11	2	1	11	5	1	3	30

The fourth Table is disposed altogether as the former, and the use thereof in like sort being damageable.

1 Example.

There is an annuity or rent of 20s: per annum *to endure* 25 *yeers, it is required what it is worth ready mony?*

Look in the Table for 25 *yeers*, and right against it you shall finde 10 li. 13 s-5 d. 3 q. which is the solution.

2 Example.

What is the Lease of certaine Land valued at 140 li. per annum, *to begin presently and endure* 18 *yeers, worth ready mony?*

Search in the *Table* for 18 *yeeres*, the *term* named in the *question*, and right against it toward the left hand you shall find 9 li-7 s 5 d-1 q. which expresseth that *one pound rent* to be bought for that *terme* is worth so much; therefore that summe 140 times is the value required. Now 140 times 9 li. is 1260, and 140 times 7s. is 980 s. or 49 li; likewise 140 times 5 d. is 700 d. or 2 li-18 4d. and 140 *farthings* is 2 s-11 d. all which added together make 1312 li--1 s--3 d. for the value of the said *Lease* paying no *rent*.

li.	s.	d.
1260	0	0
49	0	0
2	18	4
	2	11
1312	1	3

3 Example

3 Example.

A Lease taken for 21 *yeers at* 13 li. 6 s. 8 d. per annum, *which after* 5 *yeers expired, the Tenant is desirous to give a fine, and bring the rent down to* 8 li. per annum, *for the rest of the term, the demand is what fine is to be payed?*

Subtract 5 *yeers* from 21, the remain 16, is the time unexpired; likewise from the present *rent* abate 8 li. the rest will be 5 li-6 s-8 d. now the drift of the *question* is, what 5 li-6 s-8 d. *yeerly rent* or *annuity* to indure 16 yeeres is worth present money.

The value of 1 li. *rent* or *annuity* answering to 16 *yeers* is, 8 li-17 s-0 d. 1 q. Now 5 times 8 li. is 40 li. and 5 times 17 s. 4 li 5 s. and 5 times one farthing, is 1 d-1 q. and because 6 s-8 d. is $\frac{1}{3}$ of 1 li. I take $\frac{1}{3}$ of 8 li-17 s-0 d. 1 q. which is 2 li-19 s. 0 d. all which added together, make 47 li-4 s-0 d-1 q. which is the *fine* that ought to be paid to bring the *rent* to 8 l. *per annum*.

li.	s.	d.	q.
40	0	0	0
4	5	0	0
2	19	0	1
47	4	0	1

The fifth Table declaring what yeerly rent or annuity of one pound ready mony will purchase for any number of yeers under 31 *at* 8 per centum per annum.

yeers.	li.	s.	d.	q.	li.	s.	d.	q.	yeers.
1	0	18	7	0	0	2	9	3	16
2	0	14	0	2	0	2	8	3	17
3	0	9	8	2	0	2	8	0	18
4	0	7	6	0	0	2	7	0	19
5	0	6	3	0	0	2	6	2	20
6	0	5	4	3	0	2	5	3	21
7	0	4	9	2	0	2	5	1	22
8	0	4	4	0	0	2	4	3	23
9	0	4	0	0	0	2	4	1	24
10	0	3	8	2	0	2	4	0	25
11	0	3	6	0	0	2	0	3	26
12	0	3	3	3	0	2	3	1	27
13	0	3	1	3	0	2	3	0	28
14	0	3	0	1	0	2	2	3	29
15	0	2	11	0	0	2	2	2	30

In

In the fifth Table *the* Numbers *and* Columns *are all diſpoſed as the former* Tables, *and needeth no further explanation but only Examples.*

1 Example.

The Table *declareth at firſt ſight what yearly rent or annuity one Pound ready mony will purchaſe for any terme in the* Table *expreſſed.*

But if the *ready mony* be above one pound, then if any *value* or *rent* ſet down in this Table, be multiplyed by the *number* belonging to the *yeers in queſtion*, the *product* will ſhew what *yeerly rent* or *annuity* that *ready mony* will purchaſe for the *time propoſed.*

2 Example.

A certain man hath 750 li. *to purchaſe an Annuity to endure* 27 *yeers, ſo as it may yeeld him the like profit, as if it were put out according to the rate in the* Table *expreſſed, it is required what that annuity ought to be?*

Becauſe the *annuity* is to endure 27 yeeres, ſeek out the value or rent ſet againſt 27 yeers, in this fifth Table, which is 2s——3d——1q. now this being the *Annuity* with 20s. ready *mony* will purchaſe for that *terme*, it muſt be multiplyed by 750 li. as followeth, becauſe 2s. is the tenth part of 20s. therefore take the tenth part of 750 li. which is 75 li.

li	s	d	q
75	0	0	0
9	7	6	0
	15	7	2
85	3	1	2

which ſet firſt down, then 750 times 3d, is 91, 7s, 6d. which ſet under the former, laſt of all 750 *farthings* is 15s, 7d, 2q. All which added together, produce 85 li. 3s. 1d. 2q. the *yeerly Annuity* required.

Deo ſoli laus, omnis honor & gloria tribuatur.

AMEN.

FINIS.

Compendious Tables *of Interest money forborn any number of Dayes Weeks, Moneths, or Years under 22; exactly calculated at* 6 l. per Cent. per Annum.

Days.	Numbers.	Yeares.	Numbers.
I	100016.	I	106000.
II	100032.	II	112360.
III	100048.	III	119102.
IV	100064.	IIII	126248.
V	100080.	V	133822.
VI	100096.	VI	141852.
Weekes.		VII	150363.
I	100112.	VIII	159385.
II	100224.	IX	168948.
III	100336.	X	179085.
		XI	189830.
Moneths.		XII	201220.
I	100487.	XIII	213293.
II	100976.	XIV	226090.
III	101467.	XV	239656.
IIII	101961.	XVI	254035.
V	102458.	XVII	269277.
VI	102956.	XVIII	285434.
VII	103457.	XIX	302560.
VIII	103961.	XX	320714.
IX	104467.	XXI	339956.
X	104976.		
XI	105486.		

The

The use of this Table *of proportionall numbers; the* Radius 100000.

This Table is divided into 4 columnes, and the first in 3 parts ascending from 1 day unto 6. secondly from 1 weeke to three, thirdly from one moneth proceeding to 11. the third columne comprehends the yeares inclusive from 1 to 21 as by their numerall letters doe appear: the second and fourth columnes are proportionall numbers, in arithmeticall characters, respectively answering the time or times compounded of Days, Weeks, Moneths and Yeers to 21, as by examples shall be evidenc'd.

An explanation.

What will 50 L. amount unto, if forborne 21 years, the Interest allow'd at 6 L. *per Cent. per An.*

Look in the columne of years for the terme of forbearance propounded, in this 21, whose *Decimall* is 339956, which multiplied by the principall forborn *viz.* 50 L. the product will be 169|97800, to be divided by the *Radius* 100000, therefore cut off 5 places (as in the margent) from the right hand, and there will appeare 169 L. the remainer 97800 multiply by 20 s. or by 2, and annex a cipher at the right hand, the product is 19 56000, cutt off 5 places as before; on the left hand you will find 19 S. the remainder 56000, which encreas'd by 12 D. produceth

		33956
		50
L	169	97800
S	-19	56000
		112000
D	-6	72000
Q	-2	88 | 000
		100

ceth 6 72000, strike off 5 places, you will discover the 6. remainder 72000 multiplied by 4 Q. and worke as before, you shall find 2 $\frac{22}{25}$ Q. so 50 L. forborn 21 yeares at 6 L. *per Cent. per An.* will amount unto the summe of 169 L. 19 S. 6 D 2 $\frac{22}{25}$ Q. the demand solv'd.

If a question consists of severall denominations *viz.* Pounds, Shillings, Pence, see the rules of Practice in the propositions of Interest; if mixt, as in respect of time, *viz.* Moneths, Weeks, Days &c. find first the encrease for the terme of years, and multiplie all that by the proportionall number found in the former Table, for the parts of a yeare the result will answer gripple expectation; for Decimall tables, (or what these Authors have not treated off) I referre the Reader unto my bookes of *Naturall* and *Artificiall* Arithmetick, or my Scales of commerce and Trade, this place being not convenient to enlarge my selfe, or build upon anothers ground neither will I lessen their workes (by me now corrected) whereby to magnifie my owne, but have inserted this Table, and here I subscribe my name, hoping my labours may give you ease I rest

Your friend although unknown

Thomas Willsford.

To every young Arithmetician, or practicioner in Numbers, who ſhall peruſe theſe Bookes.

Candid Lector,

You will here receive an old Arithmeticke *from the authoritie of* Record, *entaild upon the People, ratified and ſign'd by the approbation of Time, with multitudes of ſurviving witneſſes; ſo expect not from me to confer Encomiums, when I had written nothing here, but that through miſtakes and overſights of former Correctors, Errors have appear'd like infirmities incident to decrepit Age, involv'd within the ſheets, as if prepar'd for a funerall, the Authors Senſes departed, or in a trance confuſed and ambiguous, their names remaining like inſcriptions on a Tombe, where corruption of one have demonſtrated generations of others.*

Some places I found obliterated, other parts diſlocated, or falſe numbers have uſurp'd their

their roomes, and those established by sundrie impressions, may animate some (ignorantly) to plead Prescription for them; although multiplied into a numerous and adulterous offspring, which deterred many young beginners from their progresse in these Numbers, and invoked me to their assistance in the restitution of the Authors, notwithstanding I have published some bookes of this subiect alreadie, differing in forme, the scope I aimed at being both Speculation and Practice, my intentions dedicating that and this to the Publicke good, whereby purblinde Suspicion, or fond affection (the Parents of Partialitie) are expulsed and vanished, and I elected as an impartiall Corrector of this Treatise where many of the Tables *and* rules *of direction were direct divided from the first Composers meaning, the grounds of Truth, or wayes of Art; some of which deviations I have rectified, subtracted others, and totally cancelled many; adding numbers in their places accommodated and reduced to the Authors sence, the Questions stated, and the pristin Copies; usher'd from the Presse again by* Mr. Mellys *attended by* Hartwell, *and all these expose to publicke view, drest (without disguise) in their old attires; otherwise it would seeme absurd, as to see grave Antiquitie vested in* French *habite.*

Humanum

Humanum eſt errare; *ſo I will not promiſe that all the old errors are corrected (although above* 1000) *nor yet engage the Preß ſhall commence no new, but ſo faithfully as I could, theſe Authors recovered are here preſented to your view; which is all that was required of me, and from you (Courteous* Reader) *a friendly acceptance (in recompence of my labours) deſiring to be numberd amongſt the coadjutors of my Country-men; in teſtimonie whereof I here ſubſcribe my name*

Thomas Willsford,

FINIS.

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